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Understanding Random Effects in Group-Based Trajectory Modeling: An Application of Moffitt's Developmental Taxonomy

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Abstract

The group-based trajectory modeling approach is a systematic way of categorizing subjects into different groups based on their developmental trajectories using formal and objective statistical criteria. With the recent advancement in methods and statistical software, modeling possibilities are almost limitless; however, parallel advances in theory development have not kept pace. This paper examines some of the modeling options that are becoming more widespread and how they impact both empirical and theoretical findings. The key issue that is explored is the impact of adding random effects to the latent growth factors and how this alters the meaning of a group. The paper argues that technical specification should be guided by theory, and Moffitt's developmental taxonomy is used as an illustration of how modeling decisions can be matched to theory.

Keywords

Group-based trajectory modeling; Random effects; Developmental theories of crime; Life-course theory

Introduction

When Terrie Moffitt (1993) first proposed her developmental taxonomy of delinquency 15 years ago, there were limited methods to test her theory. A group-based trajectory modeling approach with the ability to group developmental trajectories into meaningful clusters while simultaneously predicting group membership was not widely practiced in criminology until Nagin and Land (1993) introduced their technique. Since its presentation, many life course and developmental criminologists have utilized the methodology successfully. A variation of the technique, general growth mixture modeling, was also catching on in other disciplines, such as education and psychology (e.g., Muthén, 1989; Muthén et al., 2002; Stoolmiller, 2001; Stoolmiller et al., 2005; van Lier, Muthén, van der Sar, & Crijnen, 2004). With the development of the new modeling theories and statistical software, increasingly advanced and complex methods became available for developmental theory testing. The differences between the modeling strategies may appear small and insignificant, but they have the potential to significantly impact both theoretical and empirical findings if they are integrated into the literature.

Currently, statistical models can be very sophisticated, which brings up the question as to whether the methodology is currently more advanced and developed than the theories it is testing. When testing a developmental theory, in any discipline, that characterizes groups, it is important to consider the theoretical meaning of the group that is being tested—is it a group of people who change in an identical manner, or will group members mature in a

similar way, but differ in their development along some known or unknown distribution? If the group members vary from one another, would each group demonstrate an equivalent amount of variation? These questions have been largely ignored in the literature, most likely because developmental theories do not explicitly specify how a group is defined in respect to its heterogeneity (e.g., Loeber, 1991; Moffitt, 1993; Patterson, 1996).

This paper examines the theoretical meaning of a trajectory group and the impact of including random effects. As this can be a rather ambiguous discussion, Moffitt's (1993) developmental taxonomy is used as an example as to how theory can guide model selection. Moffitt's taxonomy is now 15 years old and has been widely tested in both criminology and psychology, but some of the details that are important for an appropriate test have yet to be worked out, in particular, the degree of within-group heterogeneity, if any, that should be expected. This paper will examine how modeling decisions can be made using Moffitt's theory as an illustration of a theory-based model selection process. However, the dependent variable and application of Moffitt's theory are secondary to this paper's main goal, which is to illustrate the theoretical and empirical importance of thinking through the variance specification prior to utilizing these methodologies.

Literature Review

Over 80 studies on delinquency and criminal behavior have used some form of group-based trajectory modeling, with most using Moffitt's theory or undefined developmental taxonomy as an organizing theoretical framework (Piquero, 2008). The general approach of dividing subjects into groups based on developmental trajectories of any particular outcome is an extension of two statistical methodologies, multi-level modeling and group-based approaches. Group-based trajectory modeling separates individuals into latent groups of similar developmental trajectories to examine group differences in the change of a particular outcome over time.

These empirical tests have identified different developmental trajectories of antisocial behavior and emphasized the importance of examining various phases of the offending cycle from onset to desistence (e.g., Brame, Nagin, & Tremblay, 2001; Broidy *et al.*, 2003; Bushway, Thornberry, & Krohn, 2003; Chung, Hill, Hawking, & Gilchrist, 2002; D'Unger, Land, McCall, & Nagin, 1998; Lancourse, Nagin, Vitaro, Claes, & Tremblay, 2003; Laub, Nagin, & Sampson, 1998; Loeber, 1982; Maughan, Pickles, Rowe, Costello, & Angold, 2000; Nagin, Pagani, Tremblay, & Vitaro, 2003; Nagin & Tremblay, 1999; 2001; Piquero, Brame, Mazerolle, & Haapanen, 2002; Sampson & Laub, 2003; Tremblay *et al.*, 2004). Distinct groups of offending trajectories have been replicated in multiple longitudinal studies on criminal behavior during various historical periods and using diverse sampling methods. Longitudinal studies on delinquency and criminal behavior in the United States, Canada, Puerto Rico, England, Scotland, Denmark, Sweden, Finland, China, Japan, Switzerland, Australia, New Zealand, all of which employ different methodologies, support the conclusion that groups of individuals that follow similar patterns of behavioral development can be empirically identified (see Piquero, 2008, for a review).

An important subject that has not been discussed in these studies is the empirical and theoretical definition of a trajectory group. Almost all of the studies employ an estimation technique that defines trajectory group in a very narrow manner, that is as a group of people that follow an identical developmental pathway where only random error accounts for each individual's deviation from their group average. Regardless of the manner in which the model is defined, trajectory group members will never follow their trajectory perfectly (Nagin, 2005; Nagin & Tremblay, 2005a, 2005b), but the question of modeling within-group heterogeneity has only begun to be addressed and has never been specifically discussed in

conjunction with a developmental theory (Kreuter & Muthén, 2007, 2008). Kreuter and Muthén (2008) demonstrate the utility of applying different modeling techniques, some that incorporate within-group variation using criminological data, but acknowledge that ultimately model selection should rely on substantive theory. This paper will use Moffitt's developmental taxonomy as an example of how theory can provide direction in making such modeling decisions.

Terrie Moffitt's dual taxonomy of offenders (Moffitt, 1993; Nagin, Farrington, & Moffitt, 1995) is the most cited developmental theory that has been connected with this analytic approach in the criminological literature. Her theory was innovative in that it was the first to put forth the idea that there are distinct developmental clusters of trajectories of antisocial behavior that are the result of divergent etiologies. The most striking difference between the offender types is their continuity and discontinuity of antisocial behavior across age and environmental context. Life-course-persistent (LCP) offenders begin their antisocial behavior at a young age and continue to display these characteristics over their lives, whereas adolescence-limited (AL) offenders are involved in antisocial behavior only through their adolescent years. The two types of offenders have very different developmental trajectories and causal factors. LCP offenders begin manifesting antisocial behavior in infancy or childhood, and their etiology lies in a confluence of psycho-physiological and environmental deviance. AL offenders begin their deviant behavior in adolescence when their peers expose them to antisocial behavior and it is reinforced by coveted social rewards. Inherent in this theory is that there is one other group, those who abstain from any involvement in criminal behavior (AB).

