Subzone based magnetic resonance elastography using a Rayleigh damped material model

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Purpose: Recently, the attenuating behavior of soft tissue has been addressed in magnetic resonance elastography by the inclusion of a damping mechanism in the methods used to reconstruct the resulting mechanical property image. To date, this mechanism has been based on a viscoelastic model for material behavior. Rayleigh, or *proportional*, damping provides a more generalized model for elastic energy attenuation that uses two parameters to characterize contributions proportional to elastic and inertial forces. In the case of time-harmonic vibration, these two parameters lead to both the elastic modulus and the density being complex valued (as opposed to the case of pure viscoelasticity, where only the elastic modulus is complex valued).

Methods: This article presents a description of Rayleigh damping in the time-harmonic case, discussing the differences between this model and the viscoelastic damping models. In addition, the results from a subzone based Rayleigh damped elastography study of gelatin and tofu phantoms are discussed, along with preliminary results from *in vivo* breast data.

Results: Both the phantom and the tissue studies presented here indicate a change in the Rayleigh damping structure, described as *Rayleigh composition*, between different material types, with tofu and healthy tissue showing lower Rayleigh composition values than gelatin or cancerous tissue.

Conclusions: It is possible that Rayleigh damping elastography and the concomitant Rayleigh composition images provide a mechanism for differentiating tissue structure in addition to measuring elastic stiffness and attenuation. Such information could be valuable in the use of Rayleigh damped magnetic resonance elastography as a diagnostic imaging tool. © 2011 American Association of Physicists in Medicine. [DOI: [10.1118/1.3557469](https://doi.org/10.1118/1.3557469)]

Key words: magnetic resonance elastography, viscoelasticity, Rayleigh damping, image reconstruction

I. INTRODUCTION: DAMPING EFFECTS IN MR ELASTOGRAPHY

Attenuation and damping in elastography are naturally of great interest as the presence of these effects in biological tissue goes without question and therefore must be addressed if quantitative assessment of tissue elastic properties is to be achieved. Additionally, given the change in the tissue structure present in the diseases that elastographic imaging seeks to detect and diagnose, there is every reason to expect that the resulting lesions will also exhibit a change in their attenuation behavior, indicating diagnostic value to any description of the damping property distribution elastographic methods are able to provide. Indeed, evidence of this diagnostic merit has already been demonstrated in several *in vivo* studies.¹⁻⁹ Important in this group is the work of Sinkus *et al.* and Huwart *et al.*, who found significant variation in the

viscoelastic properties between diseased and healthy breast tissue¹ and liver tissue.⁴

While some of these studies have involved investigation of different rheological classifications for the resulting attenuation behavior,¹⁰⁻¹² they all rely on a damping model that equates to a viscoelastic (VE) description for material behavior, i.e., a model where the attenuation forces are proportional to the elastic forces present within the material. Alternative attenuation models exist for elastic materials, particularly Rayleigh, or proportional, damping (RD) models where the attenuation forces are proportional to both the elastic and the inertial forces within the material.^{13,14} In the time-harmonic case commonly found in dynamic elastography, RD corresponds to the case where both the density and the modulus of the material are complex valued, whereas time-harmonic VE leads to a real valued density and a com-

plex valued modulus. With two parameters to represent the damping behavior of the material, RD is able to differentiate between two classes of attenuation, compared to VE, which contains only a single damping parameter and thus cannot differentiate between attenuation originating from different structural effects. This expanded damping description is appropriate for soft-tissue elastography as the true attenuation characteristics of biological tissue are generated from multiple levels of tissue component interactions and display correspondingly complex frequency dependency.¹⁵

Elastographic reconstruction of RD properties is non-trivial, however, given that both the shear modulus and the density contain unknown imaginary components. This makes RD reconstruction impossible in the so-called *direct inversion* methods where elastic properties are calculated by direct application of the equilibrium equations to the measured data assuming known inertial terms and making use of various filtering methods. In addition, it will be shown that the dynamics of a RD material model can be matched exactly in homogeneous cases by an equivalent VE shear modulus, leading to ill-posedness in optimization based iterative property reconstruction methods. Given these issues, the ability to image both VE and RD based property distributions has merit, with the VE reconstructions generating valuable damping level information through a reasonably well-posed optimization process while the RD reconstructions provide a more sophisticated material structure description at the cost of some condition in the underlying inverse problem.

