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Error Estimation of Nanoindentation Mechanical Properties Near a Dissimilar Interface via Finite Element Analysis and Analytical Solution Methods

Y. Zhao and T. C. Ovaert^{a)}

Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, IN 46556

Abstract

Nanoindentation methods are well suited for probing the mechanical properties of a heterogeneous surface, since the probe size and contact volumes are small and localized. However, the nanoindentation method may introduce errors in the computed mechanical properties when indenting near the interface between two materials having significantly different mechanical properties. Here we examine the case where a soft material is loaded in close proximity to an interface of higher modulus, such as the case when indenting bone near a metallic implant. Results are derived from both an approximate analytical quarter-space solution and a finite element model, and used to estimate the error in indentation-determined elastic modulus as a function of the distance from the apex of contact to the dissimilar interface, for both Berkovich and spherical indenter geometries. Sample data reveal the potential errors in mechanical property determination that can occur when indenting near an interface having higher stiffness, or when characterizing strongly heterogeneous materials. The results suggest that caution should be used when interpreting results in the near-interfacial region.

I. INTRODUCTION

Determination of the elastic modulus and hardness of materials during the nanoindentation test relies on use of the load vs. indentation depth curve,^{1–5} and is based on the mathematical solutions for contact of the semi-infinite half space. However, it can be expected that the results may not reflect the true mechanical properties when the indentation test is conducted near a dissimilar interface. An example of this is the case where one desires to determine the mechanical properties of bone near a metallic implant, such as titanium (as in the case of a threadless dental implant^{6–8}), or typical cobalt-chrome alloys used in orthopedic implants.^{9–10} Since bone remodels itself and adapts to applied loads,^{11–13} its properties very near the implant interface may be considerably different than its properties further away. Nonetheless, the mechanical properties at the (load-bearing) bone-implant interface are important for proper implant fixation and performance. Discrepancies in properties in the region of a dissimilar interface are the result of partial development of the displacement field that would normally occur in the half space contact problem. One method proposed to solve this is to experimentally account for structural compliances introduced by the transition across a dissimilar interface.¹⁴ Another method is to solve the quarter space problem, however, due to the complexity of the boundary conditions and the non-axisymmetry of typical three-dimensional (point contact) indentation probe contact geometries, quarter space solutions are difficult to derive.

^{a)}Address all correspondence to this author. tovaert@nd.edu.

The spatial wedge problem, with displacements prescribed on the boundary, is known as the second boundary value problem, and was solved by Uflyand.¹⁵ Later, he constructed the solutions with stresses prescribed on the boundary, as well as the sliding problem with both prescribed normal displacements and tangential stresses.¹⁶ In this investigation, analytical solutions are derived for the indentation contact problem of spherical and Berkovich indenter geometries on an elastic quarter space near a dissimilar interface (assumed rigid). The results were compared with finite element simulations assuming the properties of bone. The potential error in the mechanical properties of bone in the near-interface region were then addressed. The results show that the error increases substantially as one approaches the interface.

II. METHODOLOGY

A. Analytical solutions

Three-dimensional (point load) contact solutions for both (rigid) spherical and Berkovich (approximated as a 140.6° axisymmetric cone) indenters were used to model the indentation geometry, and are shown in Fig. 1 (right-handed coordinate system specified). Although the three-dimensional half space contact problems for the spherical indenter (or Hertz contact problem) and the axisymmetric cone have been solved previously, the quarter space problems have received less attention, due to their complex boundary conditions and solution asymmetry. By converting the above two contact geometries into their corresponding elasticity problems, it is possible to construct the solutions and estimate the error due to the dissimilar interface.

The left portion of both models in Fig. 1 is assumed to be rigid, compared to the elastic constituent on the right. This is reasonable for a bone-implant interface given that a typical modulus value for bone is on the order of 15 GPa, while those for titanium and steel alloys are roughly 150 and 230 GPa, respectively. The bone contact, which deforms during indentation, is assumed to be elastic. This assumption is reasonable given that unloading (used to estimate Young's modulus) is a predominantly elastic event. In addition, two cases were examined which set the interface boundary conditions. First, the interface was assumed to be perfectly bonded. Studies have shown that the adhesive strength (179 to 224 MPa) at the interface can exceed the tensile strength of bone, thus, the cohesive failure occurs away from the interface.¹⁷ Second, the interface was allowed to slip freely, which assumes an unbonded, gap-free interface. This situation may occur, for example, when the shear traction forces exerted during specimen preparation cause a debonding at the interface in the near-surface region.