Moffitt's theory does not specifically address the issue of group heterogeneity, and therefore this paper will attempt to translate her hypotheses into modeling strategies. First, the AB group is assumed to never engage in delinquent or antisocial behavior regardless of age or environmental context. Implicit in this statement is that the AB group would be homogeneous in their displays of no delinquency. The AL group, on the other hand, displays antisocial and delinquent behavior due to their experience of the "maturity gap" and exposure to delinquent peer role models. They might not display antisocial behavior across all situations because antisocial behavior is environmentally motivated and used as a means to gain social status. Each individual in this group would have a unique experience with the maturity gap because there is variation in ages at which physical and social maturity occur. The implications for modeling would be that there would be a substantial amount of trajectory heterogeneity within this group.

The most ambiguity is in the LCP group, whose members, it is hypothesized, begin their antisocial behavior at a young age and display antisocial behavior across environmental and developmental stages due to a convergence of ecological and psycho-physiological difficulties. Moffitt does not specify the absolute level of delinquency that should be observed at any period, except that it remains "high." It is not clear how high their levels should be, especially when considering different manifestations of antisocial behavior. It is less clear as to whether this group would be homogenous; for instance, could an LCP offender display antisocial behavior at lower levels throughout his/her life course? Could there also be an age-graded process, such that their antisocial behavior declines over time but is still exhibited? It is conceptually unclear how much variation would be expected in the LCP trajectory group.

In the literature, the within-group heterogeneity issue has been largely overlooked in tests of Moffitt's theory, and the groups have been modeled with no variation, with the notable exception of one recent paper (Odgers et al., 2008). In this recent test of Moffitt's theory, group random effects were added, but their empirical and theoretical importance was not

discussed. In fact, random effects were fixed to be class invariant for model parsimony, which means that each group was as homogenous as the others. As discussed above, this seems theoretically incongruent. It appears that the field is moving in the direction of including variation in the latent growth terms, but the manner in which random effects are added remains unspecified, other than remarks that the decisions should be theory-guided (Kreuter & Muthén, 2007; 2008). This study seeks to fill that gap and demonstrate how a theory could guide model selection.

Current Study

Sample

The current research study uses data from the second generation of the Johns Hopkins University Prevention and Intervention Research Center's (PIRC) prevention intervention trials.¹ Two classroom-based, universal preventive intervention programs were fielded in Baltimore City schools with a focus on the early risk behaviors of poor achievement and aggressive and shy behavior and their distal correlates of antisocial behavior, substance abuse, and anxious and depressive symptoms. In the fall of 1993, an epidemiologically defined sample of 678 urban first-graders were recruited from Baltimore and followed for 10 years. There was complete information on an average of 77% of the individuals at each of the data collection periods used for this paper (Range=100%–58%, SD=12%). It does not appear that missingness is related to early aggression scores, and in terms of more distal outcomes, there were no differences in those surveyed and those lost to follow-up at waves six through eight with respect to teacher ratings, academic achievement, race/ethnicity, sex, or free lunch status (Furr-Holden, Ialongo, Anthony, Petras, & Kellam, 2004). Therefore missing values were generated under the assumption that they were missing at random (MAR). For a review of the major findings from this study, see Furr-Holden et al., 2004, Ialongo et al., 1999, Lambert, Ialongo, Boyd, & Cooley, 2005, and Storr, Ialongo, Kellam, & Anthony, 2002.

Data

The main dependent variable in this paper is teacher reports of delinquency, measured by The Teacher Observation of Classroom Adaptation - Revised (Werthamer-Larsson, Kellam, & Wheeler, 1991), which was administered annually (except during the 4th and 5th grades) from 1st through 12th grades. Eleven developmentally appropriate items were assessed in first through third grade (stubborn, breaks rules, harms or hurts others physically, harms or damages property on purpose, breaks things, yells at others, takes others' property, fights, lies, talks back to adults/disrespectful, teases classmates) and five items in 6th through 12th grades (breaks rules, hurts others physically, damages other people's property on purpose, takes others' property, and lies) to determine the delinquency score. While the scales vary across the two developmental periods, previous research has validated that they represent the same latent construct (Werthamer-Larsson et al., 1991). The coefficient alpha for the scale was .94 for the 1st generation of JHU PIRC trials (Kellam, Rebok, Ialongo, & Mayer, 1994).

¹The JHU PIRC second generation work has been supported by grants from the National Institute of Mental Health (Epidemiologic Prevention Center for Early Risk Behaviors, NIMH 5 PO MH38725, Sheppard G. Kellam, P.I.; Periodic Follow-up of Two Preventive Intervention Trials, RO 1 MH57005-02A, Nicholas S. Ialongo, P.I.) and the National Institute on Drug Abuse (NIDA RO1 DA11796-01A1, Nicholas S. Ialongo, P.I.). The principal collaborators have included Drs. Lisa Werthamer, Hendricks Brown, Sheppard G. Kellam, and Nicholas S. Ialongo. Nancy Karweit, Ph.D., Mary Alice Bond, M.A., Carolyn Webster-Stratton, Ph.D., Joyce Epstein, Ph.D., Irving Sigel, Ph.D., and Ruth Kandel, Ed.D. Each made significant contributions to the development of the second generation JHU interventions.

Models

Group-based trajectory models of delinquency are estimated to successively test different variance structures in the model constraints using MPlus (Muthén, 1989; Muthén & Muthén 1998–2004).² There are many different ways in which to model the variance and covariance structures of the latent growth factors that determine how strictly group members follow the overall group pattern, including: (1) permitting the variance of the intercept and the slope growth factors to differ across class, (2) freeing the variance of the intercept and slope, but constraining them to be equal across classes, and (3) restricting the variance in the intercept and slope factor to zero.³ The initial model estimates the least restrictive model with the rest of the models successively restricting different variance components within and between the groups.⁴ The final model is the most restricted, with all of the variances set to zero (i.e., no random effects are specified), which is the traditional group-based trajectory model used by criminologists.⁵

One of the greatest challenges in group-based trajectory modeling is the actual model selection, since there is no single generally accepted statistical test of model fit. Nagin (2005) and Nylund, Asparouhov, & Muthén (2007) recommend examining a wide variety of statistical criteria in order to select a model: the fit statistics, model parsimony, estimation problems, meaningfulness of class prevalence, and the theoretical fit of trajectories. The specific fit indicators that are used for model identification and other statistical measurements used in the following analyses are all described in published papers: Akaike Information Criterion (AIC: Akaike, 1987), Bayesian Information Criterion (BIC: Weakliem, 1999), Entropy (Muthén et al., 2002), Lo-Mendell-Rubin Test (LMRT: Lo, Mendell, & Rubin, 2001), Bootstrap Likelihood Difference Test (BLRT: McLachlan & Peel, 2000; Nylund et al., 2007), and Pseudo Class Membership (Bandein-Roche, Miglioretti, & Rathouz, 1997).

