Published in final edited form as:

Phys Med Biol. 2011 April 7; 56(7): 2245–2264. doi:10.1088/0031-9155/56/7/021.

Lamb Wave Dispersion Ultrasound Vibrometry (LDUV) Method for Quantifying Mechanical Properties of Viscoelastic Solids

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Abstract

Diastolic dysfunction is the inability of the left ventricle to supply sufficient stroke volumes under normal physiological conditions and is often accompanied by stiffening of the left-ventricular myocardium. A noninvasive technique capable of quantifying viscoelasticity of the myocardium would be beneficial in clinical settings.

Our group has been investigating the use of Shearwave Dispersion Ultrasound Vibrometry (SDUV), a noninvasive ultrasound based method for quantifying viscoelasticity of soft tissues. The primary motive of this study is the design and testing of viscoelastic materials suitable for validation of the Lamb wave Dispersion Ultrasound Vibrometry (LDUV), an SDUV-based technique for measuring viscoelasticity of tissues with plate-like geometry. We report the results of quantifying viscoelasticity of urethane rubber and gelatin samples using LDUV and an embedded sphere method. The LDUV method was used to excite antisymmetric Lamb waves and measure the dispersion in urethane rubber and gelatin plates. An antisymmetric Lamb wave model was fitted to the wave speed dispersion data to estimate elasticity and viscosity of the materials. A finite element model of a viscoelastic plate submerged in water was used to study the appropriateness of the Lamb wave dispersion equations. An embedded sphere method was used as an independent measurement of the viscoelasticity of the urethane rubber and gelatin. The FEM dispersion data were in excellent agreement with the theoretical predictions. Viscoelasticity of the urethane rubber and gelatin obtained using the LDUV and embedded sphere methods agreed within one standard deviation. LDUV studies on excised porcine myocardium sample were performed to investigate the feasibility of the approach in preparation for open-chest *in vivo* studies. The results suggest that the LDUV technique can be used to quantify mechanical properties of soft tissues with a plate-like geometry.

Introduction

According to the American Heart Association, the estimated economic burden of treating heart failure in the United States in 2010 is close to \$40 billion [1]. Between 30 and 50% of patients diagnosed with heart failure present with preserved systolic function, suggesting that close to half of the heart failures in the United States are caused by diastolic dysfunction [2–8]. Diastolic dysfunction can be caused by impaired ventricular relaxation, reduced ventricular compliance and pericardial constriction. Decreased left-ventricular (LV) compliance and relaxation can compromise the ability of the LV to fill passively under physiological conditions and supply sufficient stroke volumes into the system circulation. If left untreated, insufficient stroke volumes and increased filling pressures can cause heart failure. A noninvasive technique capable of quantifying material properties of the myocardium would improve clinicians' ability to evaluate diastolic function and would be highly beneficial in clinical settings.

Alterations in tissue tactile stiffness due to pathophysiological changes have been known for centuries and have been an important part of medical examination. In recent decades, significant effort has been directed towards producing a non-invasive technique for

quantifying material properties of soft tissues [9, 10]. These techniques are mostly based on either magnetic resonance [11–13] or ultrasound imaging [14–23], and some of them have been extended to use in myocardial stiffness measurements [12, 13, 17–20, 23]. Since the ultrasound methods typically use an ultrasound transducer at the surface of the body close to the tissue of interest, they are limited by the acoustic window and distance from the organ. While magnetic resonance methods generally do not suffer from this limitation, they are more expensive, subject to high operational costs, often uncomfortable for patients and less likely to be widely available.

The growing field of ultrasound elastography offers diverse approaches to quantifying soft tissue elasticity [9]. Several of these approaches are based on measuring shear wave propagation velocity, generated by either physiological motion [17–20] or external radiation force [13–15, 21–23]. The advantage of shear wave velocity is that it is directly related to the elastic modulus, and the shear moduli of elasticity of body tissues such as muscle, liver, dermis, cartilage and bone differ by several orders of magnitude [15].

Recent liver fibrosis studies [24, 25] demonstrate that viscosity serves as an important indicator of tissue pathology and should be considered in addition to tissue elasticity. Ignoring tissue viscosity gives incomplete information about material properties of the tissue, and inherently underestimates the elasticity. A study by Schmeling *et al.* [26] reports significant changes in myocardial viscosity resulting from myocardial ischemia. The study reports an increase in myocardial viscosity due to ischemia caused by coronary occlusion and return to control values following reperfusion.

Recently, our group has proposed the use of Shearwave Dispersion Ultrasound Vibrometry (SDUV) to quantify material properties of soft tissues. SDUV is a noninvasive method that uses focused ultrasound radiation force to excite harmonic or transient shear waves in the tissue of interest and a pulse-echo transducer to detect the motion. SDUV is based on measuring shear wave velocity over several frequencies (shear wave dispersion) and fitting the shear wave dispersion equation to obtain elasticity and viscosity of the given tissue [21, 22]. The successful use of SDUV for *in vivo* liver studies [21] has motivated us to explore the feasibility of using SDUV to quantify viscoelasticity of the myocardium.

The SDUV studies in the liver provided tools for exciting and detecting shear waves in soft tissues. While successful use of the technique in the liver is encouraging, the shear wave dispersion in the liver, vessels, kidneys, spleen, heart and other organs is strongly influenced by the density, perfusion, physiological motion and boundary conditions of each organ. SDUV in the liver usually excites shear waves in the regions far from the surface of the organ, which allows one to neglect the boundary conditions and treat the motion as shear wave propagation in infinite medium.

The thickness of the LV free wall myocardium is 10 – 20 mm which is on the order of magnitude of the focal length of the ultrasound radiation force (2 – 10 mm), with blood and pericardial fluid on either side. Thus, the free wall myocardium has been approximated as a solid plate submerged in fluid rather than an infinite or semi-infinite medium. A study by Kanai [20] has reported the use of a high sensitivity ultrasound method to detect shear wave propagation in the septum due to closure of the aortic valve. The motion of the septum was modeled by an antisymmetric plane Lamb wave in an infinite isotropic plate with incompressible fluid on both sides. This method was used to measure shear wave dispersion in the frequency range 10 – 90 Hz and fit the Lamb wave model to estimate septal elasticity and viscosity.

Our group has been investigating the use of the SDUV technique to excite and measure dispersion of anti-symmetric Lamb waves in the myocardium in order to estimate elasticity

and viscosity. Lamb waves are a type of shear waves, but the term ‘shear wave’ has been extensively used when discussing shear waves in infinite media unaffected by boundary conditions. In this paper, we will refer to the use of SDUV in the heart for the purpose of exciting Lamb waves as Lamb wave Dispersion Ultrasound Vibrometry, or LDUV.

While LDUV has some similarities with the method proposed by Kanai [20], our method has several important advantages. Due to high attenuation of Lamb waves in the myocardium, sub-millimeter spatial and around 50 microsecond temporal resolution, LDUV has the capacity to produce biopsy-like measurements of viscoelasticity of any region of the myocardium accessible to ultrasound. In addition, LDUV includes an excitation that is independent of the endogenous motion in the region of interest which allows for more control over the excitation frequency.