The mathematical boundary conditions for the contact problems are as follows. At the face $x = 0$, the displacements $u_x(0, y, z) = u_y(0, y, z) = u_z(0, y, z) = 0$; and at the free surface $z = 0$, frictionless contact occurs with the spherical or Berkovich indenters. In this case, we solve for the indentation depth h_t of the apex of the indenter beneath the original specimen free surface. This will be a function of the bone material properties, the geometry of the indenter, the applied load, and the indentation distance, Δ (the distance from the interface to the apex of the indenter).

The next step is to solve for the indentation elastic modulus using the load vs. indentation depth curve assuming a half space geometry. The result from the quarter space solution can then be compared with that from the half space model, with the difference being the mathematical error caused by the effect of the interface.

The above two three-dimensional quarter space contact problems can be solved by first establishing an alternative three-dimensional quarter space elasticity model as noted below.

A quarter space with the coordinate system $0 \leq x < +\infty$, $-\infty < y < +\infty$, $0 \leq z < +\infty$, the face $x = 0$ is fixed, such that the displacements:

$$u_x(0, y, z) = u_y(0, y, z) = u_z(0, y, z) = 0. \quad (1)$$

On the free surface $z = 0$, only an arbitrary normal pressure distribution, σ_{zz} , is prescribed, with the interfacial shear stress terms, τ_{zx} and τ_{zy} , equal to zero:

$$\sigma_{zz}(x, y, 0) = -p(x, y), \tau_{zx}(x, y, 0) = \tau_{zy}(x, y, 0) = 0. \quad (2)$$

The exact solution of the displacement field in the z -direction within the quarter space, $u_z(x, y, z)$, where $0 \leq x < +\infty$, $-\infty < y < +\infty$, $0 \leq z < +\infty$, was found using select results from Popov.¹⁸

First, the Lamé equations in the absence of mass forces can be written as:

$$\begin{aligned} \nabla^2 u + \mu_0 \frac{\partial}{\partial x} \left(Z + \frac{\partial w}{\partial z} \right) &= 0 \\ \nabla^2 v + \mu_0 \frac{\partial}{\partial y} \left(Z + \frac{\partial w}{\partial z} \right) &= 0 \\ \nabla^2 w + \mu_0 \frac{\partial}{\partial z} \left(Z + \frac{\partial w}{\partial z} \right) &= 0 \end{aligned} \quad (3)$$

Where $u = 2Gu_x$, $v = 2Gu_y$, $w = 2Gu_z$, G is the shear modulus of the isotropic elastic quarter space, $\mu_0 = 1/(1 - 2\mu)$, and μ is Poisson's ratio. The functions Z and Z^* are defined as:

$$Z = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, Z^* = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (4)$$

Equations (3) and (4) can be combined into simultaneous equations:

$$\begin{aligned} \nabla^2 Z + \mu_0 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(Z + \frac{\partial w}{\partial z} \right) &= 0 \\ \nabla^2 w + \mu_0 \frac{\partial}{\partial z} \left(Z + \frac{\partial w}{\partial z} \right) &= 0 \end{aligned} \quad (5)$$

The boundary conditions on $x = 0$ are rewritten as:

$$u(0, y, z) = v(0, y, z) = w(0, y, z) = 0. \quad (6)$$

By using the equations:

$$\begin{aligned} (1 - 2\mu)\sigma_{zz} &= \mu Z + (1 - \mu) \frac{\partial w}{\partial z} \\ 2 \left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} \right) &= \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + \frac{\partial Z}{\partial z} \\ 2 \left(\frac{\partial \tau_{zy}}{\partial x} - \frac{\partial \tau_{zx}}{\partial y} \right) &= \frac{\partial Z^*}{\partial z} \end{aligned} \quad (7)$$

