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A Symmetrized Theil index measure of health disparities: An example using dental caries in U.S. children and adolescents

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SUMMARY

While the health status of Americans has generally improved over time, health disparities between groups of the population have been pervasive. Designing a measure that tracks the resulting disparities remains a challenge. In this paper, we propose a new measure of health disparities, the Symmetrized Theil Index (STI), and derive its design-based sampling variance in grouped survey data. Because STI is symmetric, it circumvents the drawback of the Theil Index in how groups are weighted: indeed, the latter is mostly influenced by groups with high disease frequencies. Moreover, STI is related to Pearson's chi-square test of independence for binary data, and to the F-test in one-way analysis-of-variance for continuous data. We illustrate our approach using data on dental caries for children and adolescents from the third National Health and Nutrition Examination Surveys (NHANES III; 1988–1994) and NHANES 1999–2004. Tooth decay remains the most common chronic disease in US children and adolescents. Although their oral health has generally improved, we find no change in the prevalence or severity of untreated tooth decay in US children and adolescents between surveys. These findings are consistently observed for both the overall STI and its between-group component across race/ethnicity, country of birth, survey respondent's education, income and poverty-income ratio.

Keywords

Epidemiologic methods; health disparities; dental caries; race/ethnicity; socioeconomic position; Taylor series linearization

1. INTRODUCTION

As underscored by Healthy People 2000 in September 1990 [1], and, almost ten years later, by Healthy People 2010 in January 2000 [2], the health status of Americans has been improving over time. However, despite this improvement, health disparities between racial/ethnic and socioeconomic groups have been pervasive over time [3–12]. Oral health is no exception to this trend [11,13,14]. For example, while the oral health of non-Hispanic Whites and high socioeconomic position (SEP) individuals has improved, racial/ethnic minority and low SEP groups continue to bear a higher burden of disease throughout this period [11]. Consistent with these findings, one of the two overarching goals of Healthy People 2010 is “to eliminate health disparities among segments of the population, including

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differences that occur by gender, race or ethnicity, education or income, disability, geographic location or sexual orientation” [2]. To parallel Healthy People 2010’s goal of eliminating health disparities, the publication of the first Surgeon General’s Report on Oral Health [11] not only underscores the importance of oral health but also calls attention to disparities in the oral health status of the American population. For instance, disparities in dental caries and periodontal disease, the most common oral conditions in children and adults, by gender, race/ethnicity and SEP have been pervasive in the US [11,13–25].

Although the investigation of health disparities has generated an extensive body of research, it is not clear how findings could be used to track disparities across groups of the population over time. While several measures have been proposed [26–33], measuring health disparities remains an arduous task, as illustrated in recent studies on lung cancer [34] and untreated caries [35], and criteria for best measures remain controversial [34,36,37]. Although the Theil Index (TI) and Mean Log Deviation (MLD) were originally proposed as measures of income inequality [38–42], they are now common measures of disparity in health research [28,32–34,43,44] and of inequity in environmental justice research [43,45]. Indeed, TI and MLD are well-established statistical measures of *divergence* (or relative entropy) between two probability distributions [46,47]. In the context of this paper, they measure the disproportionality between shares of disease and shares of population. TI can be looked upon as a log-likelihood ratio statistic [46,47] for testing the null hypothesis that an individual’s or a group’s population share should be “allocated” according to that individual’s or group’s share of disease. Thus, TI is mostly influenced by high disease frequencies. Similarly, MLD tests the null hypothesis that disease share should be “allocated” according to the individual’s or group’s share of population. Thus, MLD is mostly influenced by groups with a large population share [48]. Because of this inherent asymmetry, TI and MLD require a value judgment with regards to the null distribution (disease frequencies for TI and population shares for MLD), which is a critical drawback [43].