Model Validity

As the purpose of the trajectory groups is to summarize the behaviors of a set of individuals, group membership needs to be meaningful. An important part of validating the methodology is through checks of other antisocial or delinquent behavioral patterns. Regardless of how a model is defined, group members will never follow their trajectory groups perfectly (Nagin, 2005; Nagin & Tremblay, 2005a, 2005b), but this may be especially true when the variation (random effects) is specified as part of the model. As group membership may change when group members are allowed to vary, the degree of heterogeneity within a group becomes a key issue (Kreuter & Muthén, 2007; 2008; Raudenbush, 2005). Therefore, the validity of group membership for each of the models must be evaluated separately. One generally accepted method of validating group membership is through creating profiles of the trajectory group members to characterize them.

²For all of the following analyses the time points were fixed incrementally based on the timing of their administration. Each of the models was estimated allowing for the residual variance between the fall and the spring semesters to correlate because the same teacher provided scores at both of these time points. As suggested by Muthén and Muthén (1998–2004), each of the analyses used automated multiple starting values in the optimization in order to reduce the probability that the solutions are based on local rather than global optima. Intervention status was not modeled because its inclusion did not increase model fit and as the interventions were randomized using schools as blocking factors and lasted only one year.

³Models that allowed the between-group estimation of the intercept variance only and the slope variance only were also explored. The parameters were statistically equivalent to the models in which both were free to be estimated separately. The model with varying variance in both parameters was a statistically better fit than the ones that only allowed it in one [Varying intercept invariant slope: $\chi^2(1)=49.87$, $p<.001$; varying slope invariant intercept: $\chi^2(1)=22.81$, $p<.001$] and therefore will not be further examined in this paper.

⁴The quadratic slope factor did not have enough variation to make modifications meaningful and was therefore not explored.

⁵This model mimics the SAS Proc Traj approach, where all the variances are set to zero and residuals are fixed to be equal at each time point. More details are available through the National Consortium of Violence Research at www.ncovr.org (Jones & Nagin, 2005; Jones, Nagin, & Roeder, 2001).

Group profiles are compared in their manifestations of alternative measurements of delinquency (school suspension and tobacco, alcohol, marijuana, and illicit drug use onset) to determine if the different model specifications change the concurrent validity of group membership. Research is replete with studies demonstrating the strong correlation between substance use and delinquency (e.g., Gillmore et al., 1991; Hawking et al., 2000; Loeber, 1990; Slade et al., 2008). In fact, the relationship between early age of onset of initial substance use and engaging in multiple health risk behaviors among young adolescents has been documented in multiple studies, so for this study, early onset of substance use was used as a face validity check.⁶

Results

Models

The first and least restrictive approach relaxes the between-group variance restrictions by allowing the variance in both the intercept and the slope factors to differ by class. This model is the most complex with regard to the latent variance and covariance structures. It allows the intercepts and slopes between the different classes to have different variances, and therefore one class may comprise a relatively homogeneous group of individuals with respect to their initial aggression ratings and slopes, whereas another class can be made up of a relatively heterogeneous group of individuals. Under these least restricted variance assumptions, a three-class model again appears to be the optimal solution when considering all the fit indices (see Table 1).

When allowing heterogeneity in the intercept and slope factors, generally fewer numbers of latent classes are needed to capture the different growth trajectories; however, this must be balanced against a less parsimonious model with a more complex set of parameters necessary to model the variation around the latent growth factors. For this approach, variance is allowed to be estimated in the most restricted manner around the intercept and slope factors because the variance in each of the classes is constrained to be equal. In other words, each group is as heterogeneous as the others with respect to their variation around their group intercept and slope factors. A three-class model appears to be the optimal solution when considering all the fit indices (see Table 1).

The final approach to group-based trajectory modeling that was explored is the most restricted and commonly used model by criminologists (Jones, Nagin, & Roeder, 2001; Nagin, 2005; Nagin & Land, 1993). This method does not allow variation in the latent growth factors within or across classes. The growth model was run using one through five classes and each model's fit indices are summarized in Table 1. Following the suggestions offered by Nylund et al., 2007, the five-class model was selected as the model of best fit because while most of the fit indices (AIC, BIC, and SSA BIC) continued to decrease, the LVMR is significant for this condition, meaning that the five-group model is a better fit than a four-group model.

The first two approaches, which include random effects, arrived at three-group solutions, which are nested models and therefore can be compared using a likelihood ratio test. The approach that allowed the variance to be estimated separately in the intercept and slope factors for each group was a significantly better fit, even after balancing the loss of power due to additional parameter estimation (Approach 1 vs. Approach 2: $\chi^2(2)=104.95, p<.001$). These findings are not surprising, as freeing parameters will almost inevitably result in a significantly better fitting model. There are no statistical tests with the ability to compare

⁶Each time period contains a slightly different number of subjects, based on the different students that were present in school the days that the questionnaire was filled out.

across models with different numbers of classes while changing the variance structures, which makes it impossible to make a direct comparison of model fit between the first two approaches and the third since the class enumerations are different.

Inspecting the parameters and class proportions produced by the three approaches in Table 2, interesting similarities and differences can be observed. Each model includes a group that escalates through elementary school (Group 1 “Escalators”: 24.4% of Approach 1, 9.3% of Model 2, and 17.6% of Approach 3), with Approach 3 displaying an additional group that fits this developmental pattern (Group 4: 5.4%). All of the models also have a small high and declining group, “Decliners.” This group is made up of around 10% of the sample in Approaches 1 and 2, whereas the five-group model has two groups that fit a declining pattern (Group 2 “High Decliners”: 6.9% and Group 5 “Low Decliners”: 13.0%). In each model, there is also a low non-escalating group that constitutes the majority of the sample (Group 3 “Low Null Growth”: 64.5% of Approach 1, 80.7% of Model 2, and 57.2% of Approach 3).