The boundary conditions on $z = 0$ are rewritten as:

$$\begin{aligned} [\mu Z + (1 - \mu) \frac{\partial w}{\partial z}] \Big|_{z=0} &= - (1 - 2\mu) p(x, y) \\ [(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) + \frac{\partial Z}{\partial z}] \Big|_{z=0} &= 0 \\ \frac{\partial Z^*}{\partial z} \Big|_{z=0} &= 0 \end{aligned} \quad (8)$$

Defining the Fourier transform of a function $f(x, y, z)$ with respect to y as:

$$f_\beta(x, z) = \int_{-\infty}^{+\infty} f(x, y, z) \cdot e^{i\beta y} dy, \quad (9)$$

Eq. (5) becomes:

$$\begin{aligned} \frac{\partial^2 w_\beta(x, z)}{\partial z^2} + \frac{1}{\mu_*} \left(\frac{\partial^2 w_\beta(x, z)}{\partial x^2} - \beta^2 w_\beta(x, z) \right) + \frac{\mu_0}{\mu_*} \frac{\partial Z_\beta(x, z)}{\partial z} &= 0 \\ \frac{\partial^2 Z_\beta(x, z)}{\partial z^2} + \mu_* \left[\frac{\partial^2 Z_\beta(x, z)}{\partial x^2} - \beta^2 Z_\beta(x, z) \right] + \mu_0 \frac{\partial}{\partial z} \left[\frac{\partial^2 w_\beta(x, z)}{\partial x^2} - \beta^2 w_\beta(x, z) \right] &= 0 \end{aligned} \quad (10)$$

where $\mu_* = 2(1 - \mu)\mu_0$, and β is the Fourier transform variable conjugate to y .

The boundary conditions then become:

$$\begin{aligned} u_\beta(0, z) &= v_\beta(0, z) = w_\beta(0, z) = 0 \\ \mu Z_\beta(x, 0) + (1 - \mu) \frac{\partial w_\beta(x, 0)}{\partial z} &= - (1 - 2\mu) p_\beta(x) \\ \left[\frac{\partial^2 w_\beta(x, z)}{\partial x^2} - \beta^2 w_\beta(x, z) + Z_\beta(x, z) \right] \Big|_{z=0} &= 0 \\ \frac{\partial Z_\beta(x, 0)}{\partial z} &= 0 \end{aligned} \quad (11)$$

Taking the sine Fourier transform of Eq. (10):

$$\begin{aligned} \frac{\partial^2 w_{\beta\alpha}(z)}{\partial z^2} - \frac{N_{\beta\alpha}}{\mu_*} w_{\beta\alpha}(z) + \frac{\mu_0}{\mu_*} \frac{\partial Z_{\beta\alpha}(z)}{\partial z} &= 0 \\ \frac{\partial^2 Z_{\beta\alpha}(z)}{\partial z^2} - N_{\beta\alpha}^2 [\mu_* Z_{\beta\alpha}(z) + \mu_0 \frac{\partial w_{\beta\alpha}(z)}{\partial z}] &= 0 \end{aligned} \quad (12)$$

where $N_{\beta\alpha}^2 = \alpha^2 + \beta^2$, and α is a parameter similar to β .

The boundary conditions may be written as:

$$\begin{aligned} -N_{\beta\alpha}^2 w_{\beta\alpha}(0) + \frac{\partial Z_{\beta\alpha}(0)}{\partial z} &= 0 \\ \mu Z_{\beta\alpha}(0) + (1 - \mu) \frac{\partial w_{\beta\alpha}(0)}{\partial z} &= - (1 - 2\mu) p_{\beta\alpha} \end{aligned} \quad (13)$$

Solving Eqs. (12) and (13), one obtains:

$$w_{\beta\alpha}(z) = p_{\beta\alpha} e^{-N_{\beta\alpha} z} [2(1 - \mu) N_{\beta\alpha}^{-1} + z] \quad (14)$$

Now using the inverse Fourier sine transform of a function $f_{\beta\alpha}(z)$ as:

$$f_{\beta}(x, z) = \frac{2}{\pi} \int_0^{+\infty} f_{\beta\alpha}(z) \cdot \sin \alpha x d\alpha$$

and the inverse Fourier transform with respect to y as:

$$f(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f_{\beta}(x, z) \cdot e^{-i\beta y} d\beta,$$