This paper discusses the evaluation of *subzone* structured, optimization based VE and RD elastography reconstruction processes intended for use with time-harmonic motion data measured by MR phase-contrast imaging methods. In particular, a review of the time-harmonic RD formulation, as compared to the VE continuum equations, is presented, followed by a discussion of some of the uniqueness issues associated with the time-harmonic RD model. A summary of results from a study of tofu and gelatin phantoms is also presented, as well as preliminary results in *in vivo* breast tissue.

II. METHODS: TIME-HARMONIC RAYLEIGH ELASTOGRAPHY

Rayleigh damping, also referred to as *proportional damping*, is a damping model that attributes attenuation to both elastic and inertial forces.^{14,16} Typically, the equilibrium statement for a damped elastic system is written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \quad (1)$$

for displacements $\mathbf{u}$, given forcing $\mathbf{f}$, and a discretized stiffness matrix, $\mathbf{K}$, mass matrix $\mathbf{M}$, and damping matrix $\mathbf{C}$. For a Rayleigh damped system, the damping matrix is directly proportional to the mass and stiffness matrices,

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \quad (2)$$

and thus has the property that it can be modally decoupled using the same spectral decomposition used for the corresponding undamped elastic system. Substitution of Eq. (2) into Eq. (1), along with the assumption of the time-harmonic

form, where $\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x})e^{i\omega t}$ and $\mathbf{f}(\mathbf{x}, t) = \bar{\mathbf{f}}(\mathbf{x})e^{i\omega t}$, leads to the relationship

$$\left[-\omega^2 \left(1 - \frac{i\alpha}{\omega} \right) \mathbf{M} + (1 + i\omega\beta) \mathbf{K} \right] \bar{\mathbf{u}} = \bar{\mathbf{f}} \quad (3)$$

in terms of the original undamped stiffness and mass matrices, $\mathbf{K}$ and $\mathbf{M}$. Accepting that the terms $(1 - i\alpha/\omega)$ and $(1 + i\omega\beta)$ from Eq. (3) describe a spatial distribution of Rayleigh damping parameters, this relationship can be rewritten in the form

$$[-\omega^2 \tilde{\mathbf{M}} + \tilde{\mathbf{K}}] \bar{\mathbf{u}} = \bar{\mathbf{f}}, \quad (4)$$

where $\tilde{\mathbf{K}}$ and $\tilde{\mathbf{M}}$ have the same form as the original stiffness and mass matrices, $\mathbf{K}$ and $\mathbf{M}$, except for the use of a complex valued shear modulus, $\mu = \mu_R + i\mu_I$, and density, $\rho = \rho_R + i\rho_I$, to describe the material's elastic properties. In this case, μ_R and ρ_R represent the elastic shear modulus and mass density originally present in the undamped system, while the imaginary components can be written in terms of the Rayleigh damping parameters from Eq. (2) as

$$\rho_I = \frac{-\alpha\rho_R}{\omega} \quad \text{and} \quad \mu_I = \omega\beta\mu_R. \quad (5)$$

This is similar to the standard time-harmonic VE formulation, where a complex valued shear modulus is used to describe the storage and loss modulus for shear wave motion, except that the density in the standard VE model is assumed real valued everywhere.

From the damped elastic system model, described in Eq. (4), the damping ratio, ξ , can be written as

$$\xi = \frac{1}{2} \left(\frac{\mu_I}{\mu_R} - \frac{\rho_I}{\rho_R} \right), \quad (6)$$

which gives the relative level of attenuation within the material. Note that for VE materials, the ratio of density components in the second term of this relation will be zero, as the imaginary density, ρ_I , is zero by definition. For RD materials, an additional *Rayleigh composition (RC)* measure can be defined as

$$\text{RC} = (2\xi)^{-1} \left(\frac{\mu_I}{\mu_R} \right). \quad (7)$$

This value is interpreted as the percentage contribution of VE to the overall damping levels within the RD material, i.e., a material with a RC value of 1 could be perfectly described by a standard VE model as no attenuation effects proportional to the inertial forcing will be present. It is important to understand that the true origins of attenuation effects in soft tissue lie in the complex, multiscale, multiple-scattering events occurring within the elastic microstructure of the tissue. Both RD and VE represent continuum approximations to this multiple-scattering elastic behavior, and direct relationships between these approximations and the underlying tissue microstructure are difficult to assess, although advances in elastic homogenization theory may provide reasonable estimates.^{17,18} While a full development of the RD

continuum model through homogenization theory is still some ways away, there is a connection between the suitability of an inertially or elastically based model of damping forces and the underlying tissue microstructure so that the RC value for a particular tissue will be related to its structure in some, at this point unknown, manner.