The differences between the two approaches with three groups are subtle, but still very important (see Figure 1). The greatest differences between Approaches 1 and 2 are in the slope and class proportion of the escalating class (Group 1). A smaller proportion of the sample is classified into Group 1 when the variance is constrained to be equal across groups (Approach 2) and the slope of the group is steeper. In fact, when cross-tabulating pseudo class membership⁷ by model, it is clear that the majority of the sample is classified into the same trajectory group, with only 13.7% (N=93) of the sample switching group membership (see Table 3). Of the “switchers,” 88% are individuals who were classified in Group 1 in Approach 1 (“Escalators”) and Group 3 in Approach 2 (“Low Null Growth”).

These individuals change group membership solely as a result of the altering of the latent variance restrictions, and therefore it is instructive to look at their individual trajectories to examine how and why this happened. Figure 2 illustrates the individual growth curves of a random sample of 25% (n=22) of those who switched between “Escalators” and “Low Null Growth,” with observed means and confidence intervals at each time period.⁸ Looking at the plots, it becomes apparent that these individuals do not fit the pattern of Group 3 from Approach 2 (“Low Null Growth”), but appear more like those who belong to Group 1 from Approach 3 (“Escalators”). In fact, 11 of the 12 observed means fall within the confidence intervals of Group 1 from Approach 3, but overlap with the confidence intervals for Group 3 from Approach 2 at only the initial data collection period. The confidence intervals clearly illustrate that constraining the variance within groups to be the same across the groups, variation in the low non-escalating class had to be artificially inflated by including members of another group in order to make it equivalent to the variation in the other groups.

Figure 3 displays the estimated means from Approach 3 with five groups (no variance) and the estimated means from Approach 1 (varying variance). This graph helps to highlight one of the many issues in model selection when considering a group-based trajectory model—there are two sets of groups that follow similar developmental patterns, Groups 1 and 4 and Groups 2 and 5. Groups 1 and 4 both increase through middle school and then decline through high school, with the main difference between these groups being their intercepts. In fact, when random effects are freely estimated in Approach 1, Group 1 follows the same trajectory pattern but has an intercept between these two trajectory groups. The same is true

⁷Pseudo class assignment was used to classify individuals into actual trajectory groups (see Bandeen-Roche, Miglioretti, & Rathouz, 1997). Pseudo class assignment is determined based on a random draw using the estimated conditional posterior probabilities of group membership, a set of probabilities that collectively calculate a specific individual’s chance of belonging to each of the model trajectory groups (see Nagin, 2005).

⁸The CIs were computed using the class-specific (c) variation at each time (t) period: $95\% \text{ CI}_{ct} = \mu_{ct} \pm 1.96 \times \sqrt{(\sigma_{ct}/n_{ct})}$

for Groups 2 and 5; they both decline through the study period but differ dramatically in their intercepts with Group 2 beginning at 3.97 and Group 5 at 2.47. When variance is estimated in Approach 1, Group 2 follows the same developmental trajectory and has an intercept between Group 2 and 5. So while each of these groups can be distinguished empirically, they may belong to the same developmental thematic group.

When pseudo class membership is compared across classes, not all of those who belong to the developmentally congruent groups in Approach 1 also belong to the developmentally similar group in Approach 3 (see Table 4). Eighty-five percent of the individuals are still classified in similar groups in terms of their development; but unlike when Approaches 1 and 2 were compared, there is no obvious pattern as to how individuals shifted between groups, and examining their individual trajectories provides little additional insight. This paper is an application of Moffitt's theory, and therefore five groups were collapsed into the three developmentally similar groups. The parameters and class proportion are not statistically equivalent to either of the previous two approaches when the five groups are collapsed into three.⁹

Model Validity

When examining the percentage of individuals who were suspended one or more times during each school year, there is a difference across groups regardless of the variance structure of the model (see Figure 4). There is evidence of the "Low Null Group," which is analogous to Moffitt's Abstainer Group (AB), having a much lower suspension rate than the other groups, which provides some degree of validity of group membership. This group difference is stable across models, but the 95% confidence intervals between the two other groups ("Escalators" and "Decliners") overlap at almost every time period.

When examining group substance use by approach, the group patterns hold consistent across all measures (tobacco, alcohol, marijuana, illicit drug, any drug initiation) and therefore only the results from tobacco use initiation are presented here (see Figure 5). The age of initiation does not appear to vary by group in any of the approaches, as would be theoretically expected. The majority of the LCP and AL group members have tried tobacco by the end of middle school, and only half of the Abstainers have experimented with tobacco by the end of high school, which is anticipated by Moffitt's theory. However, it brings up the question as to whether they can truly be called "Abstainers" if they are experimenting with substances. The LCP and AL groups are not distinguishable at the majority of data collection periods, which is not fully contradictory to Moffitt's theory, as she explained that, at times, the AL offenders might be even more delinquent than the LCP during the peak of their offending (Moffitt, 1993).

Discussion

Group-based trajectory modeling has become an increasingly popular tool to test life-course and developmental theories of crime; however, there are still some concerns about the theoretical appropriateness of group-based trajectory modeling, with conventional random effects growth curve modeling advocates and developmental taxonomists disagreeing about whether there are distinct subgroups. While this study does not seek to address that specific issue, it examines the implications of adding random effects within classes, which, in effect, combines the two approaches. The examples of three different ways in which to model

⁹If groups 1 and 4 are combined, their weighted intercept, slope, and quadratic are statistically equivalent to the estimates for Group 1 of Approach 1 ($i=1.44$ vs. 1.36 , $s=.33$ vs. $.30$, and $q=-.024$ vs. $-.021$, respectively). If Group 2 and 5 are combined, their weighted intercept, slope, and quadratic are not statistically equivalent to Group 2 of Approach 1. The intercept is dramatically lower (2.99 compared to 3.67) and the slope coefficient is half the size ($-.15$ compared to $-.32$).

variation presented above demonstrate that these modifications significantly impact the empirical findings, and this section will discuss their impact on the theoretical findings in the context of Moffitt's developmental taxonomy.

Empirical Implications

When random effects are included in the models, the three-group solution is selected (Approaches 1 and 2). Group 2, the group with the highest intercept, remains remarkably stable in its class proportions across the two models, with approximately one tenth of the sample following this pattern. Group 1, which exhibits an escalating slope, was larger in the model that restricted variance to be equal between the groups. This difference has a significant impact on the parameterization of Group 1, the escalating group, in each of the models. When the variation is restricted to be the same for each group (Approach 2), the group has a much greater slope because 12% of the sample switched from the low escalating group (Group 1) to the non-escalating group (Group 3). When examining this "switching" group's observed means and a sample of their individual trajectories, Figure 3 demonstrated that this group does, in fact, appear to be more similar to the escalator group (Group 1) than the low null growth group (Group 3).