Eq. (14) becomes:

$$w(x, y, z) = \frac{1}{2\pi} \int_0^{+\infty} \int_{-\infty}^{+\infty} p(\xi, \eta) \{ 2(1-\mu) \left[\frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}} - \frac{1}{\sqrt{(x+\xi)^2 + (y-\eta)^2 + z^2}} \right] \right. \\ \left. + z^2 \left[\frac{1}{[(x-\xi)^2 + (y-\eta)^2 + z^2]^{3/2}} - \frac{1}{[(x+\xi)^2 + (y-\eta)^2 + z^2]^{3/2}} \right] \right\} d\xi d\eta \quad (15)$$

At the surface $z = 0$, this reduces to:

$$w(x, y, 0) = \frac{1-\mu}{\pi} \int_0^{+\infty} \int_{-\infty}^{+\infty} p(\xi, \eta) \left[\frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} - \frac{1}{\sqrt{(x+\xi)^2 + (y-\eta)^2}} \right] d\xi d\eta \quad (16)$$

For a spherical indenter, the pressure distribution takes the form:¹⁹

$$p(r) = \frac{3P}{2\pi a^2} \left(1 - \frac{r^2}{a^2}\right)^{1/2} \quad \text{for } 0 \leq r \leq a \quad (17)$$

where P is the indenter load, a is the radius of the circle of contact, and r is the distance from the center of the circle of contact. Thus,

$$w(\Delta, 0, 0) = \frac{1-\mu}{\pi} \int_{\Delta-a}^{\Delta+a} \int_{-\sqrt{a^2-(\xi-\Delta)^2}}^{+\sqrt{a^2-(\xi-\Delta)^2}} p(\xi, \eta) \left[\frac{1}{\sqrt{(\Delta-\xi)^2 + \eta^2}} - \frac{1}{\sqrt{(\Delta+\xi)^2 + \eta^2}} \right] d\xi d\eta \quad (18)$$

Using the relations $2Gu_z = w$, and defining $h_t = u_z(\Delta, 0, 0)$,

$$h_t = \frac{1-\mu}{2\pi G} \int_{\Delta-a}^{\Delta+a} \int_{-\sqrt{a^2-(\xi-\Delta)^2}}^{+\sqrt{a^2-(\xi-\Delta)^2}} p(\xi, \eta) \left[\frac{1}{\sqrt{(\Delta-\xi)^2 + \eta^2}} - \frac{1}{\sqrt{(\Delta+\xi)^2 + \eta^2}} \right] d\xi d\eta. \quad (19)$$

Note that in the above equation, h_t is a function of the material properties, the pressure distribution, p , the distance from the interface, Δ , and the radius of the circle of contact, a . However, a is also an unknown, thus another equation is required to solve for h_t and a .

From the Hertz contact solution, the depth of the contact circle beneath the specimen free surface, h_a , is one-half of the total elastic displacement. Thus,

$$h_a = h_p = h_t/2, \quad (20)$$

where h_p is the distance from the bottom of the contact to the contact circle.²⁰ Geometrically, one can estimate:

$$h_t = \frac{a^2}{R} \quad (21)$$

where R is the indenter radius. Thus, h_t can be obtained by solving Eqs. (19) and (21) (numerically) using MatlabTM or similar.

The indentation load-displacement compliance relationship for a spherical indenter on a half space is given by:²¹

$$E^* = \frac{dP}{dh_t} \frac{1}{2a} = \frac{dP}{dh_t} \frac{1}{2} \sqrt{\frac{\pi}{A}} \quad (22)$$

Where A is the contact area, and E^* is the well-known reduced modulus of the specimen and indenter (diamond, primed), given by:

$$\frac{1}{E^*} = \frac{1 - \mu^2}{E} + \frac{1 - \mu'^2}{E'}. \quad (23)$$

Thus, using Eqs. (22) and (23), one can obtain the calculated indentation elastic modulus E for a spherical indenter on a half space, in order to determine the discrepancy due to the dissimilar interface.