II.A. Comparing Rayleigh damping and viscoelasticity

Both the RD and the VE forms of the elastic equilibrium conditions can be investigated by considering the distortion-wave equation,

$$\nabla \cdot (\mu \nabla \mathbf{u}) + \nabla \cdot (\mu (\nabla \mathbf{u})^T) = -\rho \omega^2 \mathbf{u}, \quad (8)$$

where compressive wave terms have been left out for the sake of simplicity. Assuming a RD formulation with complex shear modulus, μ , and density, ρ , this expression can be rewritten for a real valued density, ρ_R , on the RHS by multiplying through by ρ_R/ρ so that

$$\begin{aligned} \left(\frac{\rho_R}{\rho}\right)(\nabla \mu) \cdot \nabla \mathbf{u} + \left(\frac{\rho_R}{\rho}\mu\right)\nabla^2 \mathbf{u} + \left(\frac{\rho_R}{\rho}\right)(\nabla \mu) \cdot (\nabla \mathbf{u})^T \\ + \left(\frac{\rho_R}{\rho}\mu\right)\nabla \cdot (\nabla \mathbf{u})^T = -\rho_R \omega^2 \mathbf{u}. \end{aligned} \quad (9)$$

The first term on the LHS of Eq. (9) can be expanded using the product rule relation,

$$\partial(ab) = (\partial a)b + a(\partial b) \Rightarrow a(\partial b) = \partial(ab) - (\partial a)b, \quad (10)$$

which, in combination with an *effective* shear modulus, $\tilde{\mu} = (\rho_R/\rho)\mu$, leads to

$$\begin{aligned} (\nabla \tilde{\mu}) \cdot \nabla \mathbf{u} + \tilde{\mu} \nabla^2 \mathbf{u} + (\nabla \tilde{\mu}) \cdot (\nabla \mathbf{u})^T + \tilde{\mu} \nabla \cdot (\nabla \mathbf{u})^T \\ - \left(\nabla \frac{\rho_R}{\rho}\right) \cdot \mu \nabla \mathbf{u} - \left(\nabla \frac{\rho_R}{\rho}\right) \cdot \mu (\nabla \mathbf{u})^T = -\rho_R \omega^2 \mathbf{u}. \end{aligned} \quad (11)$$

This equilibrium statement can be compared to the corresponding statement for a VE shear wave, with a real valued density, ρ_R , and complex viscoelastic shear modulus, $\hat{\mu}$,

$$\begin{aligned} \nabla \cdot (\hat{\mu} \nabla \mathbf{u}) + \nabla \cdot (\hat{\mu} (\nabla \mathbf{u})^T) = (\nabla \hat{\mu}) \cdot \nabla \mathbf{u} + \hat{\mu} \nabla^2 \mathbf{u} \\ + (\nabla \hat{\mu}) \cdot (\nabla \mathbf{u})^T + \hat{\mu} \nabla \cdot (\nabla \mathbf{u})^T \\ = -\rho_R \omega^2 \mathbf{u}, \end{aligned} \quad (12)$$

where the difference is seen to lie in the final two LHS terms of Eq. (11), involving spatial derivatives of $\rho_R/\rho = (\rho_R^2 - i\rho_I\rho_R)/(\rho_R^2 + \rho_I^2)$. Thus, in regions of material homogeneity, or at least where $\nabla(\rho_R/\rho) = 0$, a RD and a VE model will be indistinguishable, save the fact that the VE model will be based on the effective modulus,

$$\tilde{\mu} = \left(\frac{\rho_R}{\rho}\right)\mu = \frac{(\mu_R\rho_R^2 + \mu_I\rho_I\rho_R - i(\mu_R\rho_I\rho_R - m\mu_I\rho_R^2))}{(\rho_R^2 + \rho_I^2)}, \quad (13)$$

where the real component, $\tilde{\mu}_R = (\mu_R\rho_R^2 + \mu_I\rho_I\rho_R)/(\rho_R^2 + \rho_I^2)$, is seen to differ from μ_R when $\rho_I \neq 0$.

These considerations lead to two important characteristics of RD materials: the presence of damping forces proportional to inertial terms in an elastic system will lead to arti-

TABLE I. DMA results for 10% gelatin at 100 Hz based on TTS methods.