In this paper, we propose a new measure, the Symmetrized Theil Index (STI), which is simply the average of TI and MLD. Unlike TI and MLD, STI is symmetric, and therefore, it avoids the aforementioned value judgment that TI and MLD suffer from. In addition, while TI and MLD are only pre-metrics, STI is a semi-metric measure between two probability distributions [46,47]: STI satisfies all the properties of a distance measure except for the triangle inequality. Further, just like TI and MLD, STI is decomposable into the sum of two independent components: a so-called *within-group* component STI_W and a *between-group* component STI_B [41,49,50]. However, for continuous data such as on the severity of disease amongst groups in the population, we argue (cf. section 3) that the between-group component STI_B of STI is closely related to the between-group sum-of-squares (SS_B) in an analysis-of-variance decomposition. As a function of the *differences* in the group means relative to the overall mean, SS_B is an *absolute* measure of health disparity, also known as “Between-Group Variance” (BGV) [34,51]. The between-group index STI_B is a function of *ratios* of group means relative to the overall mean, thus STI_B remains a *relative* measure of health disparity. Nonetheless, STI_B can be looked upon as a *quasi*-absolute measure when it is appropriately rescaled, in that it becomes an approximate function of differences in means. **For dichotomous variables**, we argue that an approximation of the between-group component STI_B of STI is akin to Pearson’s chi-square statistic for testing the independence of an individual’s disease status and his/her socio-demographic position.

Thus, we present STI as a single measure that captures most of the desirable features of existing measures of health disparities, while at the same time compensating for some of their drawbacks. In particular, a critical drawback of TI and MLD is that they are non-symmetric in the two distributions whose divergence (or relative entropy) is being evaluated.

The sampling variances for both the overall TI and the overall MLD were derived for complex survey data using a Taylor series linearization technique [40]. In section 3, we, further, extend this linearization technique to the case of grouped complex survey data, which will provide the sampling variance of the between-group component STI_B of STI. We illustrate our approach using dental caries data from the National Health and Nutrition Examination Surveys (NHANES III and NHANES 1999–2004) for participants 2 to 19 years of age. Specifically, we examine disparities in the prevalence of tooth decay, as well as in the severity of tooth decay, i.e. the number of untreated primary and permanent dental caries lesions, in US children and adolescents who suffer from untreated caries, according to gender, race/ethnicity, country of birth, education, income, and poverty-income ratio within and between surveys.

2. DENTAL CARIES IN U.S. CHILDREN AND ADOLESCENTS

Data Source

Data for this analysis came from the NHANES III (Adult and Examination files) and the NHANES 1999–2004 (Demographic, Questionnaire and Examination files) public-use data files. NHANES III, conducted from 1988 to 1994, represents the seventh in a series of national examination studies in the US. NHANES 1999–2004 represents the ongoing collection of the NHANES from March 1999 to December 2004. Both surveys assessed the health status of a nationally representative sample of the civilian non-institutionalized US population, selected through a stratified multistage probability sampling design. Full descriptions of the sample design and methodology for NHANES III [52,53] and NHANES 1999–2004 [54] have been reported elsewhere. NHANES III yielded 13,830 records for age 2 to 19 years, of whom 10,737 had a complete dental examination. However, 60 records did not have information on dental caries, yielding an analytical sample of 10,677. NHANES 1999–2004 yielded 13,329 records for age 2 to 19 years, of whom 12,127 had a complete dental examination, with 5 records missing information on dental caries, for an analytical sample of 12,122.

Study Variables

During the dental examination in the surveys, trained dentists performed a surface-specific evaluation of coronal caries for persons two years and older [52,55]. A maximum of 20 deciduous and 28 permanent teeth were examined in children and adults. From these data, we derive the number of decayed primary teeth (dt) for children 2–11 years of age. For persons aged 12 years and older, we also calculate the numbers of decayed permanent teeth (DT). As a result, we consider the following two variables: 1) a binary variable indicating tooth decay, i.e., the presence/absence of at least one decayed primary (children aged 2 to 11 years) or permanent (adolescents aged 12 to 19 years) tooth; and 2) a numerical variable counting the number of untreated dental caries lesions among children and adolescents suffering from primary and permanent tooth decay, respectively. These two variables allow us to examine inequalities in the prevalence and the severity of tooth decay in US children and adolescents, within selected socio-demographic characteristics.

We selected gender, race/ethnicity, country of birth, education, income, and poverty-income ratio because, in the US, oral health disparities have already been documented for these characteristics over time [11,14–25]. In NHANES III, race/ethnicity was self-reported by the survey participants as non-Hispanic White, non-Hispanic Black or African-American, Mexican-American, and Other Hispanic. In NHANES 1999–2004, an additional category was added: Other Race-Including Multiracial. Hereafter, non-Hispanic Black and non-Hispanic White will be referred to as Black and White, respectively. For our analysis, we re-coded country of birth as U.S.-born (born in the 50 U.S. states or Washington, D.C.) and