In this paper, we present studies aimed at validating the use of Lamb wave Dispersion Ultrasound Vibrometry to quantify elasticity and viscosity of the LV free-wall myocardium. In addition, we briefly review the basic principles of the SDUV. A mathematical model for antisymmetric cylindrical Lamb wave dispersion in isotropic plates submerged in a noncompressible fluid is derived. Finite element analysis (FEA) of the described model is used to investigate the appropriateness of the model. The LDUV method was used to quantify elasticity and viscosity of gelatin and urethane rubber plates. An embedded sphere method [27] was used for comparison and validation of the LDUV method. LDUV measurements of elasticity and viscosity in excised porcine myocardium samples are also reported.

Methods

A. Principles of Shearwave Dispersion Ultrasound Vibrometry

In a basic implementation of SDUV, amplitude modulated (AM) ultrasound beam from a push transducer is focused in the medium of interest to generate monochromatic shear waves in the frequency range 50 – 500 Hz [21, 22]. A pulse-echo, or detect, transducer is used to detect motion parallel to the excitatory beam (Figure 1) [22]. Ultrasound pulses are transmitted at the location of interest at a pulse repetition rate of few kilohertz. Each point in the time-domain of the returning echo signal corresponds to a specific region of the tissue along the beam axis. Cross-spectral analysis of the echoes is used to calculate tissue displacement as a function of time [28]. A specialized Kalman filter is applied to the displacement versus time data to extract the motion at the excitation frequency of the push transducer and estimate the shear wave amplitude and phase [29].

The beamwidth of the radiation force in the focal point is fairly narrow and uniform so the resulting shear wave can be assumed to be cylindrical [22]. Particle displacement due to monochromatic cylindrical shear waves is mostly along the z -axis (Figure 1), which for homogenous viscoelastic media is approximated by:

$$u_z(r, t) \approx \frac{i}{4} \sqrt{\frac{2}{\pi h r}} e^{i(hr - \omega t + \pi/4)}, \quad (1)$$

where u_z is the particle displacement, r is the distance away from the excitation source, ω is the angular frequency of the shear wave, t is the time and h is the shear wavenumber

$h = \sqrt{\rho\omega^2/\mu}$. Equation (1) shows that for large distance r (about one tenth of the shear wavelength), the phase delay varies linearly with the distance from the excitation point.

Since $\omega = 2\pi f$, one can estimate the shear wave speed by measuring the phase shift $\Delta\phi = \phi_2 - \phi_1$ over distance traveled Δr :

$$c_s = \omega \frac{\Delta r}{\Delta\phi}. \quad (2)$$

Equation (2) requires phase measurements at two or more locations, with more locations improving the quality of the shear wave velocity estimates.

B. Anti-symmetric Cylindrical Lamb wave Dispersion in Plates

We are treating the plate as an incompressible, homogenous, isotropic solid submerged in an incompressible nonviscous fluid. The basic wave equations, equilibrium relations and displacement field theorems (i.e., starting point of this derivation), (3) – (13) can be found in any of the book chapters [30–33]. The Stokes-Helmholtz decomposition theorem states that a displacement field can be written as a sum of the gradient of a scalar and the curl of a vector potential (4) [30]. Applying this to the Navier's equation with zero equivalent body force (3) [30] allows for separation of variables resulting in two wave equations, one for the compressional (5) and one for the shear wave (6):

$$(\lambda + 2\mu)\nabla(\nabla \cdot \underline{u}) - \mu\nabla \times \nabla \times \underline{u} = \rho \frac{\partial^2}{\partial t^2} \underline{u}, \quad (3)$$

$$\underline{u} = \nabla\varphi + \nabla \times \underline{\psi}, \quad (4)$$

$$(\lambda + 2\mu)\nabla^2\varphi = \rho \frac{\partial^2}{\partial t^2} \varphi, \quad (5)$$

$$\mu\nabla^2\underline{\psi} = \rho \frac{\partial^2}{\partial t^2} \underline{\psi}, \quad (6)$$

Here, $\underline{u}$ represents the displacement field vector, $\nabla \cdot$ the divergence operator, $\nabla \times$ the curl

operator, ∇ the gradient operator and $\frac{\partial^2}{\partial t^2}$ second partial derivative with respect to time. A more convenient way of expressing (5) and (6) is in terms of the compressional and shear wave velocities c_p and c_s

$$\nabla^2\varphi = \frac{1}{c_p^2} \frac{\partial^2}{\partial t^2} \varphi, \quad (7)$$

$$\nabla^2\underline{\psi} = \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} \underline{\psi}, \quad (8)$$

$$c_p = \sqrt{(\lambda + 2\mu)/\rho}, \quad (9)$$

$$c_s = \sqrt{\mu/\rho}, \quad (10)$$

where ρ is the material density, λ the first Lamé constant and μ the second Lamé constant, commonly referred to as the shear moduli of elasticity.

The LV free-wall myocardium surrounded by blood and pericardial fluid was modeled as a homogenous solid plate submerged in a noncompressible fluid of mass density similar to that of blood (Figure 2). The frequency response of the myocardium was assumed to be that of a Voigt material, so the shear modulus (μ) is expressed as $\mu = \mu_1 + i\omega\mu_2$, where μ_1 and μ_2 are the shear elastic and viscous moduli, respectively. The Voigt model has been extensively used to model the behavior of the myocardium and other soft tissues [20–22].

Because the shear waves propagating in the myocardium are cylindrical and the solid is assumed to be isotropic, the condition of axial symmetry allows us to consider the motion independent of the angle θ , so that $u_\theta = \partial/\partial\theta = 0$. To clarify, the angle θ is the angle of rotation of the r -axis around the z -axis in Figure 2. Expressing the gradient and the curl operators in cylindrical coordinates and introducing this simplification into (4), the following equations are obtained:

$$\frac{\partial^2 \psi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \psi_2}{\partial r} + \frac{\partial^2 \psi_2}{\partial z^2} - \frac{\psi_2}{r^2} = \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} \psi_2, \quad (11)$$

$$\frac{\partial^2 \varphi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_2}{\partial r} + \frac{\partial^2 \varphi_2}{\partial z^2} = \frac{1}{c_p^2} \frac{\partial^2}{\partial t^2} \varphi_2, \quad (12)$$

$$\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_1}{\partial r} + \frac{\partial^2 \varphi_1}{\partial z^2} = \frac{1}{c_F^2} \frac{\partial^2}{\partial t^2} \varphi_1, \quad (13)$$

where c_p and c_s are the compressional and shear wave speeds in the solid, and c_F is the speed of the compressional wave in the fluid. Subscripts 1 and 2 refer to the fluid and the solid, respectively. Shear waves do not propagate in fluids so the vector potential for the fluid is ignored.