μ_R (Pa)	ξ (%)
8800 ± 900	6.09 ± 0.13

facts in a VE based property reconstruction of the real shear modulus and the presence of these forces in regions of material inhomogeneity will mean that the VE equilibrium equations will not fully characterize the system dynamics. It is important to understand that *both* RD and VE represent simplified models of the damping behavior in complex materials such as soft tissue and that, in general, damping forces can be expected to have components proportional to both elastic and inertial forces as well as additional components that are not proportional to either forcing component. Thus, both RD and VE models will be expected to produce artifacts when used as the basis for elastographic property reconstruction in real tissue, although RD offers the capability to account for one additional component of the overall damping effect.

II.B. Subzone based Rayleigh damping MR elastography

The so-called *subzone* based MR elastography has been previously documented in phantoms and breast tissue for elastic material models,^{19,20} while global RD property reconstruction methods have been discussed previously in the context of simulation studies.¹⁴ Here, we present a combination of the subzone based elastographic method with a nearly incompressible RD elastic material model, implemented in finite elements using a sparse distributed memory matrix solver,^{21,22} to investigate the damped elastic structure of both gelatin and tofu based phantoms as well as *in vivo* breast tissue. For details of the reconstruction methods, the reader is referred to previous works, particularly Refs. 14 and 23–25.

II.C. Phantom and *in vivo* studies

Two phantom configurations were considered for the study presented here. The first consisted of a tofu background (*Mori-Nu Silken Soft*[®]) with a single, stiff gelatin inclusion (10% *Sigma Aldrich*[®] porcine skin gelatin, 300 bloom, St. Louis, MO) at the center. The second phantom was homogeneous 10% gelatin. Both phantoms were imaged at a frequency of 100 Hz motion excitation on a Phillips 1.5T MRI scanner using a spin echo based phase-contrast

TABLE II. Tissue data taken from Ref. 1.

Material	μ_R (Pa)	ξ (%)
Healthy tissue at 65 Hz	870 ± 150	12.8 ± 0.61
Cancer tissue at 65 Hz	2900 ± 300	158.31 ± 103.33

FIG. 1. Image results for RD reconstruction of the single gelatin inclusion tofu phantom: (a) MR image, (b) μ_R image (kPa), (c) ξ image (%/100), and (d) RC image (%/100).

sequence, 2 mm isotropic voxels, TR=480 ms, TE=40 ms, and 16 coronal slices with a 128×128 FOV.

To provide comparable elastic property measurements for the gelatin phantom material, dynamic mechanical analysis (DMA) was performed using a TA Instruments[®] Q800 device, with measurements at 100 Hz computed using time-temperature superposition (TTS).²⁶ A summary of these results is given in Table I. Difficulty in achieving similar conditions between the DMA and MRE analysis for the tofu phantom material means that no DMA results for the tofu material are presented.

In addition to the phantom data, RD reconstructions were also performed on motion data obtained from a breast cancer patient undergoing neoadjuvant chemotherapy. These *in vivo* tissue data were captured at 85 Hz motion excitation using

similar scan settings to those used for the phantom experiments, as outlined above. While Samani *et al.*²⁷ provided some mechanical measurements of breast tissue material properties, these were done at frequencies far from the 85 Hz used in the experiments here; thus, the best comparison for the damping properties of breast tissue comes from the VE based MRE results of Sinkus *et al.*,¹ taken at 65 Hz, which are summarized in Table II in terms of μ_R and ξ .

III. RECONSTRUCTION RESULTS

The figures below present RD reconstructions for the phantom configurations and breast tissue data described in Sec. II C. In addition, VE reconstructions were performed on each of the phantoms for comparison with the RD reconstruc-

FIG. 2. Image results for VE reconstruction of the single gelatin inclusion tofu phantom: (a) μ_R image (kPa) and (b) ξ image (%/100).

tion results. In order to quantifiably compare each reconstruction, *regions of interest* (ROIs) were selected from areas of each image representing particular material or tissue types, and the mean and standard deviation of each property within these ROI were calculated.

Figures 1 and 2 show the results from RD and VE reconstructions, respectively, from the single inclusion tofu phantom. For comparison, Fig. 3 shows a noniterative, *direct* VE (DVE) reconstruction of this phantom based on the algorithm proposed by Sinkus *et al.*,¹ where the use of filtering eliminates reconstruction results from top and bottom slices as well as a region around the exterior of the phantom. Table III

summarizes these results for the soft tofu background and stiffer gelatin inclusion. A single slice of the RD reconstruction for this phantom is highlighted in Fig. 4, where the background and inclusion areas selected for ROI analysis from the available T2* MR images are highlighted in Figs. 4(b) and 4(c), respectively. In addition, Table IV shows the mean values of μ_R and μ_I for this single inclusion phantom for the RD, VE, and DVE reconstructions as well as the effective VE shear modulus $\tilde{\mu}$, given by Eq. (13), of the RD reconstruction.