Under the third approach where within-group variation is restricted to zero (the traditional group-based trajectory approach), a greater number of groups are needed to capture the variation in trajectories. While this is not surprising, it is important to note because if the model definitions are, in part, driving the number of classes, their theoretical validity is suspect. Of course, it is important to consider the guidelines for model selection, which include theoretical significance (Nagin, 2005; Nylund et al., 2007), and recognize that this is not an exact science. It is not unusual for a criminologist to rely on theory to guide their model selection even when it does not result in a superior empirical fit; in fact, some argue that a statistical model can never solve what is fundamentally a theoretical issue (Sampson, 2006).

Comparing the pseudo class membership of the three-group models (Approaches 1 and 2), it is clear that the majority of the sample remains in the same group (87%), and that those who switch, do so in a predictable manner. When the variance is constrained to be equal for each of the classes (Approach 2), members of the escalating group (Group 1) are pulled into the low null growth group (Group 3). This is due to the fact that the model is requiring equal variation in each of the groups when the low null growth group (Group 3) is more homogenous than the other groups when it is free to be estimated separately (Approach 1). By requiring each group to have equal variation, the model must pull members into the more homogeneous group to create equality, which significantly alters the class parameterization.

The final approach, which arrived at a five-class solution, illustrates another of the many difficulties faced by empirical researchers who approach group-based trajectory modeling without a strong theoretical framework. Even when groups can be distinguished mathematically, they could still belong to a theoretically similar group, which illustrates the main research question of this paper: Are groups homogenous? There were two sets of groups that followed similar developmental patterns: two groups increased through middle school before beginning to decline in their delinquency and another two groups decreased in their displays of delinquency throughout the entire study period. As developmental taxonomic theories seek to explain the trajectory groups developing in a similar manner, it may be appropriate to consider them together. Should these sets of groups be considered separately, or are two of those groups simply variations on a theme, which would make it a three-group model?¹⁰ Theoretical relevance and importance should dictate these decisions. This exercise is based on an application of Moffitt's theory, so the groups are combined into a three-group model, but other theoretical perspectives may disagree with this decision.

With the fundamental issue of the validity and reliability of trajectory group membership at the center of dispute (see 2005: *American Academy of Political and Social Sciences Annals* (November), *Volume 602*: 6–258 and *Criminology*, *Volume 23*(4): 873–918), the validity of group membership is exceedingly important. Theoretically, a no-variance model should produce groups that are the most homogeneous and would display the largest differential manifestations of other delinquency measures and patterns, but this was not the case. Adding in random effects did not decrease the validity of group membership, which admittedly, was suspect in all the approaches. However, it is important to note that adding in these effects did not make it worse, as researchers have speculated that it might.

Most of the alternate measures of delinquency differed equally by group in each of the approaches, although the differences between group initiation rates were not tremendously large. The low null growth trajectories in each model (Group 3) consistently demonstrated lower incidence rates of substance use (tobacco, marijuana, and any illicit drug use), as well as a markedly reduced suspension rate by grade; however, their rates were well above what Moffitt's theory would predict if they were truly "Abstainers." Group 1, the declining group, consistently exhibited the highest rates of substance use initiation by grade, as well as suspension rates from fourth through seventh grade. It appears that this group experimented with substances at an earlier age, which suggests that membership in this group does have meaning in alternative measures of delinquency in the direction hypothesized by Moffitt. Overall, the concurrent validity of group membership in any of the approaches is still questionable at this point—while trajectory group does appear to predict suspension and tobacco and drug initiation rates, the differences are not as great as would be expected if group membership were a large predictor of these alternative delinquent behaviors. Nevertheless, it is clear that allowing variance, especially varying variance, does not reduce concurrent validity using these measures.

This discussion of concurrent validity all hinges on the assumption that these alternative delinquency patterns should be related to the patterns of delinquency; however, these might not be the most appropriate measures of validity. The largest group differences in delinquency appear in elementary school, and therefore it would follow that the largest differences in alternative delinquency measures may appear during this time frame as well, which were not explored in this paper. Additionally, there are no long-term outcomes to test group validity yet since this sample is only in their early 20s.

Theoretical Implications

The question of freeing additional parameters for estimation gets very tricky as the estimation technique is retrospective and data-driven, and consequently the more parameters that are freed, the more likely the findings could be based on data artifact. However, if it can be guided by a theory, this concern is less troublesome. These different variance structures have implications beyond the empirical changes discussed above; they alter the theoretical definition of a group by specifying how group members are similar with respect to their developmental trajectories.

For the no-variance restriction (Approach 3), a latent class is a collection of individuals who follow the same pattern, with only random error producing differences in the trajectories of group members. In other words, group membership alone (along with their theoretical predictors) is the only factor driving trajectories. While no one would claim that each member of the trajectory group will follow their group flawlessly, theoretically the zero

¹⁰The groups considered as a variation on a theme is a similar approach to the non-parametric growth mixture model proposed in Kreuter & Muthén, 2008

variance method will produce group members that are as similar as possible (Nagin, 2005). If that theoretical assumption is modified and group members are allowed to vary from one another along a distribution, the next question becomes, how much heterogeneity should be allowed and how will that affect the validity of the group?

While it might seem that restricting variance within each group to be equal would be an artificial assumption (Approach 2), if there is no clear theoretical reason to allow variance to be estimated for each group separately, it would make sense to make them equal both for the sake of parsimony and the fact that freeing this parameter could leave the model vulnerable for overestimation. This variance-invariance approach was used in the last big test of Moffitt's theory (Odgers et al., 2008). However, in the current set of analyses, it can be seen that requiring the variances to be the same in each group, the variation in the low null growth group is artificially inflated. This simply illustrates how a minor modeling specification can alter the empirical findings, which needs to be considered and worked out before complicating models further (e.g., adding covariates, additional dependent measures, outcomes, etc.).