The boundary conditions for the Lamb wave model at $z = \pm h$ are $\sigma_{zz1} = \sigma_{zz2}$, $\sigma_{rz2} = 0$ and $u_{z1} = u_{z2}$, where the displacements (u) and stresses (σ) are defined as follows:

$$u_{r1} = \frac{\partial \varphi_1}{\partial r}, \quad (14)$$

$$u_{r2} = \frac{\partial \varphi_2}{\partial r} - \frac{\partial \psi_2}{\partial z}, \quad (15)$$

$$u_{z1} = \frac{\partial \varphi_1}{\partial z}, \quad (16)$$

$$u_{z2} = \frac{\partial \varphi_2}{\partial z} + \frac{1}{r} \frac{\partial(r\psi_2)}{\partial r}, \quad (17)$$

$$\sigma_{zz} = (\lambda + 2\mu) \frac{\partial u_z}{\partial z} + \frac{\lambda}{r} \frac{\partial(r u_r)}{\partial r}, \quad (18)$$

$$\sigma_{zr} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right). \quad (19)$$

In order to proceed, we need to solve for potential functions ϕ_1 , ϕ_2 and ψ in equations (11)–(13). These differential equations are not easy to solve in the real time and space domain, so following Zhu *et al* [34], we perform the Laplace transform with respect to time and Hankel transform with respect to the radial component.

$$\bar{f}(p) = \int_0^{\infty} f(t) e^{-pt} dt \quad (20)$$

$$f^{H_n}(\xi) = \int_0^{\infty} f(r) J_n(\xi r) r dr \quad (21)$$

One-sided Laplace (20) and n th order Hankel transforms (21) were used to transform the differential equations (11) – (13) into the Laplace – Hankel domain. In (20) and (21), $f(t)$ and $f(r)$ are functions time (t) and distance (r) and J_n is the Bessel function of first kind. More details on Hankel transforms and transform tables can be found in [35]. Following the Hankel and Laplace transforms equations (11) – (13) become:

$$\frac{\partial^2}{\partial z^2} \bar{\varphi}_1^{H_0} = \alpha_1^2 \bar{\varphi}_1^{H_0}, \quad (22)$$

$$\frac{\partial^2}{\partial z^2} \bar{\varphi}_2^{H_0} = \alpha_2^2 \bar{\varphi}_2^{H_0}, \quad (23)$$

$$\frac{\partial^2}{\partial z^2} \overline{\psi}_2^{H_1} = \beta^2 \overline{\psi}_2^{H_1}, \quad (24)$$

where α_1 , α_2 and β are defined as $\alpha_1^2 = \frac{p^2}{c_f^2} + \xi^2$, $\alpha_2^2 = \frac{p^2}{c_p^2} + \xi^2$ and $\beta^2 = \frac{p^2}{c_s^2} + \xi^2$, where p and ξ are, respectively, the variables of the Laplace and Hankel domains.

The solutions of the potential functions in the Hankel-Laplace space (22) – (24) are of the following form:

$$\overline{\varphi}_1^{H_0} = \Phi_{11}(\xi, p)e^{\alpha_1 z} + \Phi_{12}(\xi, p)e^{-\alpha_1 z}, \quad (25)$$

$$\overline{\varphi}_2^{H_0} = \Phi_{21}(\xi, p)e^{\alpha_2 z} + \Phi_{22}(\xi, p)e^{-\alpha_2 z}, \quad (26)$$

$$\overline{\psi}_2^{H_1} = \Psi_{21}(\xi, p)e^{\beta z} + \Psi_{22}(\xi, p)e^{-\beta z}. \quad (27)$$

Due to geometry, we have to consider the potentials for the fluid above and below the solid separately. In addition, we use definitions of hyperbolic sine and cosine functions $2\sinh(x) = (e^x - e^{-x})$ and $2\cosh(x) = (e^x + e^{-x})$ to rewrite equations (26) and (27). The following equations are obtained:

$$\overline{\varphi}_2^{H_0} = \Phi_{21}e^{\alpha_2 z} + \Phi_{22}e^{-\alpha_2 z} = A\sinh(\alpha_2 z) + B\cosh(\alpha_2 z), \quad (28)$$

$$\overline{\psi}_2^{H_1} = \Psi_{21}e^{\beta z} + \Psi_{22}e^{-\beta z} = C\sinh(\beta z) + D\cosh(\beta z), \quad (29)$$

$$\overline{\varphi}_L^{H_0} = \Phi_{L1}e^{\alpha_1 z} + \Phi_{L2}e^{-\alpha_1 z} = Ne^{\alpha_1 z}, \quad (30)$$

$$\overline{\varphi}_U^{H_0} = \Phi_{U1}e^{\alpha_1 z} + \Phi_{U2}e^{-\alpha_1 z} = -Ne^{-\alpha_1 z}. \quad (31)$$

The potential functions must be stable in the region of interest (i.e. cannot approach infinity), so the unstable solutions are discarded. Subscripts L and U represent the lower and upper fluid on either side of the solid. In order to use (28) – (31), the displacements and stresses have to be expressed in the Hankel-Laplace domain:

$$\overline{u}_{r1}^{H_1} = -\xi \overline{\varphi}_1^{H_0}, \quad (32)$$

$$\overline{u_{z1}}^{H_0} = \frac{\partial \overline{\varphi_1}^{H_0}}{\partial z}, \quad (33)$$

$$\overline{u_{z2}}^{H_0} = \frac{\partial \overline{\varphi_2}^{H_0}}{\partial z} + \xi \overline{\psi}^{H_1}, \quad (34)$$

$$\overline{u_{r2}}^{H_1} = -\frac{\partial \overline{\psi}^{H_1}}{\partial z} - \xi \overline{\varphi_2}^{H_0}, \quad (35)$$

$$\overline{\sigma_{zz2}}^{H_0} = \mu_2 \left(\left(\frac{p^2}{c_s^2} + 2\xi^2 \right) \overline{\varphi_2}^{H_0} + 2\xi \frac{\partial \overline{\psi}^{H_1}}{\partial z} \right), \quad (36)$$

$$\overline{\sigma_{zz1}}^{H_0} = \rho_1 p^2 \overline{\varphi_1}^{H_0}, \quad (37)$$

$$\overline{\sigma_{zr2}}^{H_1} = -\mu_2 \left(2\xi \frac{\partial \overline{\varphi_2}^{H_0}}{\partial z} + \left(\frac{p^2}{c_s^2} + 2\xi^2 \right) \overline{\psi}^{H_1} \right). \quad (38)$$

In anti-symmetric Lamb wave motion (Figure 2), the displacement of the solid in the z -direction is anti-symmetric relative to the r -axis, so the potential functions for the anti-symmetric Lamb wave are as follows:

$$\overline{\varphi_2}^{H_0} = A \sinh(\alpha_2 z), \quad (39)$$

$$\overline{\psi_2}^{H_1} = D \cosh(\beta z), \quad (40)$$

$$\overline{\varphi_t}^{H_0} = N e^{\alpha_1 z}, \quad (41)$$

$$\overline{\varphi_u}^{H_0} = -N e^{-\alpha_1 z}. \quad (42)$$

Introducing the potential functions (39) – (42) into the boundary conditions expressed in the Hankel – Laplace domain results in three equations with three unknowns. This can be expressed as a 3×3 matrix multiplied by a 3×1 vector containing the unknowns. The determinant of the matrix must be equal to zero in order for the system of equations to have a nontrivial solution which yields the following expression:

$$\left(\frac{p^2}{c_s^2} + 2\xi^2\right)^2 \sinh(\alpha_2 h) \cosh(\beta h) - 4\xi^2 \beta \alpha_2 \cosh(\alpha_2 h) \sinh(\beta h) = \frac{p^4 \alpha_2 \rho_1}{c_s^4 \alpha_1 \rho_2} \cosh(\beta h) \cosh(\alpha_2 h) \quad (43)$$