Figure 5 shows the results from RD reconstruction of the homogeneous 10% gelatin phantom, while Table V summa-

FIG. 3. Image results for DVE reconstruction of the single gelatin inclusion tofu phantom: (a) μ_R image (kPa) and (b) ξ image (%/100).

TABLE III. Single inclusion tofu phantom, RD, VE, and DVE reconstruction ROI results. Both the mean ($\bar{x}$) and the standard deviation (s_x) values are given.

RD	$\bar{\mu}_R$ (Pa)	s_{μ_R} (Pa)	$\bar{\xi}$ (%)	s_{ξ} (%)	$\overline{RC}$ (%)	s_{RC} (%)
Tofu background	4 863	556	4.75	2.54	14.00	12.30
10% gelatin inclusion	9 021	1039	1.06	0.95	33.86	22.93
VE	$\bar{\mu}_R$ (Pa)	s_{μ_R} (Pa)	$\bar{\xi}$ (%)	s_{ξ} (%)	$\overline{RC}$	s_{RC}
Tofu background	4 810	667	2.35	1.17	NA	NA
10% gelatin inclusion	9 470	1176	0.81	0.80	NA	NA
DVE	$\bar{\mu}_R$ (Pa)	s_{μ_R} (Pa)	$\bar{\xi}$ (%)	s_{ξ} (%)	$\overline{RC}$	s_{RC}
Tofu background	7 203	989	11.20	4.45	NA	NA
10% gelatin inclusion	10 236	3860	10.94	14.90	NA	NA

rizs these results. Due to the ill-posed nature of the reconstruction problem for RC information in these homogeneous phantoms, predicted by Eq. (11) and evidenced by the random appearance of the RC images in Fig. 5(d), calculation of effective VE shear modulus $\tilde{\mu}$ is of little value and therefore was not performed.

Figure 6 shows the results for the RD reconstruction of the *in vivo* data from the patient undergoing neoadjuvant chemotherapy, while Table VI summarizes these results for ROIs selected from the reconstructed data based on manual segmentation of the real shear modulus image [Fig. 6(d)]. The ROIs used for the *healthy* and *cancerous* tissue samples for

FIG. 4. Single slice image results for the single gelatin inclusion tofu phantom, with ROI selections indicated (a) MR image, (b) MR image (*background ROI indicated*), (c) MR image (*inclusion ROI indicated*), (d) μ_R image (kPa), (e) ξ image (%/100), and (f) RC image (%/100).

TABLE IV. Single inclusion tofu phantom, RD, VE, DVE, and $\tilde{\mu}$ complex shear modulus values.

RD	μ_R (Pa)	μ_I (Pa)
Soft tofu BG	4 863	46
10% gelatin INC	9 021	63
VE	μ_R (Pa)	μ_I (Pa)
Soft tofu BG	4 810	221
10% gelatin INC	9 470	145
$\tilde{\mu}$	μ_R (Pa)	μ_I (Pa)
Soft tofu BG	4 823	460
10% gelatin INC	9 018	189
DVE	μ_R (Pa)	μ_I (Pa)
Soft tofu BG	7 203	1574
10% gelatin INC	10 237	2011

these comparisons are highlighted in Figs. 6(b) and 6(c), respectively. A single slice of the RD reconstruction for these patient data is highlighted in Fig. 7, where Fig. 7(a) indicates the region of tissue selected for MRE reconstruction against the corresponding slice of a full field MR image.

IV. DISCUSSION

IV.A. μ_R results

Qualitatively, the images obtained for the μ_R reconstructions of the single inclusion tofu phantom were relatively similar for the RD and VE methods, with the location of the inclusion clearly visible in the μ_R images seen in Figs. 1(b), 2(a), and 4(d). In general, the diameter of the gelatin inclusion reconstructed by both RD and VE methods appears larger than the diameter seen in the T2* MR images, a likely result of mixing between the stiff gelatin material and the surrounding soft tofu as well as continuity requirements of the finite element based elastic modulus description. The variation in the μ_R values within both materials is seen to be relatively low, as confirmed by the standard deviation values given in Table III. For the DVE reconstruction result, high variation is seen around the region of the inclusion, where local homogeneity assumptions in the direct reconstruction method would be inaccurate. The RD reconstructions of the homogeneous gelatin phantom shows a relatively smooth distribution for μ_R , as seen in Fig. 5(b), with variation mostly appearing at the physical edges of the phantom. Finally, the stiffness distribution for the *in vivo* breast tissue seen in Figs. 6(d) and 7(b) show some variation in the background stiffness values within the healthy tissue region, while the cancerous tumor is easily located based on visual inspection of the μ_R image.