Similarly, for a Berkovich indenter, we begin by considering the contact pressure distribution:²²

$$p(r) = \frac{P}{\pi a^2} \cosh^{-1} \frac{a}{r} \quad \text{for } 0 \leq r \leq a.$$

Thus,

$$h_t = \frac{1 - \mu}{2\pi G} \int_{\Delta-a}^{\Delta+a} \int_{-\sqrt{a^2 - (\xi - \Delta)^2}}^{+\sqrt{a^2 - (\xi - \Delta)^2}} P(\xi, \eta) \left[\frac{1}{\sqrt{(\Delta - \xi)^2 + \eta^2}} - \frac{1}{\sqrt{(\Delta + \xi)^2 + \eta^2}} \right] d\xi d\eta. \quad (24)$$

Geometrically, for a Berkovich tip:

$$h_t = \frac{\pi}{2} \cot \varphi \cdot a \quad (25)$$

where $\varphi = 70.3^\circ$.²⁰ Again, h_t can be obtained by numerically solving Eqs. (24) and (25).

The indentation load-displacement compliance relationship method for a Berkovich indenter on a half space is given by:²

$$E^* = \frac{dP}{dh_t} \frac{1}{2h_p} \frac{1}{\zeta} \sqrt{\frac{\pi}{24.5}} \quad (26)$$

where ζ is a correction factor for a Berkovich indenter, equal to 1.128.

Using Eqs. (23) and (26), one can obtain the calculated indentation elastic modulus E for a Berkovich indenter on a half space, in order to determine the discrepancy due to the interface.

Finally, solutions were obtained for the case where the interface was allowed to slip freely, assuming an unbonded, gap-free boundary condition. In this case, the normal displacements and tangential stresses on the face $x = 0$ are defined as:

$$u_x(0, y, z) = 0 \text{ and } \tau_{xy}(0, y, z) = \tau_{xz}(0, y, z) = 0$$

The indentation depth h_t may be divided into two parts:

$$h_t = h_{t1} + h_{t2} \quad (27)$$

Where h_{t1} corresponds to the traditional half space solution, and h_{t2} corresponds to the additional depth caused by the interface boundary effect. For the spherical indenter case,

$$h_{t1} = \frac{3}{4} \cdot \frac{1}{E^*} \cdot \frac{P}{a} \quad (28)$$

and h_{t2} is equal to the normal displacement of a point on the free surface, twice the indentation distance, Δ , away from the indenter apex on the half space. In this case, h_{t2} is given by:

$$h_{t2} = \frac{1-\mu}{2\pi G} \iint_{\Omega} \frac{p(\xi, \eta) d\xi d\eta}{\sqrt{(x-\xi)^2 + (y-\eta)^2}}. \quad (29)$$

With $r = \sqrt{x^2 + y^2} = 2\Delta$, and p the pressure distribution for a spherical indenter,

$$h_{t2} = \frac{1}{\pi R} \cdot \left[(2a^2 - 4\Delta^2) \sin^{-1}\left(\frac{a}{2\Delta}\right) + 2\Delta \cdot a \cdot \sqrt{1 - \frac{a^2}{4\Delta^2}} \right]. \quad (30)$$

Thus, h_{t2} is a function of the indentation distance, Δ , as expected. Similarly, for the Berkovich indenter,

$$h_{t1} = \frac{\pi}{2} \cot \varphi \cdot a, \text{ and} \quad (31)$$

$$h_{t2} = a \cot \varphi \cdot \left[\sin^{-1}\left(\frac{a}{2\Delta}\right) + \sqrt{\frac{4\Delta^2}{a^2} - 1} - \frac{2\Delta}{a} \right]. \quad (32)$$

Note that the free interface boundary condition yields a greater indentation depth than that from the perfectly bonded solution, as expected. Again, using Eqs. (22), (23), and (26), one may obtain the elastic modulus E for spherical and Berkovich indenters on a half space, for comparison purposes.