This is the dispersion relationship for cylindrical anti-symmetric Lamb waves of a viscoelastic solid submerged in a noncompressible fluid. Note that the bilateral Laplace transform (44) is similar in form to the Fourier transform (45) and the two transforms differ by $p = i\omega$ factor.

$$F_{BL}(p) = \overline{f(p)} = \int_{-\infty}^{\infty} f(t) e^{-pt} dt \quad (44)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (45)$$

By introducing the substitution $\xi = \eta p$ and recognizing that $\eta = \pm i/c_L$, where c_L is the Lamb wave velocity ([34, 36]), equation (43) becomes (46).

$$4k_L^2 \eta \beta \cosh(\eta h) \sinh(\beta h) - (2k_L^2 - k_s^2)^2 \sinh(\eta h) \cosh(\beta h) = \frac{\rho_1 \eta k_s^4}{\rho_2 \eta_f} \cosh(\eta h) \cosh(\beta h) \quad (46)$$

In (46), $\beta = \sqrt{k_L^2 - k_s^2}$, $\eta = \sqrt{k_L^2 - k_p^2}$, $\eta_f = \sqrt{k_L^2 - k_f^2}$ and k_f is the wave number of the compressional wave in the fluid. For the case of tissue and blood, the compressional wave numbers (k_p and k_f) are very similar and about 4 orders of magnitude smaller than the shear wave number (k_s) which is similar to the Lamb wave number (k_L). Applying these

assumptions, we can write $\eta_f \approx \eta = \sqrt{k_L^2 - k_p^2} \approx k_L$. Since the densities for the blood and tissue are fairly similar, the dispersion equation (44) can be expressed as follows:

$$4k_L^3 \beta \cosh(k_L h) \sinh(\beta h) - (k_s^2 - 2k_L^2)^2 \sinh(k_L h) \cosh(\beta h) = k_s^4 \cosh(k_L h) \cosh(\beta h) \quad (47)$$

In (45) $k_L = \omega/c_L$ is the Lamb wave number, ω is the angular frequency, c_L is the frequency dependent Lamb wave velocity, $k_s = \omega \sqrt{\rho_m/\mu}$ is the shear wave number, ρ_m is the density of the sample and h is the half-thickness of the sample. Here, the shear modulus (μ) is written in terms of the elastic (μ_1) and viscous (μ_2) components so that $\mu = \mu_1 + i\omega\mu_2$. Equation (47) is fit to the experimentally measured Lamb wave dispersion curves (velocity versus frequency) to obtain elasticity and viscosity coefficients μ_1 and μ_2 . Note that equation (47) is model-independent and that any rheological model can be used to fit the dispersion data by expressing the shear modulus (μ) in terms of that model.

C. Finite Element Analysis of a Plate in a Fluid

A finite element model of a solid viscoelastic plate ($0.5 \text{ m} \times 0.5 \text{ m} \times 2 \text{ cm}$) submerged in an incompressible fluid was designed using ABAQUS 6.8-3 (SIMULIA, Providence, RI). The

fluid surrounding the plate was represented with acoustic elements with a bulk modulus of 2.2 GPa and density of 1 g/cm³. The material properties of the plate were as follows: density of 1.08 g/cm³, Poisson's ratio of 0.495, Young's modulus of 90 kPa and the mechanical properties defined in terms of the two parameter Prony series where $g_1 = 0.5$ and $\tau_1 = 10^{-5}$. The two parameter Prony series is directly related to the generalized Maxwell model with three parameters, a spring, G_∞ , in parallel with a Maxwell element (spring, G_1 , and dashpot, η_1 , in series) [37]. Using the values in the Prony series and the theory presented by Vappou, *et al.*, the values for the two springs were set to $G_\infty = G_1 \approx 30.1$ kPa. The dashpot was set to $\eta_1 = 0.301$. A line source through the entire thickness of the plate was used as a source of vibrations. Amplitudes of the vibrations were around 10 μ m. Motion characteristic of anti-symmetric Lamb waves was produced by oscillating the line source with four cycles of sinusoidal waves in the frequency range 25 – 600 Hz with the maximal displacements on the order of tens of micrometers. Shear wave displacements were recorded at 40 points, 1 mm apart, along a line away from the source in the middle of the plate. Displacement was recorded in the direction parallel to the excitation at several depths of the sample. The data were used to calculate the Lamb wave velocity at each frequency (dispersion). The dispersion data were compared to theoretical predictions to study the validity of the Lamb wave equations.

D. Materials

Urethane rubber samples were prepared by mixing 25% of Part A, 25% of Part B of Reoflex 20 (Smooth-On, Inc., Easton, PA), and 50% of softener (So-Flex Flexibilizer, Smooth-On Inc., Easton, PA), all by mass. In order to provide ultrasound scatterers, 3% by mass of Sigmacell Cellulose Type 20 (Aldrich Chemical Company, Inc., Milwaukee, WI), was added to the mixture. Gelatin samples consisted of 75% water, 10% glycerol and 15% of 300 Bloom gelatin (all manufactured by Sigma-Aldrich, St. Louis, MO), all by volume. Plate phantoms were prepared by pouring the mixtures into an 11 cm $\times$ 8 cm $\times$ 1.2 cm plastic mold and allowed to cure for 24 hours. Embedded sphere phantoms were prepared by pouring the mixtures into cylindrical molds (2.5 cm in radius, 5 cm in height) with a solid stainless steel sphere, 0.75 mm in radius, suspended in the middle of the sample. Pig hearts were obtained from a local butcher shop. The hearts were obtained a few hours after the animals were sacrificed. Left ventricular free wall myocardium was excised from the pig hearts in our laboratory. The porcine myocardium sample was embedded in gelatin and stored overnight in a refrigerator before use.