To provide an unbiased measurement of the relative difference between property values, a *mean difference* (MD) calculation

was used such that the MD between measurements *A* and *B* is given by

$$MD(A, B) = 100(\|A - \overline{AB}\| + \|B - \overline{AB}\|)/\overline{AB},$$

where $\overline{AB} = \frac{1}{2}(A+B)$. MD comparison of the property values obtained from RD and VE reconstructions of three phantoms are given in Tables VII and VIII, along with comparison against the mechanical measurements taken by DMA. Analysis of the results from the two reconstruction methods shows relatively good agreement on the values of μ_R for the single inclusion tofu phantom, with differences less than 10%. Agreement between the two reconstruction methods on the values of μ_R for the homogeneous gelatin phantom was reasonable, with a difference of 11%. Agreement between the μ_R values for both methods and those from DMA measurements was good for the 10% gelatin, with differences ranging from 2% to 9% for the VE and RD methods, respectively. Agreement between the DVE reconstruction results and DMA measurements for μ_R in the single inclusion phantom was reasonable, with a difference of 15%.

Table IX shows a comparison of the RD and corresponding $\tilde{\mu}$ values with the VE and DVE reconstruction results for the single inclusion tofu phantom. In terms of stiffness values, all three iterative reconstruction methods (RD, VE, and $\tilde{\mu}$) were in relatively close agreement for both the background and the inclusion materials, with RD reconstruction values differing from those obtained by VE reconstruction by 1%–5%, while $\tilde{\mu}$ stiffness values differed from the VE reconstructed values by 0.3%–5%. Disagreement with the DVE reconstruction result is higher, varying from 13% to 40% for the μ_R values.

For the tissue stiffness values, agreement between μ_R values observed here and those observed by Sinkus *et al.*¹ were relatively poor, with differences ranging from 109% to 115% for the healthy and cancerous tissues, respectively (see Table X). Given that the tissue property values here were generated by RD reconstruction, Eq. (13) can be used to develop the equivalent $\tilde{\mu}$ values to those RD properties given in Table VI, allowing better comparison with the values reported by Sinkus *et al.*, which were generated by VE reconstruction. These values, along with MD comparison with the values obtained by Sinkus *et al.*, are shown in Table XI. Disagreement with the stiffness values obtained by Sinkus *et al.* is seen to improve significantly with the use of the equivalent $\tilde{\mu}_R$ stiffness, with differences ranging from 23% to 74% for healthy and cancerous tissues, respectively. This level of disagreement, with the *in vivo* tissue data used here measured at 85 Hz, while the images from Sinkus *et al.* were taken at 65 Hz, is not unreasonable, given the observed power-law frequency dependence for viscoelastic properties in the breast tissue.²⁸

IV.B. ξ results

Qualitatively, the damping ratio reconstructions for the single inclusion tofu phantom again show similar results for both the VE and the RD methods, although the RD reconstruction seems to provide stronger contrast between the

FIG. 5. Image results for RD reconstruction of the homogeneous 10% gelatin phantom: (a) MR image (*ROI indicated*), (b) μ_R image (kPa), (c) ξ image (%/100), and (d) RC image (%/100).

TABLE V. Homogeneous 10% gelatin phantom, RD, and VE reconstruction results. Both the mean ($\bar{x}$) and the standard deviation (s_x) values are given.

RD	$\bar{\mu}_R$ (Pa)	s_{μ_R} (Pa)	$\bar{\xi}$ (%)	s_{ξ} (%)	$\overline{RC}$ (%)	s_{RC} (%)
10% gelatin	8056	623	7.52	5.07	64.18	25.08
VE	$\bar{\mu}_R$ (Pa)	s_{μ_R} (Pa)	$\bar{\xi}$ (%)	s_{ξ} (%)	$\overline{RC}$	s_{RC}
10% gelatin	8952	1634	9.56	10.43	NA	NA

FIG. 6. Image results for RD reconstruction of a patient undergoing neoadjuvant chemotherapy at 85 Hz excitation: (a) MR image, (b) MR image (*healthy ROI indicated*), (c) MR image (*cancer ROI indicated*), (d) μ_R image (kPa), (e) ξ image (%/100), and (f) RC image (%/100).

more highly attenuating tofu background and the gelatin. For the DVE reconstruction result, high variation is again seen around the region of the inclusion, where local homogeneity assumptions in the direct reconstruction method would be inaccurate. The location of the inclusion is not clearly characterized by the ξ images for this noniterative method. For the homogeneous phantoms, the reconstructed damping ratio distribution looks reasonably smooth, save for some variation at the boundaries of the phantom, while for the tissue results, the damping ratio values in the background healthy tissue show expected inhomogeneity while the tumor location is clearly visible as an area of relatively low damping.

