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Effect of Regularization Parameter and Scan Time on Crossing Fibers with Constrained Compressed Sensing

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Abstract

Diffusion tensor imaging (DTI) is an MR imaging technique that uses a set of diffusion weighted measurements in order to determine the water diffusion tensor at each voxel. In DTI, a single dominant fiber orientation is calculated at each measured voxel, even if multiple populations of fibers are present within this voxel. A new approach called Crossing Fiber Angular Resolution of Intra-voxel structure (CFARI) for processing diffusion weighted magnetic resonance data has been recently introduced. Based on compressed sensing, CFARI is able to resolve intra-voxel structure from limited number of measurements, but its performance as a function of the scan and algorithm parameters is poorly understood at present. This paper describes simulation experiments to help understand CFARI performance tradeoffs as a function of the data signal-to-noise ratio and the algorithm regularization parameter. In the compressed sensing criterion, the choice of the regularization parameter β is critical. If β is too small, then the solution is the conventional least squares solution, while if β is too large then the solution is identically zero. The correct selection of β turns out to be data dependent, which means that it is also spatially varying. In this paper, simulations using two random tensors with different diffusivities having the same fractional anisotropy but with different principle eigenvalues are carried out. Results reveal that for a fixed scan time, acquisition of repeated measurements can improve CFARI performance and that a spatially variable, data adaptive regularization parameter is beneficial in stabilizing results.

Keywords

Compressed sensing; Diffusion tensor Imaging

I. INTRODUCTION

Traditional DTI faces a big challenge as it fails to detect the presence of multiple fibers with different orientations within a voxel. Using minimum least squared error criteria with pseudo inverse techniques, the eigenvalues and the eigenvectors of the diffusion tensor may be calculated. In case of multiple fiber orientations, an average fiber direction is calculated [1-5].

A new processing method called Crossing Fiber Angular Resolution of Intra-voxel (CFARI) has been recently introduced to resolve fiber structures in diffusion weighted Magnetic Resonance Imaging (DW-MRI) [6-8]. By using a compressed sensing framework, CFARI is

able to resolution crossing fibers in conventional DW-MRI acquisition scenarios that are typically used to compute diffusion tensors. To date, however, the effects of the underlying acquisition parameters as well as the parameters of the algorithm itself have been explored only in a limited fashion. In this work, a more complete model for the underlying diffusion process is used and the effects of multiple repetitions in data acquisition and regularization coefficient in the algorithm are studied. The performance of CFARI in resolving the correct angles and fractional contributions of fibers within the intra-voxel structure is determined via simulation

In the compressed sensing criterion, a spatially adaptive regularization parameter is used; this is essential to evaluate performance at different experimental conditions. To start with, the choice of the regularization parameter β is critical. If it is chosen to be too small, then the problem solution will yield a conventional least squares solution; while if it is chosen to be too large, the optimal solution might be zero [9, 10]. This is why adaptive regularization parameter is used; which means that β will be dependent on the data within each voxel. Following the same logic as DTI, the b-value resulting in minimal error for each single tensor is found. Then, the use of more repetitions is compared to the use of multiple b-values. The angular error becomes smaller as more repetitions are used as expected.

II. THEORY

In the multi-compartment diffusion model, the Stejskal-Tanner (ST) tensor formulation can be extended to model each voxel as a finite mixture of discrete, independent compartments [11] :

$$S_{NF} = S_0 \sum_{i=1}^N f_i e^{-b g_k^T D_i g_k} \quad (1)$$

where S_{NF} is the noise free signal along the k^{th} diffusion weighting direction g_k , S_0 is the noise-free reference signal in the absence of diffusion weighting, N is the number of possible compartments or tensors within each voxel, f_i is the unknown mixture component for each compartment, b is the diffusion weighting parameter, and D_i is the tensor associated with the i^{th} compartment. However, MR signals suffer from Rician noise [12], and therefore the observed signal S_k is related to the noise free signal as:

$$S_k = \sqrt{(S_{NF} + \eta)^2 + (\eta)^2} \quad (2)$$

where η is a zero-mean Gaussian random variable. When the observed signal has an SNR greater than 5:1, it can be modeled more simply as a true signal corrupted by additive Gaussian noise. In this case, the collection of observed signals can be modeled as:

$$y_{K \times 1} = S_{K \times N} f_{N \times 1} + \eta_{K \times 1}^* \quad (3)$$

where $y_{K \times 1}$ is the vector of measured signal, $S_{K \times N}$ is the sensing matrix, $f_{K \times 1}$ is the vector of unknown mixture fractions, K is the number of diffusion directions, η^* is a scaled noise

term. In order to estimate the N unknown mixture fractions, the CFARI compressed sensing method with L1-regularized least squares is used as follows [9],

$$f = \arg \min_{f: f_i \in [0, \infty)} \| Sf - y \|_{l_2}^2 + \beta \| f \|_{l_1} \quad (4)$$

where β is the regularization parameter. By solving (4), CFARI finds a parsimonious collection of mixture fractions whose largest elements define the dominant directions (from among a prespecified reconstruction basis) that best represent the acquired data y (Figure 1).

III. REGULARIZATION PARAMETER SELECTION

Previously, the choice of β in CFARI has been carried out in an ad hoc manner to produce a satisfactory result. It was set to be constant over the whole image domain and was the same for all data acquisition scenarios. But its proper selection is actually quite critical. In fact, it has been shown that β might be best chosen in a data adaptive manner [9 10]. Following [9], we define:

$$\beta_{\max} = \| 2S^T y \|_{\infty} \quad (5)$$

where $\beta_{\max}$ defines the maximum possible value of β . If $\beta > \beta_{\max}$, the optimal solution might become $f=0$. However, when $\beta < 10^{-4} \beta_{\max}$, this will yield a conventional least squares solution [5]; this is why the choice of β is critical. There are many experimental parameters that affect the value of $\beta_{\max}$ through both the sensing matrix that is implied and the resulting “size” of the observations y . In this work, we explore four critical values: b -value, fractional anisotropy, the number of data repetitions, and the SNR.

We evaluated this maximum regularization parameter $\beta_{\max}$ for a scenario in which three parameters were fixed to their nominal value while one parameter was varied. The nominal values were $b = 1000 \text{ s/mm}^2$, $FA = 0.71$, number of repetitions = 1, and $SNR = 25:1$. From the plots in Figure 3, we can conclude that β should depend on the experimental parameters and the resultant data itself. For the experiments described in this paper, the regularization parameter β was picked to be $0.1\beta_{\max}$ empirically, where $\beta_{\max}$ was evaluated for each voxel independently. Thus, CFARI is herein a spatially varying, data adaptive algorithm.

IV. SIMULATION DATA

In all simulations, the reconstruction basis functions (D_i 's) are cylindrically symmetric of equal diffusivities ($FA=0.71$, and $\lambda_1=1 \times 10^{-3} \text{ mm}^2/\text{s}$), and regularly distributed on a sphere (5th order tessellated dodecahedron, i.e., 241 symmetric orientations) [7]. The error criterion used to evaluate all simulations is the mean angular error described as [7]

$$E = \frac{\sum_{i=1}^5 f_i \min_{j \in \text{model}} \angle(\nu_i, \nu_j)}{\sum_{i=1}^5 f_i} \quad (6)$$

where v_i is each vector of the basis set (estimated model), and v_j is each vector in the truth model. In this work, the largest 5 mixture components are used for error calculations. The following describes the various simulations used in this work.

Optimal b-value for each diffusion simulations

One random tensor was simulated at two different diffusions. First, the tensor has $FA=0.71$, and $\lambda_1=2\times 10^{-3}$ mm²/s. Second, the tensor has $FA=0.71$, and $\lambda_1=1\times 10^{-3}$ mm²/s. 1000 Monte Carlo iterations at $SNR=25:1$ were used with 30 diffusion weighting directions.