E. Experimental Setup for LDUV

The following procedure was followed for urethane rubber plates, gelatin plates and excised porcine LV free-wall myocardium samples. The samples were embedded in a gelatin mixture (80% water, 10% glycerol, 10% 300 Bloom gelatin, all by volume and 10 g/L potassium sorbate preservative, all manufactured by Sigma-Aldrich, St. Louis, MO) inside a plastic container. The container was mounted on a stand (not shown) in a water tank (Figure 3a). A window was cut out on the bottom of the container to allow for pulse-echo ultrasound measurements for motion detection. The gelatin was used as a stabilizer and was contained to the edges to minimize the affect of mechanical coupling. A mechanical shaker (V203, Ling Dynamic Systems Limited, Hertfordshire, UK) was used instead of a push transducer (Figure 3a) in order to ensure large motion and avoid ultrasound wave interference. A glass rod coupled with the shaker was glued to the hole bored through the thickness of the sample. Four cycles of sinusoidal waves were used to drive the shaker at different frequencies ranging from 40 to 500 Hz to induce cylindrical shear waves in the samples. More Lamb wave speed measurements were made below 200 Hz since the Lamb wave curve plateaus at higher frequencies and the lower frequencies provide more insight into the curvature of the Lamb wave dispersion. The motion of the shaker was parallel to the rod and the maximum

amplitude of the displacement was on the order of tens of micrometers near the rod and exponentially decaying with distance (Figure 3a). At each excitation frequency, motion along the z -axis was measured at 31 points, 0.5 mm apart along the r -axis where $r = \sqrt{x^2 + y^2}$, using a 5 MHz pulse-echo transducer with a pulse repetition rate of 4 kHz. Phase measurements at these points were used to fit a regression curve and calculate the Lamb wave speed at each frequency, as described in the SDUV methods [21, 22]. The pulse-echo transducer was mounted on a robotic arm capable of micrometer size steps in three Cartesian directions. The robotic arm was used to move the transducer in four orthogonal directions in the r -plane at each excitation frequency (Figure 3a). For simplicity, we will refer to the four orthogonal directions in the r -plane at $0, \pi/2, \pi$ and $3\pi/2$ radians as $+x, +y, -x$ and $-y$ directions. The long axis of the plate shaped samples was aligned parallel to the x -axis.

F. Embedded Sphere Method

The embedded sphere method is a technique developed by our group for measuring viscoelastic properties of organic and inorganic materials [27]. The method estimates the shear elasticity and viscosity of a given medium by measuring the response of a solid sphere in the middle of the phantom under harmonic radiation force (Figure 3b). The motion of an oscillating sphere is governed by its radiation impedance, which is governed by the mechanical properties of the surrounding medium. The magnitude of the vibrating sphere depends on the resonant frequency of the sphere in the surrounding material and since each material/sphere combination has a specific resonance, this approach provides a “fingerprint” for each medium. The ultrasound radiation force from the push transducer is used to oscillate the sphere at different frequencies. The vibration magnitude is measured at each frequency of vibration by the pulse-echo transducer. The measurements are then combined with the vibration theory to solve for the complex stiffness of the medium. The approach used in this study assumes that the frequency response of the tested material can be represented by the Voigt model, but is readily modified for other models and model-free approaches. The typical percent error of the embedded sphere method compared to the dynamic mechanical testing is around 5% so this method has been used as an independent validation of the estimates of elasticity and viscosity obtained using the LDUV approach [27].

Results

A finite element model of a viscoelastic isotropic plate submerged in a water-like fluid was used to study the deformation of an infinite plate due to harmonic excitation from a line source perpendicular to the plate (z -axis in Figure 2). A typical displacement as a function of time of several points due to harmonic excitation at 200 Hz is shown in the motion map in Figure 4a. The phase gradient as a function of distance from the excitation was used to calculate the Lamb wave velocity at each frequency using (2). Prior to calculating the Lamb wave velocity, a discrete Fourier transform of the displacement as a function of time of each point was used to create a 2D k -space of the displacement field. The coordinates of the k -space are frequency and wave number ($1/\lambda$). The absence of peaks in quadrants I and III in Figure 4b confirmed absence of reflections from the boundaries [38]. This procedure was followed for all the excitation frequencies. Conveniently, the velocity of the propagated wave can be also obtained by dividing the frequency and wave number coordinate, i.e. $c = f/k_x = f\lambda$, for the peaks at the appropriate frequency. The two methods produce nearly identical results. This process was repeated for several frequencies between 25 and 600 Hz. The Lamb wave velocity at each excitation frequency is shown as red circles in Figure 5.

The Prony series parameters used to define material properties of the plate were inserted into the Lamb wave dispersion equation (47) and compared to the FEM Lamb wave velocities at

different frequencies. The blue line in Figure 5 is the analytical Lamb wave dispersion based on the Prony series parameters and is in excellent agreement with the FEM plate model velocities shown as red circles.

The LDUV experimental setup described in the Methods section (Figure 3a) was used to measure Lamb wave velocity of gelatin plates at different frequencies. The speed was measured at each millimeter of thickness and averaged throughout the thickness of the sample. The Lamb wave velocities at different frequencies in four orthogonal directions in the r -plane are shown as red circles in Figure 6b. Figure 6a shows the phase throughout the thickness of the plate (z -axis) and at different distances from the excitation point (r -axis) for three different frequencies in the $+x$ direction. The preserved phase throughout the thickness of the sample is characteristic of the zeroth mode anti-symmetric (A_0) Lamb wave. The A_0 Lamb wave motion in a plate can be thought of as bulk motion, i.e., the entire plate moves up and down, as shown in Figure The phase gradient as a function of distance along the r -axis is used to calculate the velocity of the propagated wave. Other frequencies and directions show a similar pattern. The Lamb wave dispersion equation (47) was fit to the experimental velocity versus excitation frequency data to estimate the material properties of the gelatin plate in each of the four directions. Shear modulus μ was expressed in terms of elasticity (μ_1) and viscosity (μ_2) and (47) was evaluated numerically for values of μ_1 in the range 0–200 kPa and μ_2 in the range 0–100 Pa·s; the mean square error (MSE) between the experimental data and the numerical solutions of (47) was the criterion in the curve fitting process. The Lamb wave fit is shown as the blue line in Figure 6b with the results of the fit shown above the figures for each direction.

The LDUV method was also used to quantify the material properties of a urethane rubber plate. The protocol used for measuring viscoelasticity of gelatin plates was followed. For simplicity, Figure 7 shows the average experimental Lamb wave velocities in all directions with the average elasticity and viscosity parameters. The phase map pattern seen in the gelatin plates (Figure 6a) was also seen in the urethane rubber (not shown).

The embedded sphere method was used as an independent measurement of viscoelasticity of the gelatin and urethane rubber mixtures. The results are summarized in Table I with the mean and standard deviations. The values of elasticity and viscosity of gelatin and urethane mixtures obtained using the Lamb wave Dispersion Ultrasound Vibrometry (LDUV) in plates and the embedded sphere method agree within one standard deviation.

This result gave us the confidence to investigate the use of LDUV in excised porcine LV free wall myocardium. The phase map at several different frequencies (Figure 8a) shows that the phase is fairly preserved throughout the thickness of the sample. Figure 8a represents typical phase maps at 100, 150 and 200 Hz. This pattern persists for other frequencies and other directions. Relatively preserved phase throughout the thickness suggests that the anti-symmetric Lamb wave propagates through the excised free wall myocardium. Thus, the Lamb wave dispersion equation is appropriate to fit the experimental data to estimate the elastic and viscous moduli. The results are shown in Figure 8b.