Bone was modeled with an assumed modulus $E = 15.6$ GPa, Poisson's ratio, $\mu = 0.3$,²³ and the implant is assumed to be rigid. In order to simulate a typical nanoindentation test, for the spherical indenter case, the tip radius was arbitrarily fixed at $1 \mu\text{m}$, and the maximum load was set at $100 \mu\text{N}$. For the Berkovich indenter, an axisymmetric 140.6° cone was used since it approximates the Berkovich shape with the same projected area to depth ratio. The maximum load for this case was set at $10,000 \mu\text{N}$, which is a commonly used maximum load for nanoindentation of bone²³ using a Berkovich indenter. The combination of 100 and $10,000 \mu\text{N}$ maximum loads yielded significantly different indentation depths, and thus permitted some exploration of load effects.

B. Finite element verification

Using the AbaqusTM finite element software, a three-dimensional model was developed for each of the two indenter geometries, to compare with the analytical results. Again, bone was modeled with $E = 15.6$ GPa, and Poisson's ratio, $\mu = 0.3$. Three-dimensional models were developed since the displacement field is asymmetric about the centerline of contact. However, symmetry was employed along the x-z plane. As in the analytical case, the implant was assumed to be rigid. A typical mesh consisted of approximately 15000 8-noded hexahedral elements, and resulted in mesh dimensions of $40 \times 10 \times 30$ mm in the x, y, and z directions, respectively.

III. RESULTS

A. Analytical results

Figures 2–5 show the variation and error for the analytical solution for the $1 \mu\text{m}$ radius spherical indenter at a load of $100 \mu\text{N}$, as a function of the distance Δ . Figures 2 and 3 correspond to the case of the spherical indenter, and Figs. 4 and 5 to the Berkovich indenter. In Figs. 2–3, the indentation depth, h_t , is plotted for the quarter space and half space problems. As noted earlier, their differential results in the error in the computed elastic modulus. In Fig. 2, for the bonded interface, the error in modulus, E , when compared to the

“true” value (15.6 GPa), goes from 0.53% to 37.1% as Δ decreases from 10 μm to 0.16 μm . Note that at $\Delta = 0.16 \mu\text{m}$, the radius of the contact circle between the spherical indenter and the bone surface is 0.15 μm . Thus, the analysis was terminated at $\Delta = 0.16 \mu\text{m}$ to avoid geometrical interference between the indenter and the interface. For the unbonded interface, the error in modulus goes from -0.69% to -25.6% . In general, the bonded interface has a “stiffening” effect, thus, the effect of the interface is to overestimate the modulus E . The opposite is true for the unbonded interface, where, as expected, the computed modulus is less than that obtained from the half space geometry, as shown in Fig. 3.

In Fig. 4, for the bonded interface, the error in computed modulus goes from 0.08% to 28.5% as Δ decreases from 10 μm to 1 μm . At $\Delta = 1 \mu\text{m}$, the radius of the contact circle between the Berkovich indenter and the bone surface is 0.94 μm . Thus, the analysis was terminated at $\Delta = 1 \mu\text{m}$ to avoid geometric interference between the indenter and the interface. In Fig. 5, for the unbonded interface, the error in E goes from -0.72% to -36.2% .

B. Finite element results

Figures 6 and 7 shows a comparison between the analytical (quarter space), half space, and finite element solutions. For the spherical indenter, the agreement is good when Δ is greater than 0.5 μm . For the Berkovich indenter, the two methods converge when Δ is greater than 5 μm . Figures 8 and 9 show the normal displacement (in the z-direction) in the bone for the (bonded) spherical and Berkovich indenter finite element model, respectively. The coordinate axes are referenced in Fig. 1.

IV. DISCUSSION

As shown in Figs. 2–5, the indentation modulus error caused by the effect of the stiffer interface is 5% or less when Δ is greater than approximately 40 times the indentation depth, h_i , for the spherical indenter, and greater than approximately 10 times the indentation depth for the Berkovich geometry. The difference between the spherical and Berkovich geometries can be seen in Figs. 8 and 9. For the spherical indenter, the effective displacement field extends further along the x axis (toward the interface) than that of the Berkovich indenter, which tends to be much more localized near the apex of the indenter tip (Fig. 9). In general, the effect of the implant is to “stiffen” the contact by inhibiting displacements in the x direction. As Δ decreases, this effect becomes magnified.