More repetitions simulations

Two different crossing tensors (the first one with $FA=0.71$, and $\lambda_1=2\times 10^{-3}$ mm²/s, and the second one with $FA=0.71$, and $\lambda_1=1\times 10^{-3}$ mm²/s) were randomly generated for 1000 Monte Carlo iterations at $SNR=25:1$. The simulations were done using one repetition, two repetitions or three repetitions of 30 diffusion weighting directions.

Multiple b-values simulations

Two different crossing tensors (the first one with $FA=0.71$, and $\lambda_1=2\times 10^{-3}$ mm²/s, and the second one with $FA=0.71$, and $\lambda_1=1\times 10^{-3}$ mm²/s) were randomly generated for 1000 Monte Carlo iterations at $SNR=25:1$. The simulations were done using one repetition of the 30 diffusion weighting directions.

V. RESULTS

Optimal b-value for each diffusion

From Figure 4, we can conclude that the optimal b-value for the first tensor ($FA=0.7$, and $\lambda_1=1\times 10^{-3}$ mm²/s) is 1600 s/mm², and for the second tensor ($FA=0.7$, and $\lambda_1=2\times 10^{-3}$ mm²/s) is 1000 s/mm². We can also see that there is generally only a small variation in the error when $b > 800$ s/mm² in both cases.

More repetitions results

From Figure 5, we can conclude that the error decreases as we use more repetitions for the 30 gradient directions. This is entirely consistent with the improved SNR that is achieved with more repetitions.

Multiple b-values results

According to Table 1, the last row represents the minimum error which is smaller than the error when a single b-value=1000 or 1600 s/mm² is used (see Figure 3 for details). However, the use of multiple b-values=600 and 1000 s/mm² didn't result in any error improvement compared to the error when a single b-value=600 or 1000 s/mm² is used.

VI. CONCLUSION

CFARI shows a great promise in helping to resolve crossing fibers from limited measurements. In the compressed sensing criterion, the choice of β is critical. This is why adaptive regularization parameter is used; in which β will be dependent on the data within

each voxel. In this paper, a more realistic model is developed; two random crossing tensors with different diffusivities are simulated. Similar to DTI logic, the b-value resulting in minimal error for each single tensor is found. Like traditional DTI, CFARI performance is expected to improve by increasing the experimental acquisition time. This can be achieved by using more b-values or by using more repetitions. In general, the more repetitions, the better the CFARI performance. Nevertheless, for the same experimental time (scan time), multiple b-values can be better than having more repetitions for limited conditions. It is important to note that the choice of the values of the diffusion weighting parameter used for multiple b-values is critical and needs more investigations, since the use of multiple b-values is not generally superior over multiple repetitions.

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Figure 1.

Comparison between Traditional DTI and CFARI at SNR of 25:1 and b-value of 1000 s/mm². (a) Traditional DTI, where fibers having $FA \leq 0.25$ are considered to be isotropic component (shown in black). (b) CFARI can resolve multiple crossing fibers.

Figure 2.

A diagram of the CFARI method using spatially adaptive regularization parameter. A set of traditional DW images is used. The regularization parameter β is calculated at each voxel using the empirical relation $\beta = 0.1 \times \beta_{max}$. The reconstruction model is characterized by defining the eigenvalues of the tensor model, as well as the tessellated shape and the tessellated model order. Performing compressed sensing at each voxel, mixture fractions can be estimated.

Figure 3.

The maximum value of regularization parameter β_{max} depends on many parameters. (a) β_{max} decreases as the b-value increases. (b) β_{max} increases with the fractional anisotropy of the fiber. (c) β_{max} increases with the number of repetitions of the 30 gradient directions used (as expected). (d) For high SNR (>25:1), β_{max} is almost constant.

Figure 4.
Averaged angular error versus b-value.

Figure 5.
Averaged angular error versus b-value at different repetitions.

Table 1

Averaged angular error versus multiple b-values

B-values used	Error (degrees)
b-value=(600, 1000)s/mm ² (30 + 30 directions)	12.0367 ± 5.3654
b-value=(1000, 1600)s/mm ² (30 + 30 directions)	9.5955 ± 3.9699
b-value=(1000, 1600)s/mm ² (30 + 2rep × 30 directions)	8.5832 ± 3.3296