Discussion

The FEM simulations of the Lamb wave dispersion of a viscoelastic plate submerged in water are in good agreement with the theoretically predicted results. This gives us confidence in the dispersion equation and the ability to measure viscoelasticity of a plate-like solid. The vertical displacements in our LDUV experiments on urethane and gelatin plates were on the order of tens of microns. Due to large attenuation, the waves died out (sub-micron displacements) within 2 cm from the excitation point. This pattern was

observed in gelatin plates, urethane rubber plates and excised myocardium samples. Shear wave attenuation is often due to viscosity of the material, but in case of Lamb waves, the geometric dispersion is also large. The small propagation distance compared to the dimensions of the plates and LV free-wall myocardium is an advantage because we can ignore the possible reflections from the boundaries and interference pattern formation. In essence, this allows us to assume that the LDUV samples can be treated like infinite plates. While *in vivo* LV free-walls present with curvature, that curvature is small on the scale over which these measurements were made (1–2 cm) and can be largely neglected because of the high attenuation of the shear waves. Complete 3D heart models obtained by meshing CT scans of an *in vivo* heart will be used for FEM exploration of Lamb wave propagation in a more realistic environment in future studies.

Lamb wave dispersion and elasticity and viscosity of the gelatin samples are fairly similar in all four directions. This is to be expected given that the gelatin plate is an isotropic material. The urethane rubber plate displayed similar behavior. The uniform phase throughout the thickness of the samples suggests that the A_0 mode carries most of the wave energy.

The results of quantifying elasticity and viscosity of gelatin and urethane rubber samples using the LDUV and the embedded sphere methods are in good agreement (Table I). This result suggests that the LDUV method has the potential to be used as a method for quantifying material properties of solid viscoelastic materials; further studies on different materials with varying viscoelastic properties would be necessary.

It is important to note that the frequency response of the gelatin and urethane rubber was assumed to obey the Voigt model. The Voigt model is widely used to model the frequency response of soft tissues and gelatin phantoms [22, 39], but its appropriateness for the urethane rubbers has not been extensively studied. The Voigt model consists of an elastic term and a frequency dependent viscous term. More complex generalized Maxwell models with multiple parameters could be used to describe the frequency response of any material and the Voigt model is a two parameter simplification of such models. In the future, we will be investigating a model free approach; this would require us to measure the attenuation as a function of frequency and relate the material properties to the Lamb wave velocity and attenuation. In addition, we are currently investigating the use of direct inversion algorithms on the displacement data in order to obtain the Lamb wave velocity and attenuation.

The LDUV method was used to study the elasticity and viscosity of the excised myocardium. It is important to note that the phase is relatively preserved throughout the thickness of the sample (z -direction) and changes in a linear fashion in the x -direction as predicted by (1). Phase maps at other excitation frequencies are similar to the ones shown in Figure 8a. This trend persists in all four directions and throughout experiments on different samples of LV myocardium. Thus, the anti-symmetric Lamb wave model was fit to the dispersion data to estimate viscoelastic coefficients μ_1 and μ_2 .

Our method is similar to that of Kanai [20] used to measure dispersion velocities in the heart septum due to closure of the aortic valve in the frequency range 10 – 90 Hz. Our method is capable of measuring Lamb wave dispersion over a larger frequency range, 40–500 Hz, and has the potential to make measurements at any point in the myocardium accessible to radiation force. Elasticity and viscosity of the LV myocardium vary during a heart cycle. While we are reporting *ex vivo* results, we expect our method to be capable of detecting variations in myocardial viscoelasticity periodic with a heart cycle. At this point, we are unsure of the effect a region of ischemic or necrotic tissue would have on myocardial viscoelasticity. This question will be addressed in future research.

The LDUV method has the ability to measure the Lamb wave velocity at different layers and directions, and evaluate the velocity as a function of fiber orientation in different layers of the heart wall. In this paper, we averaged the Lamb wave speeds through the thickness of the gelatin and rubber plates as well as the myocardium sample. While this is valid for the gelatin and rubber plates, this averaging may not be entirely accurate for the myocardium. Shear wave measurements in skeletal muscle have shown different wave speed dispersion based on the orientation of the fibers [21, 40–42]. A similar type of measurements has been made in an explanted sheep heart, which showed a variation of measured wave speeds through the thickness of the myocardial wall [43]. We performed this present study to assess the feasibility of using Lamb wave speed dispersion to assess the viscoelastic properties of the LV free wall. Analysis of the anisotropy and its effects on Lamb wave speed dispersion are beyond the scope of this study but is a subject of ongoing work.

The smallest vibration our method can detect in an *ex vivo* water bath environment is a sinusoidal signal with an amplitude of 100–200 nm [44]. The threshold might be higher *in vivo*, but we do not expect a drastic difference. We used a mechanical actuator to excite Lamb waves in different materials. Use of a mechanical actuator to excite shear waves in the heart has been reported before in cardiac MRE studies [16]. In the future, we plan to test the use of radiation force for shear wave excitation. This approach would mimic the SDUV method and that of Bouchard *et al.* [23] and would be necessary for clinical use.

Due to the lack of reported measures of viscoelasticity of excised myocardium, it is difficult to compare our results to those obtained by other groups with different methods. To the best of our knowledge, there are no reports of estimates of viscoelasticity of the excised free-wall using the Voigt model in the frequency range 40 – 500 Hz. Kanai [20] reported *in vivo* estimates the elasticity and viscosity of the human ventricular septum using Lamb wave theory applied to endogenous heart motion in the frequency range 10 – 90 Hz with the values $\mu_1 = 30$ kPa and $\mu_2 = 70 - 400$ Pa·s. The values of μ_2 are fairly high which is due to the narrow frequency band. Kolipaka *et al.* [45] reported *in vivo* changes of pig myocardial elasticity using MRE throughout the heart cycle in range 5 – 10 kPa. Couade *et al.* [43] reported similar *in vivo* measurements in sheep hearts using Supersonic Shear Imaging [46] with the elasticity ranging from 2.17 kPa to 38.7 kPa throughout the heart cycle and along the short and long axis. All these methods have one thing in common: they induce guided waves in the myocardium and measure the speed of propagation. The estimates of elasticity or viscoelasticity of the heart based on the speeds depends on the employed mathematical model. Thus, it is more appropriate to compare the wave velocities obtained in our study to those in [20], [43] and [45]. The velocities obtained in excised porcine myocardium sample using the LDUV method are 2 – 5 m/s in the range 40 – 500 Hz. Kanai [20] reported velocities of 1 – 6 m/s in the range 10 – 90 Hz. Kolipaka, *et al.* [45] reported velocities of 2.2 – 3.2 m/s at 80 Hz. Couade, *et al.* [43] reported velocities of 1.45 – 4.8 m/s along the long axis and 3.5 – 6.2 m/s along the short axis of the heart in the frequency range 100 – 400 Hz.

Conclusion

Lamb wave Dispersion Ultrasound Vibrometry (LDUV) method for quantifying elasticity and viscosity of solids has been proposed. The approach has been compared to independent measurements made using an embedded sphere method in urethane rubber and gelatin phantoms. The measurements of elasticity and viscosity using LDUV are in good agreement with those obtained using the embedded sphere method. We also reported studies aimed at investigating the feasibility of using LDUV to quantify material properties of the left ventricular free wall myocardium.