Moffitt's theory (1993) does not expressly address the issue of variability; however, some of her hypotheses allude to the type of heterogeneity that would be expected in the groups. Moffitt describes a group of abstainers (AB) who never engage in delinquent behavior, and therefore the group should be relatively or entirely homogeneous. Her next group, the adolescence-limited offenders (AL), shifts in and out of delinquency due to a disconnect between social and physical maturity along with deviant peer role models. Implicit in this statement is that there would be within-group variability because each individual will have a different experience with the maturity gap and level of exposure to deviant peers from whom they mimic the delinquent behavior. Finally, the life-course-persistent offenders (LCP) should exhibit antisocial behavior throughout the course of their lives and across their environmental domains. Her theory does not provide many hints as to how diverse this group would be expected to be.

Taken together, it seems logical that in order to test Moffitt's theory, within-group heterogeneity should be estimated separately for the AL and LCP groups because there is good reason to believe that there will be a large amount of variation in the AL group. The AB group may not need any variation modeled at all. Empirical support was found for this deduction as one group was found to be more homogeneous than the others once variation is allowed to be estimated separately (Approach 1).¹¹ The standard error of the intercept is half as large in Group 3 (the group that most closely resembles Moffitt's AB group), compared to the other groups, and the standard error of the slope in this group is too small to even be estimated. In other words, individuals who did not exhibit aggressive/disruptive behavior over the course of the study had more similar trajectories than those who either escalated (the group that approximates Moffitt's AL offenders) or deescalated in their behavior (the group that most closely follows Moffitt's hypothesized trajectory for LCP offenders).

It makes theoretical sense that groups that change in their displays of antisocial behavior over time would be more heterogeneous because there is the potential for many theoretically relevant time-varying and time-invariant factors to influence the intercept and slope of an individual's trajectory. In accordance with Moffitt's theory, factors such as exposure to deviant peers and experiences with the maturity gap may influence their individual trajectories, in addition to predicting their group membership in the AL group. For the LCP offenders, individual differences, either in their environmental or psych-physiological

¹¹Some may argue that the abstainer group should be modeled with zero variation. This approach was attempted, but the models failed to converge.

dysfunction, or other constructs may vary over time and influence their trajectories as they change.

Conclusions

With the advancement in modeling theory and statistical software in group-based trajectory modeling, it is now possible to manipulate models in very complex ways. This is a remarkable case of a methodology being developed to answer a theoretical question that has, in turn, outpaced the theory. What began as a practice inspired by theory became a technique that has surpassed theory in its sophistication, and methodological decisions are being dictated by empirical, rather than theoretical, appropriateness. It should be noted that this paper explored only a narrow range of modeling options available to developmental theorists and almost every conceivable combination of variance restrictions and population distributions can now be estimated. For examples of some of the more complex models, such as the non-parametric growth mixture models, see Kreuter & Muthén (2008). These options challenge criminologists to think creativity, and it is important that model selection is based on theory and not solely on empirical fit.

Allowing variation within and/or between groups also opens modeling opportunities that are not possible with the traditional models that do not specify variation. With these more complex models, differences within groups can be explored, and there is great potential for theory growth and integration. While covariates can predict group membership using any of the models presented in this paper, only those with estimated variance can include mediating effects and explore within-group differences. Using these models, one set of covariates may predict group membership, while a completely different set may be related to differences in intercept or slope within groups. The modeling opportunities are almost endless, which yet again emphasizes that the statistical methods currently available are more advanced than the theories that are being tested. Perhaps the most important implication of adding within-group variation is the additional flexibility that random effects models provide. The issue of within-group differences is never discussed in Moffitt's theory, but it is possible that one group of factors may predict group membership and an entirely different group of variables is influencing within-group variation. Modeling these within-group effects can only be explored if random effects are included.

Biography

Jessica Saunders is an Associate Social/Behavioral Scientist at the RAND Corporation in Santa Monica. She received her Ph.D. in criminal Justice from John Jay College of Criminal Justice in 2007. Her research interests include developmental criminology, immigration and crime, and quantitative methods.

References

- Akaike H. Factor analysis and AIC. *Psychometrika*. 1987; 52:317–332.
- Bandeen-Roche K, Miglioretti DL, Rathouz PJ. Latent variable regression for multiple discrete outcomes. *Journal of the American Statistical Association*. 1997; 92:1375–1386.
- Brame R, Nagin DS, Tremblay RE. Developmental trajectories of physical aggression from school entry to late adolescence. *Journal of Child Psychology and Psychiatry*. 2001; 42:503–512. [PubMed: 11383966]
- Broidy LM, Nagin DS, Tremblay RE, Bates JE, Brame R, Dodge KA, Fergusson DM, Horwood JL, Laird R, Lynam DR, Moffitt TE, Petit GS, Vitaro F. Developmental trajectories of childhood disruptive behaviors and adolescent delinquency: A six-site, cross-national study. *Developmental Psychology*. 2003; 39:222–245. [PubMed: 12661883]