The analytical solution is an approximation at best, mainly due to the assumption in the pressure distribution. As in the half space contact problem, the pressure distribution assumes the mathematical forms discussed in Section 2. However, due to the effect of the implant interface, the pressure distribution assumed in the quarter space elastic problem is not strictly axisymmetric. This approximation affects the indentation depth, to a greater extent, when Δ is relatively small, further affecting the indentation elastic modulus calculated under this assumption, since the effects of the actual pressure distribution asymmetry will be larger as one approaches the interface. This effect can be seen in Figs. 6 and 7, where the analytical and finite element solutions diverge as Δ decreases.

The analytical results for the perfectly bonded interface show an overestimation of the indentation elastic modulus, while the unbonded results show an underestimation. These two cases represent two extreme boundary conditions. This suggests that the degree of interfacial bonding is a critical factor in the estimation of the error in indentation elastic modulus near the interface. A third interface condition, which is not considered here, is the case where a significant gap or crack forms between the bone and implant, when even at the maximum indentation load, an interfacial gap remains. This condition could occur, for example, when

traction forces during the cutting and/or polishing process are sufficient to open the interface. Thus, the elastic counterpart would be treated as having a free surface normal to the surface of contact, which would be free to deform under load. In that case, a significant underestimation of the indentation modulus would also occur, depending on Δ .

The error in indentation modulus, for both the spherical and Berkovich indenter geometries, will depend on a number of factors. The equations presented in Section 2, as well as the finite element model, contain several parametric values that affect the indentation depth and therefore the error in modulus. For example, for the spherical and Berkovich indenters with a bonded interface, decreasing the assumed modulus of bone from 15.6 GPa to 1 GPa (as in the case of weaker bone microstructure caused by lower mineral content) increases the magnitude of the percent error in the indentation modulus by a factor of four. Thus, the stiffening effect of an implant will be exacerbated as the modulus of the bone sample decreases.

V. CONCLUSIONS

Nanoindentation is a novel method used to determine the mechanical properties of heterogeneous materials at small length scales. When indenting near a dissimilar interface, care must be exercised due to errors in the modulus computation introduced by an assumed half space contact geometry, the standard used in nanoindentation analysis.

In this investigation, we have presented the analysis for the quarter space elastic contact near a rigid interface for spherical and Berkovich indenter geometries. The Berkovich indenter was approximated as an axisymmetric cone. Finite element models were also developed for verification purposes, and both the analytical and finite element models were in good agreement except when approaching the interface. This variation was due to the fact that the analytical solutions assume a fully-developed axisymmetric pressure distribution, which is not the case when one is near the interface.

The interface bonding condition also affects the resulting error. For a perfectly bonded interface, the half space solution will overestimate the elastic modulus by approximately 30% when approaching the geometrical limits of contact interference. For an unbonded interface, the half space solution underestimates the elastic modulus by a similar percentage. For the spherical indenter, maintaining a separation distance of approximately 40 times the maximum indentation depth minimizes the error to a few percent. For the Berkovich indenter, maintaining a separation distance of approximately 10 times the maximum indentation depth is sufficient.

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FIG. 1.
Contact geometry, spherical (left) and Berkovich (right).

FIG. 2. Analytical solution results of indentation depth and modulus as a function of apex separation, Δ , for a spherical indenter with a bonded interface.

FIG. 3. Analytical solution results of indentation depth and modulus as a function of apex separation, Δ , for a spherical indenter with an unbonded interface.

FIG. 4.

Analytical solution results of indentation depth and modulus as a function of apex separation, Δ , for a Berkovich indenter with a bonded interface.

FIG. 5. Analytical solution results of indentation depth and modulus as a function of apex separation, Δ , for a Berkovich indenter with an unbonded interface.

FIG. 6.

Comparison between analytical and finite element solutions as a function of the apex separation, Δ , for a spherical indenter with a bonded interface.

FIG. 7. Comparison between analytical and finite element solutions as a function of the apex separation, Δ , for a Berkovich indenter with a bonded interface.

FIG. 8. Normal (z axis) displacements (μm), finite element model, spherical indenter, bonded interface, $\Delta = 0.5 \mu\text{m}$.

FIG. 9. Normal (z axis) displacements (μm) from finite element model, Berkovich indenter, bonded interface, $\Delta = 2 \mu\text{m}$.