MD comparison of the results from the two reconstruction methods shows somewhat poor agreement on the values of ξ for the tofu background of the single inclusion phantom, with a difference of 68%, while agreement on the 10% gelatin inclusion ξ value was slightly better, with a difference of 27%. For the gelatin inclusion material, the differences in the ξ values between MRE and DMA methods were high, ranging from 147% to 158%. For the homogeneous 10% gelatin phantom, disagreement on the values for ξ between the two reconstruction methods was reasonable, at 24%, while agree-

ment with the DMA results varied from 21% to 44%. Agreement between the DVE reconstruction result in the single inclusion phantom and DMA measurements for ξ values was poor, with 84% MD for gelatin inclusion material.

The comparison of the RD and the corresponding $\tilde{\mu}$ values with the VE reconstruction results for the single inclusion tofu phantom given in Table IX shows improved agreement between values for μ_I between the effective viscoelastic shear, $\tilde{\mu}$, and the VE reconstruction results. For this imaginary shear component, RD reconstruction values differed from those obtained by VE reconstruction by 79%–131%, while $\tilde{\mu}_I$ values differed from the VE reconstructed values by 26%–70% for the 10% gelatin and tofu materials, respectively. This noticeably better match in μ_I values between the calculated $\tilde{\mu}$ distribution and the reconstructed VE values indicates that the equivalence of these viscoelastic measures predicted by Eq. (13) is observable and that purely viscoelastic reconstructions in these materials may lead to artificially high values for μ_I . Disagreement with the DVE reconstruction result is higher, varying from 110% to 166% for μ_I values.

TABLE VI. *In vivo* breast tissue and RD reconstruction ROI results. Both the mean ($\bar{x}$) and the standard deviation (s_x) values are given.

RD	$\bar{\mu}_R$ (Pa)	s_{μ_R} (Pa)	$\bar{\xi}$ (%)	s_{ξ} (%)	$\bar{RC}$ (%)	s_{RC} (%)
Healthy tissue	2 964	3100	173.45	186.13	43.57	37.58
Cancer tissue	10 681	4247	73.36	62.99	63.81	28.61

FIG. 7. Single slice image results for RD reconstruction of a patient undergoing neoadjuvant chemotherapy at 85 Hz excitation: (a) MR image, (b) μ_R image (kPa), (c) ξ image (%/100), and (d) RC image (%/100).

For the tissue values, the damping ratio results observed here were considerably higher than those found by Sinkus *et al.*,¹ as shown in Table II. Disagreement in damping levels between the two results differed by 179% for healthy tissue

TABLE VII. MD comparison of μ_R and ξ values measured by RD, VE, DVE, and DMA (for 10% gelatin only) in the single inclusion tofu phantom.

Soft tofu BG	μ_R (%)	ξ (%)
RD vs VE	1	68
RD vs DVE	39	81
10% gelatin INC	μ_R (%)	ξ (%)
RD vs DMA	2	147
VE vs DMA	7	158
DVE vs DMA	15	84
RD vs VE	5	27
RD vs DVE	13	16

and 73% for cancerous tissue. Notably, Sinkus *et al.*¹ found increased damping in cancerous tissue, while the results here show noticeably lower damping in cancerous tissue. Considering only the imaginary shear modulus component, μ_I , the healthy tissue results reported here have an average value of $\mu_I=2539$ Pa, compared to the values of $\mu_I=225$ Pa reported by Sinkus *et al.*, a MD of 167%. Likewise, the cancerous tissue results reported here give $\mu_I=7144$ Pa, compared to the values of $\mu_I=980$ Pa reported by Sinkus *et al.*, a difference of 152%. Comparing the equivalent VE stiffness values,

TABLE VIII. MD comparison of μ_R and ξ values measured by RD, VE, and DMA in the homogeneous 10% gelatin phantom.