- Bushway SD, Thornberry T, Krohn MD. Desistence as a developmental process: A comparison of static and dynamic approaches. *Journal of Quantitative Criminology*. 2003; 19:289–297.
- Chung JJ, Hill KG, Hawking D, Gilchrist LD. Childhood predictors of offense trajectories. *Journal of Research in Crime and Delinquency*. 2002; 39:60–90.
- D'Unger AV, Land KC, McCall PL, Nagin DS. How many latent classes of delinquent/criminal careers? Results from Mixed Poisson Regression Analysis of the London, Philadelphia, and Racine Cohort Studies. *American Journal of Sociology*. 1998; 103:1593–1630.
- Furr-Holden DD, Ialongo NS, Anthony JC, Petras H, Kellam S. Developmentally inspired drug prevention: Middle school outcomes in a school-based randomized prevention trial. *American Journal of Community Psychology*. 2004; 27:599–641.
- Gillmore MR, Hawking JD, Catalano RF, Day LE, Moore M, Abbott R. Structure of problem behaviors in preadolescence. *Journal of Consulting and Clinical Psychology*. 1991; 59(4):499–506. [PubMed: 1918552]
- Hawking, JD.; Herrenkohl, TL.; Farrington, DP.; Brewer, D.; Catalano, RF.; Harachi, TW.; Cothorn, L. Predictors of youth violence. Washington, DC: Office of Juvenile Justice and Delinquency Prevention NCJ; 2000. p. 179065
- Ialongo NS, Werthamer L, Kellam SG, Brown CH, Wang S, Lin Y. Proximal impact of two first-grade preventive interventions on the early risk behaviors for later substance abuse, depression, and antisocial behavior. *Drug and Alcohol Dependence*. 1999; 66:51–60. [PubMed: 11850136]
- Jones, R.; Nagin, DS. Advances in group-based trajectory modeling and a SAS procedure for estimating them. Pittsburgh, PA: Carnegie Mellon University; 2005. Working Paper
- Jones R, Nagin DS, Roeder K. A SAS procedure based on mixture models for estimating developmental trajectories. *Sociological Research and Methods*. 2001; 29:374–393.
- Kellam SG, Rebok GW, Ialongo N, Mayer LS. The course and malleability of aggressive behavior from early first grade into middle school: Results of a developmental epidemiologically-based preventive trial. *Journal of Child Psychological, Psychiatry, and Allied Disciplines*. 1994; 35:259–281.
- Kreuter, F.; Muthén, B. Longitudinal modeling of population heterogeneity: Methodological challenges to the analysis of empirically derived criminal trajectory profiles. In: Hancock, GR.; Samuelsen, KM., editors. *Advances in latent variable mixture models*. Charlotte, NC: Information Age Publishing, Inc; 2007.
- Kreuter F, Muthén B. Analyzing criminal trajectory profiles: Bridging multilevel and group-based approaches using growth mixture modeling. *Journal of Quantitative Criminology*. 2008; 24:1–31.
- Lambert SF, Ialongo NS, Boyd RC, Cooley MR. Risk factors for community violence exposure in adolescence. *American Journal of Community Psychology*. 2005; 36:29–48. [PubMed: 16134043]
- Lancourse E, Nagin D, Vitaro F, Claes M, Tremblay R. Developmental trajectories of boys' delinquent group membership and facilitation of violent behaviors during adolescence. *Developmental and Psychopathology*. 2003; 15:228–240.
- Laub JH, Nagin DS, Sampson RJ. Trajectories of change in criminal offending: Good marriages and the desistance process. *American Sociological Review*. 1998; 63:225–238.
- Lo Y, Mendell NR, Rubin DB. Testing the number of components in a normal mixture. *Biometrika*. 2001; 88:767–778.
- Loeber R. The stability of antisocial and delinquent child behavior: A review. *Child Development*. 1982; 53:1431–1446. [PubMed: 6756808]
- Loeber R. Development and risk factors of juvenile antisocial behavior and delinquency. *Child Psychology Review*. 1990; 10:1–41.
- Loeber R. Antisocial behavior: More enduring than changeable? *Journal of the American Academy of Child and Adolescent Psychiatry*. 1991; 30:393–397. [PubMed: 2055875]
- Maughan B, Pickles A, Rowe R, Costello EJ, Angold A. Developmental trajectories of aggressive and non-aggressive conduct problems. *Journal of Quantitative Criminology*. 2000; 16:199–221.
- McLachlan, G.; Peel, D. *Finite mixture models*. New York: John Wiley & Sons; 2000.
- Moffitt TE. Adolescence-limited and life-course-persistent antisocial behavior: A developmental taxonomy. *Psychological Review*. 1993; 100:674–701. [PubMed: 8255953]

- Muthén B. Latent variables modeling in heterogeneous populations. *Psychometrika*. 1989; 54:557–585.
- Muthén B, Brown CH, Masyn K, Jo B, Khoo ST, Yang CC, Wang CP, Kellam S, Carlin J, Liao J. General growth mixture modeling for randomized preventive interventions. *Biostatistics*. 2002; 3:459–475. [PubMed: 12933592]
- Muthén, B.; Muthén, L. *Mplus user's guide*. 3rd ed.. Los Angeles, CA: Muthén & Muthén; 1998–2004.
- Nagin, DS. *Group-based modeling of development*. Cambridge, MA: University Press; 2005.
- Nagin DS, Farrington D, Moffitt T. Life-course trajectories of different types of offenders. *Criminology*. 1995; 33:111–139.
- Nagin DS, Land KC. Age, criminal careers, and population heterogeneity: Specification and estimation of a nonparametric, Mixed Poisson Model. *Criminology*. 1993; 31:327–362.
- Nagin DS, Pagani L, Tremblay R, Vitaro F. Life course turning points: A case study of the effects of school failure on interpersonal violence. *Development and Psychopathology*. 2003; 15:343–361. [PubMed: 12931832]
- Nagin DS, Tremblay RE. Trajectories of boys' physical aggression, opposition, and hyperactivity on the path to physically violent and nonviolent juvenile delinquency. *Child Developmental*. 1999; 70:1181–1196.
- Nagin DS, Tremblay RE. Analyzing developmental trajectories of distinct but related behaviors: A group-based method. *Psychological Methods*. 2001; 6:18–34. [PubMed: 11285809]
- Nagin DS, Tremblay RE. What has been learned from group-based trajectory modeling? Examples from physical aggression and other problem behaviors. *Annals, AAPSS*. 2005a; 602:82–117.
- Nagin DS, Tremblay RE. Developmental trajectory groups: Fact or a useful statistical fiction? *Criminology*. 2005b; 43:873–904.
- Nylund KL, Asparouhov A, Muthén B. Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo study. *Structural Equation Modeling*. 2007; 14:535–569.
- Odgers CL, Moffitt TE, Broadbent JM, Dickson N, Hancox RJ, Harrington H, Poulton R, Sears MR, Thomson WM, Caspi A. Female and male antisocial trajectories: From childhood origins to adult outcomes. *Development and Psychopathology*. 2008; 20:673–716. [PubMed: 18423100]
- Patterson, GR. Some characteristics of a developmental theory for early-onset delinquency. In: Lenzenweger, MF.; Haugaard, JJ., editors. *Frontiers of developmental psychopathology*. New York: Oxford University Press; 1996. p. 81-124.
- Piquero, AR. Taking stock of developmental trajectories of criminal activity over the lifecourse. In: Liberman, A., editor. *The long view of crime: A synthesis of longitudinal research*. New York: Springer; 2008. p. 23-78.
- Piquero AR, Brame R, Mazerolle P, Haapanen R. Crime in emerging adulthood. *Criminology*. 2002; 40:137–169.
- Raudenbush SW. How do we study "What happens next"? *Annals, AAPSS*. 2005; 602:131–144.
- Sampson, RJ. Collective Efficacy Theory: Lessons learned and directions for future inquiry. In: Cullen, FT.; Wright, JP.; Blevins, KR., editors. *Taking stock: The status of criminological theory, Advances in criminological theory*. New York: Transaction Publishers; 2006. p. 147-167.
- Sampson RJ, Laub JH. Life-course desisters? Trajectories of crime among delinquent boys followed to age 70. *Criminology*. 2003; 41:555–592.
- Slade EP, Stuart EA, Salkever DS, Karakus M, Green KM, Ialongo N. Impacts of age of onset of substance use disorders on risk of adult incarceration among disadvantaged urban youth: A propensity score matching approach. *Drug and Alcohol Dependence*. 2008; 95:1–13. [PubMed: 18242006]
- Stoolmiller M. Synergistic interaction of child manageability problems and parent-discipline tactics in predicting future growth in externalizing behavior for boys. *Developmental Psychology*. 2001; 37:814–825. [PubMed: 11699755]
- Stoolmiller CM, Petras H, Ialongo N, Masyn KE, Hubbard S, Poduska J, Kellam S. A comparison of girls' and boys' aggressive - disruptive behavior trajectories across elementary school: Prediction

to youth adult antisocial outcomes. *Journal of Consulting & Clinical Psychology*. 2005; 74:500–510.