Acknowledgments

This work was supported in part by grant EB002167 from the National Institutes of Health. The authors would like to thank to Randall R. Kinnick for experiment support, Thomas Kinter for computer support, and Jennifer Milliken for administrative support. We would also like to express our gratitude to Miguel Bernal for inspiring scientific discussions.

References

1. Heart Disease & Stroke Statistics - 2010 Update. American Heart Association; [cited 2010 June 7]; American Heart Association]. Available from:<http://www.americanheart.org/statistics>
2. Gary R, Davis L. Diastolic heart failure. *Heart Lung*. 2008; 37(6):405–16. [PubMed: 18992623]
3. Redfield MM, et al. Burden of systolic and diastolic ventricular dysfunction in the community: appreciating the scope of the heart failure epidemic. *Jama*. 2003; 289(2):194–202. [PubMed: 12517230]
4. Zile MR, Brutsaert DL. New concepts in diastolic dysfunction and diastolic heart failure: Part I: diagnosis, prognosis, and measurements of diastolic function. *Circulation*. 2002; 105(11):1387–93. [PubMed: 11901053]
5. Maeder MT, Kaye DM. Heart failure with normal left ventricular ejection fraction. *J Am Coll Cardiol*. 2009; 53(11):905–18. [PubMed: 19281919]
6. Borbely A, et al. Cardiomyocyte stiffness in diastolic heart failure. *Circulation*. 2005; 111(6):774–81. [PubMed: 15699264]
7. Gaasch WH, Zile MR. Left ventricular diastolic dysfunction and diastolic heart failure. *Annu Rev Med*. 2004; 55:373–94. [PubMed: 14746527]
8. Paulus WJ, et al. How to diagnose diastolic heart failure: a consensus statement on the diagnosis of heart failure with normal left ventricular ejection fraction by the Heart Failure and Echocardiography Associations of the European Society of Cardiology. *Eur Heart J*. 2007; 28(20):2539–50. [PubMed: 17428822]
9. Greenleaf JF, Fatemi M, Insana M. Selected methods for imaging elastic properties of biological tissues. *Annu Rev Biomed Eng*. 2003; 5:57–78. [PubMed: 12704084]
10. Gao L, et al. Imaging of the elastic properties of tissue--a review. *Ultrasound Med Biol*. 1996; 22(8):959–77. [PubMed: 9004420]
11. Zerhouni EA, et al. Human heart: tagging with MR imaging--a method for noninvasive assessment of myocardial motion. *Radiology*. 1988; 169(1):59–63. [PubMed: 3420283]
12. Muthupillai R, et al. Magnetic resonance elastography by direct visualization of propagating acoustic strain waves. *Science*. 1995; 269(5232):1854–7. [PubMed: 7569924]
13. Kolipaka A, et al. Evaluation of a rapid, multiphase MRE sequence in a heart-simulating phantom. *Magn Reson Med*. 2009; 62(3):691–8. [PubMed: 19572388]
14. Sandrin L, et al. Shear elasticity probe for soft tissues with 1-D transient elastography. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2002; 49(4):436–46. [PubMed: 11989699]
15. Sarvazyan AP, et al. Shear wave elasticity imaging: a new ultrasonic technology of medical diagnostics. *Ultrasound Med Biol*. 1998; 24(9):1419–35. [PubMed: 10385964]
16. Nightingale K, McAleavey S, Trahey G. Shear-wave generation using acoustic radiation force: in vivo and ex vivo results. *Ultrasound Med Biol*. 2003; 29(12):1715–23. [PubMed: 14698339]
17. Pernot M, et al. ECG-gated, mechanical and electromechanical wave imaging of cardiovascular tissues in vivo. *Ultrasound Med Biol*. 2007; 33(7):1075–85. [PubMed: 17507146]
18. Konofagou EE, et al. Noninvasive electromechanical wave imaging and conduction-relevant velocity estimation in vivo. *Ultrasonics*. 50(2):208–15. [PubMed: 19863987]
19. Konofagou EE, D'Hooge J, Ophir J. Myocardial elastography--a feasibility study in vivo. *Ultrasound Med Biol*. 2002; 28(4):475–82. [PubMed: 12049961]
20. Kanai H. Propagation of spontaneously actuated pulsive vibration in human heart wall and in vivo viscoelasticity estimation. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2005; 52(11):1931–42. [PubMed: 16422405]

21. Chen S, et al. Shearwave dispersion ultrasound vibrometry (SDUV) for measuring tissue elasticity and viscosity. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2009; 56(1):55–62. [PubMed: 19213632]
22. Chen S, Fatemi M, Greenleaf JF. Quantifying elasticity and viscosity from measurement of shear wave speed dispersion. *J Acoust Soc Am*. 2004; 115(6):2781–5. [PubMed: 15237800]
23. Bouchard RR, et al. In vivo cardiac, acoustic-radiation-force-driven, shear wave velocimetry. *Ultrason Imaging*. 2009; 31(3):201–13. [PubMed: 19771962]
24. Salameh N, et al. Hepatic viscoelastic parameters measured with MR elastography: correlations with quantitative analysis of liver fibrosis in the rat. *J Magn Reson Imaging*. 2007; 26(4):956–62. [PubMed: 17896384]
25. Huwart L, et al. Liver fibrosis: non-invasive assessment with MR elastography. *NMR Biomed*. 2006; 19(2):173–9. [PubMed: 16521091]
26. Schmeling TJ, et al. Changes in passive but not active mechanical properties predict recovery of function of stunned myocardium. *Ann Biomed Eng*. 1999; 27(2):131–40. [PubMed: 10199689]
27. Chen S, Fatemi M, Greenleaf JF. Remote measurement of material properties from radiation force induced vibration of an embedded sphere. *J Acoust Soc Am*. 2002; 112(3 Pt 1):884–9. [PubMed: 12243175]
28. Hasegawa H, Kanai H. Improving accuracy in estimation of artery-wall displacement by referring to center frequency of RF echo. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2006; 53(1):52–63. [PubMed: 16471432]
29. Zheng Y, et al. Detection of tissue harmonic motion induced by ultrasonic radiation force using pulse-echo ultrasound and Kalman filter. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2007; 54(2):290–300. [PubMed: 17328326]
30. Rose, JL. *Ultrasonic Waves in Solid Media*. Cambridge University Press; New York: 1999. p. 5-39.
31. Kundu, T. *Ultrasonic Nondestructive Evaluation*. CRC Press; New York: 2003. p. 1-95.
32. Graff, KF. *Wave Motion in Elastic Solids*. Oxford University Press; London, UK: 1975. p. 273-310.
33. Achenbach, JD. *Wave Propagation in Elastic Solids*. Elsevier Science Publishers B.V; Amsterdam, The Netherlands: 2005. p. 310-318.
34. Zhu JY, Popovics JS, Schubert F. Leaky Rayleigh and Scholte waves at the fluid-solid interface subjected to transient point loading. *Journal of the Acoustical Society of America*. 2004; 116(4): 2101–2110.
35. Gerardi FR. Application of Mellin and Hankel Transforms to Networks with Time-Varying Parameters. *IRE Trans Circuit Theory*. 1959; CT-6:197–208.
36. Achenbach, JD. *Wave Propagation in Elastic Solids*. Elsevier Science Publishers B. V; Amsterdam, The Netherlands: 2005. p. 10-165.
37. Vappou J, Maleke C, Konofagou EE. Quantitative viscoelastic parameters measured by harmonic motion imaging. *Phys Med Biol*. 2009; 54(11):3579–94. [PubMed: 19454785]
38. Manduca A, et al. Spatio-temporal directional filtering for improved inversion of MR elastography images. *Med Image Anal*. 2003; 7(4):465–73. [PubMed: 14561551]
39. Catheline S, et al. Measuring of viscoelastic properties of homogeneous soft solid using transient elastography: an inverse problem approach. *J Acoust Soc Am*. 2004; 116(6):3734–41. [PubMed: 15658723]
40. Urban MW, Greenleaf JF. A Kramers-Kronig-based quality factor for shear wave propagation in soft tissue. *Phys Med Biol*. 2009; 54(19):5919–33. [PubMed: 19759409]
41. Urban MW, Chen S, Greenleaf JF. Error in estimates of tissue material properties from shear wave dispersion ultrasound vibrometry. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2009; 56(4):748–58. [PubMed: 19406703]
42. Gennisson JL, et al. Viscoelastic and anisotropic mechanical properties of in vivo muscle tissue assessed by supersonic shear imaging. *Ultrasound Med Biol*. 36(5):789–801. [PubMed: 20420970]
43. Couade M, et al. In vivo quantitative mapping of myocardial stiffening and transmural anisotropy during the cardiac cycle. *IEEE Trans Med Imaging*.