Homogeneous 10% gelatin	μ_R (%)	ξ (%)
RD vs DMA	9	21
VE vs DMA	2	44
RD vs VE	11	24

TABLE IX. MD comparison of μ_R and μ_I values measured by RD, VE, DVE, and $\tilde{\mu}$ [as per Eq. (13)] in the single inclusion tofu phantom.

RD vs VE	μ_R (%)	μ_I (%)
Soft tofu BG	1	131
10% gelatin INC	5	79
$\tilde{\mu}$ vs VE	μ_R (%)	μ_I (%)
Soft tofu BG	0.3	70
10% gelatin INC	5	26
$\tilde{\mu}$ vs DVE	μ_R (%)	μ_I (%)
Soft tofu BG	40	110
10% gelatin INC	13	166

$\tilde{\mu}$, reported in Table XI, the disagreement in terms of the imaginary shear component, μ_I , changes only slightly, with differences of 163% for healthy tissue and 157% for cancerous tissue, although the overall level of disagreement between the two results is still quite high. These differences in the level of damping could be due to a number of causes, such as the increase in frequency between the *in vivo* imaging sequences used in this study, 85 Hz versus 65 Hz used in Refs. 1 and 28. Other causes for the difference in the observed damping level could be due to the actuation methods used for these different *in vivo* imaging studies, which are distinguishable based on the level of precompression in the tissue, excitation source location, and excitation mode (i.e., compression vs shear).

IV.C. RC results

Qualitatively, the RC reconstructions for the single inclusion tofu phantom clearly distinguish the location of the gelatin inclusion within the tofu background, as shown in Figs. 1(d) and 4(f). Average results across the ROIs in this phantom, given in Table III, show that the tofu background material contains a noticeably different RC structure compared to the gelatin inclusion material, with the gelatin inclusion having noticeably higher RC values. For the homogeneous phantom, the RC results seen in Fig. 5(d) show relatively high levels of variation, as predicted by Eq. (11)

TABLE X. MD comparison of μ_R and ξ values for breast tissue as measured by RD at 85 Hz and Sinkus *et al.* (Ref. 1) at 65 Hz.

	μ_R (%)	ξ (%)
Healthy tissue		
RD	109	179
Cancer tissue		
RD	115	73

TABLE XI. MD comparison of μ_R and μ_I values for breast tissue as measured by $\tilde{\mu}$ [as per Eq. (13)] at 85 Hz and Sinkus *et al.* (Ref. 1) values at 65 Hz.

Values	$\tilde{\mu}_R$ (Pa)	$\tilde{\mu}_I$ (Pa)
Healthy tissue	1097	2194
Cancer tissue	6295	8104
MD	$\tilde{\mu}_R$ (%)	$\tilde{\mu}_I$ (%)
Healthy tissue	23	163
Cancer tissue	74	157

for the homogeneous material region where the RC value remains poorly defined.

For the tissue reconstruction, the region of the cancerous tumor is clearly visible in the RC image, as shown in Figs. 6(f) and 7(d), and is seen to have a noticeably higher RC value than the surrounding healthy tissue based on the mean values measured over the ROIs, as shown in Table VI. Based on the results observed in the single inclusion phantom, this would seem to indicate that the background tissue structure more closely matches the fluid saturated cellular structure of tofu, while the structure of the cancerous tissue region more closely matches the tightly grouped, randomly arranged calogen coil structure of gelatin. While this physiological interpretation of the RC image results is attractive and seems to make sense according to the observed cellular structure of these different tissue regions, much more work is required to be able to confirm that it is indeed these properties of the tissue structure that lead to the observed variations in the RC results.

V. CONCLUSIONS

A method for the reconstruction of both VE and RD based damped elastic properties has been developed for use with MR detected time-harmonic motion data and has been shown to lead to reasonable results in both homogeneous and heterogeneous phantoms of varying material types. Additionally, the initial results in RD imaging of *in vivo* tissue show reasonable agreement with the previous time-harmonic *in vivo* tissue stiffness values and indicate that both the overall damping level, ξ , as well as the RC information may have some diagnostic value in differentiating malignant cancers from healthy tissue. To date, this work is preliminary in nature, and a complete description of the microstructural origins of RD, most likely to be found through homogenization of the elastic scattering problem, has yet to be developed. Instead, a basic relationship between RD and VE has been established, with a link through an effective VE complex shear modulus, $\tilde{\mu}$, and initial reconstruction images have been achieved through the use of a subzone structured, optimization based process formulated in nearly incompressible finite elements.

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