Storr CL, Ialongo NS, Kellam SG, Anthony JC. A randomized controlled trial of two primary school intervention strategies to prevent early onset tobacco smoking. *Drug and Alcohol Dependence*. 2002; 73:149–158.

Tremblay RE, Nagin D, Seguin J, Zoccolillo M, Zelazo P, Boivin M, Perusse D, Japel C. Physical aggression during early childhood: Trajectories and predictors. *Pediatrics*. 2004; 114:43–50.

van Lier PAC, Muthén B, van der Sar RM, Crijnen AAM. Preventing disruptive behavior in elementary schoolchildren: Impact of a universal classroom-based intervention. *Journal of Consulting and Clinical Psychology*. 2004; 72:467–478. [PubMed: 15279530]

Weakliem DL. A critique of Bayesian Information Criterion for model selection. *Sociological Methods and Research*. 1999; 27 359-197.

Werthamer-Larsson L, Kellam SG, Wheeler L. Effects of first-grade classroom environment on child shy behavior, aggressive behavior, and concentration problems. *American Journal of Community Psychology*. 1991; 19:585–602. [PubMed: 1755437]

Figure 1.
Approaches 1 and 2

Figure 2.
Estimated Group Means and Confidence Intervals with Individual Trajectories

Figure 3.
Models 1 and 3

Figure 4.
School Suspension Validity Check

Figure 5.
Tobacco Use Initiation Validity Check

Table 1

Model Enumeration Comparisons

	AIC	BIC	SSA BIC	Entropy	LVMR LRT
Approach 1: Varying Variance					
1 Class	10812.41	10893.76	10893.78	1.0	
2 Classes	10538.14	10646.60	10570.40	.60	279.1, p<.01
3 Classes	10294.04	10420.58	10331.67	.78	242.8, p<.01
4 Classes	10204.15	10348.76	10247.16	.72	94.3, p=.15
Approach 2: Invariant Variance					
1 Class	10812.41	10893.76	10893.78	1.0	
2 Class	10610.74	10710.16	10610.31	.82	207.9, p<.01
3 Classes	10394.99	10512.49	10429.93	.89	215.5, p<.01
4 Classes	10342.89	10478.46	10383.21	.86	57.88, p=.66
Approach 3: No Variance					
1 Class	12907.09	12929.69	12913.82	1.0	
2 Classes	11482.24	11522.91	11494.33	.90	1379.94, p<.01
3 Classes	11121.55	11180.30	11139.03	.84	355.07, p=.37
4 Classes	10938.88	11015.71	10961.73	.84	183.63, p<.01
5 Classes	10749.65	10844.55	10777.88	.85	131.21, p<.01
6 Classes	10701.65	10814.63	10735.26	.86	53.93, p=.54

Table 2**Model Parameterization Comparisons**

	Approach 1 Varying Variance	Approach 2 Invariant Variance	Approach 3 No Variance
Group 1: Escalators			
Intercept μ	1.36 (.10)	1.22 (.15)	1.31 (.11)
Intercept σ	.19 (.03)	.12 (.02) =	0.0 ⁰
Slope μ	.30 (.05)	.55 (.07)	.29 (.04)
Slope σ	.002 (.000)	.001(.00)=	0.0 ⁰
Quadratic μ	-.021 (.004)	-.042 (.005)	-.021 (.002)
Class Proportion	24.4%	9.3%	17.6%
Group 2: Decliners			
Intercept μ	3.67 (.21)	3.85 (.17)	3.97 (.19)
Intercept σ	.19 (.03)	.12 (.02) =	0.0 ⁰
Slope μ	-.32 (.05)	-.31 (.05)	-.23 (.07)
Slope σ	.002 (.000)	.001 (.00)=	0.0 ⁰
Quadratic μ	.013 (.003)	.012 (.003)	.003 (.005)
Class Proportion	11.1%	10.0%	6.9%
Group 3: Low Null Growth			
Intercept μ	1.32 (.05)	1.36 (.04)	1.22 (.02)
Intercept σ	.08 (.03)	.12 (.02) =	0.0 ⁰
Slope μ	.04 (.009)	.06 (.01)	.05 (.01)
Slope σ	.000 (.000)	.001(.00)=	0.0 ⁰
Quadratic μ	-.004 (.001)	-.005 (.001)	-.004 (.001)
Class Proportion	64.5%	80.7%	57.2%
Group 4: High Escalator			
Intercept μ			1.87 (.25)
Intercept σ			0.0 ⁰
Slope μ			.47 (.09)
Slope σ	NA	NA	0.0 ⁰
Quadratic μ			-.035 (.007)
Class Proportion			5.4%
Group 5: Low Decliner			
Intercept μ			2.47 (.17)
Intercept σ			0.0 ⁰
Slope μ			-.10 (.06)
Slope σ	NA	NA	0.0 ⁰
Quadratic μ			0.00(.004)
Class Proportion			13.0%

$0_{\cdot}$ Fixed to zero

$=$ Fixed to be equal across class

Table 3

Pseudo Class Membership by Approaches 1 and 2

Approach 1: Unequal Variation					
	Group 1			Group 2	
	Group 1	Group 2	Group 3	Group 1	Group 2
Approach 2: Equal	54	0	0	0	54
Variation	Group 2	1	66	0	67
	Group 3	82	10	465	557
Total	137	76	465	678	

Number of off diagonals [**bolded**] = 93 (13.7%)

Table 4

Pseudo Class Membership by Approaches 1 and 3

Approach 1: Unequal Variation				
	Group 1	Group 2	Group 3	Total
Group 1	85	2	19	106
Group 2	6	40	0	46
Group 3	3	0	396	399
Group 4	31	4	0	35
Group 5	12	30	50	92
Total	137	76	465	678

Number of off diagonals [**bolded**] = 96 (14.2%)