44. Urban MW, Chen S, Greenleaf J. Harmonic motion detection in a vibrating scattering medium. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2008; 55(9):1956–74. [PubMed: 18986892]
45. Kolipaka A, et al. Magnetic resonance elastography as a method for the assessment of effective myocardial stiffness throughout the cardiac cycle. *Magn Reson Med*. 64(3):862–70. [PubMed: 20578052]
46. Bercoff J, Tanter M, Fink M. Supersonic shear imaging: a new technique for soft tissue elasticity mapping. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2004; 51(4):396–409. [PubMed: 15139541]

Figure 1. Principle of SDUV: a push transducer is the source of radiation force that induces harmonic shear wave propagation in tissue; the motion is measured by a pulse-echo (detect) transducer.

Figure 2.

The mechanical system consisting of the LV free wall myocardium surrounded by blood on one side and the pericardial fluid on the other was modeled as an isotropic viscoelastic plate submerged in a noncompressible fluid undergoing antisymmetric motion.

Figure 3.

Figure 3a – Experimental setup for the LDUV approach (not to scale): a mechanical actuator (shaker) was used to excite Lamb waves in the sample; pulse echo transducer was used to detect the motion. The gelatin was used as a stabilizer.

Figure 3b – Experimental setup for the embedded sphere approach: a focused ultrasound transducer (push) transducer is used to apply constant force on a solid sphere embedded in the medium of interest at varying frequencies. A pulse echo transducer was used to detect the motion.

Figure 4.

Figure 4a – FEM simulation of a viscoelastic plate submerged in water due to cylindrical antisymmetric Lamb wave excitation at 200 Hz: displacement map of a Lamb wave propagating away from the point of excitation at $(x,t) = (0,0)$.

Figure 4b – 2D FFT of the propagation data reveals strong peaks at 200 Hz, which can be used to calculate the Lamb wave speed by measuring the phase gradient and Equation (1). Another way to calculate the speed is by multiplying the frequency (f) and wave number ($1/k_x$) coordinates of the peak in the 2D FFT plot.

Figure 5. Lamb wave dispersion equation in blue is fit to the finite element simulation dispersion data shown in red. Parameters defining the material properties of the solid were used to produce the dispersion curve based on the analytic expression in Eq. 47.

Figure 6.

Figure 6a – Phase map of the gelatin plate sample for the given excitation frequencies monochromatic Lamb wave propagates in the x -direction. The phase is fairly constant in throughout the thickness of the sample (z -direction), characteristic of the zeroth mode of the anti-symmetric (A_0) Lamb wave.

Figure 6b – Lamb wave dispersion in a gelatin plate: experimentally obtained shear wave dispersion curves are shown as red circles. Lamb wave model (blue line) was fit to the data to obtain the values of elasticity and viscosity μ_1 and μ_2 . Dispersion measurements were made in four orthogonal directions in the r -plane and panels A, B, C and D correspond to $+x$, $-x$, $+y$ and $-y$ directions.

Figure 7.

Lamb wave dispersion in a urethane rubber plate: experimentally obtained shear wave dispersion curves are shown as red circles. Lamb wave model (blue line) was fit to the data to obtain the values of elasticity and viscosity μ_1 and μ_2 .

Figure 8.

Figure 8a – Phase map of an excised porcine LV free wall myocardium sample for the given excitation frequencies. Monochromatic Lamb wave propagates in the x -direction. The phase is fairly constant in throughout the thickness of the sample (z -direction), characteristic of the zeroth mode of the anti-symmetric (A_0) Lamb wave.

Figure 8b – Lamb wave dispersion in the excised porcine LV free wall myocardium: experimentally obtained shear wave dispersion curves are shown as red circles. Lamb wave model (blue line) was fitted to the data to obtain the values of elasticity and viscosity μ_1 and μ_2 . Dispersion measurements were made in four orthogonal directions in the r -plane and panels A, B, C and D correspond to $+x$, $-x$, $+y$ and $-y$ directions.

TABLE I
Elasticity and Viscosity Of Gelatin, Urethane Rubber and Excised Porcine Left-Ventricular Myocardium

Elasticity and viscosity of urethane rubber and gelatin phantoms were obtained using the Lamb wave Dispersion Ultrasound Vibrometry (LDUV) method and the embedded sphere method. The LDUV values are reported as the mean $\pm$ the standard deviation of measurements made in four orthogonal directions. The embedded sphere uncertainty was based on the percent error of the technique as described in the Methods section. Elasticity and viscosity of an excised porcine LV free-wall myocardium are shown for comparison.

Measurement Method	Elasticity (μ_1) (kPa)	Viscosity (μ_2) (Pa·s)
Gelatin, LDUV	15.9 $\pm$ 0.5	0.7 $\pm$ 0.4
Gelatin, Embedded Sphere	16.6 $\pm$ 0.3	0.5 $\pm$ 0.02
Urethane, LDUV	45.1 $\pm$ 1.4	6.5 $\pm$ 0.8
Urethane, Embedded Sphere	46.6 $\pm$ 3.2	5.7 $\pm$ 0.8
Excised Porcine Myocardium	16.5 $\pm$ 1.3	6.0 $\pm$ 0.8