foreign-born (born in Mexico or elsewhere). In NHANES III, education for the family reference person at the time of interview (hereafter, survey respondent's education) was collected as a numerical variable (number of years of education) from zero to 17 years. We categorized the survey respondent's education as follows: less than 12 years, 12 years, and more than 12 years. In NHANES 1999–2004, this variable was re-coded with the same categories as the ones created for NHANES III. In NHANES III, the total 12-month family income was recorded using increments of \$1,000 until the last category of \$50,000 and above. In NHANES 1999–2004, this variable was recorded using increments of \$5,000 until the last category of \$75,000 and above. In our analysis, we adjusted income for inflation using the Consumer Price Index [56] and then categorized it as low, medium and high. Thus, the following categories <\$14,999, \$15,000 to \$24,999, and ≥\$25,000 in NHANES III are equivalent to <\$19,999, \$20,000–\$44,999, and ≥\$45,000 in NHANES 1999–2004. In addition to income, we also examined the poverty-income ratio (PIR). PIR was reported as a continuous variable in both surveys using the USDA food assistance program (Special Supplemental Nutrition Program for Women, Infants and Children (WIC), Food Stamp Program, School Lunch and Breakfast Programs) eligibility [57,58]. For our analysis, we re-coded PIR as follows: zero to 1.850 (low), 1.851 to 3.500 (medium), and 3.501 and greater (high). Survey respondents without information on education, income, or PIR, are, nonetheless, classified and included in our analysis as a separate category. Finally, although these variables were coded ordinal, they were treated as nominal during the analysis.

3. STATISTICAL METHODOLOGY

Entropy-Based Measures of Health Disparity

With y_i representing the severity of disease for the i th individual in a sample of size n , e.g., here, the value of **dt** for the i th child in the sample, the indices TI and MLD are defined as

$$TI = \sum_{i=1}^n (y_i/y_{\bullet}) \times \ln \left(\frac{y_i/y_{\bullet}}{1/n} \right) = \frac{1}{n} \sum_{i=1}^n (y_i/\bar{y}_{\bullet}) \times \ln(y_i/\bar{y}_{\bullet}), \quad (1)$$

$$MLD = - \sum_{i=1}^n (1/n) \times \ln \left(\frac{y_i/y_{\bullet}}{1/n} \right) = - \frac{1}{n} \sum_{i=1}^n \ln(y_i/\bar{y}_{\bullet}). \quad (2)$$

Each individual contributes a relative share $y_i/y_{\bullet}$ to the sample population aggregate $y_{\bullet}$. Both TI and MLD compare that relative share to $1/n$, which is what an individual would have contributed if the sample population aggregate $y_{\bullet}$ was uniformly distributed. Further, for TI, each individual's departure from "fairness" (eq. 1) is weighted by the relative share $y_i/y_{\bullet}$: individuals with relatively large shares will contribute more to the TI than individuals with relatively small shares, i.e. the more concentrated the shares are amongst only a small number of individuals, the more inequality is ascertained. For MLD, the individual departures from "fairness" (eq. 2) are weighted by the sample population share $1/n$ instead.

Thus, TI and MLD measure the disproportionality between shares of disease and shares of population. TI can be looked upon as a log-likelihood ratio statistic [46,47] for testing the null hypothesis that an individual's or a group's population share should be "allocated" according to that individual's or group's share of disease. Thus, TI is mostly influenced by high disease frequencies. Similarly, MLD tests the null hypothesis that disease share should be "allocated" according to the individual's or group's share of population. Thus, MLD is

mostly influenced by groups with a large population share [48]. Because of this inherent asymmetry, TI and MLD require a value judgment with regards to the null distribution (disease frequencies for TI and population shares for MLD), which is a critical drawback [43]. The measure STI that we propose next is symmetric; therefore, it avoids this value judgment.

Symmetrized Theil Index (STI)

The index STI that we propose in this paper is simply the average of TI and MLD:

$$STI = \frac{1}{2} \sum_{i=1}^n (y_i/y_{\bullet} - 1/n) \times \ln \left(\frac{y_i/y_{\bullet}}{1/n} \right) = \frac{1}{2n} \sum_{i=1}^n (y_i/\bar{y} - 1) \times \ln(y_i/\bar{y}). \quad (3)$$

Note that, unlike TI and MLD, STI is, indeed, symmetric, in that, from eq. (3),

$$STI = \frac{1}{2} \sum_{i=1}^n (y_i/y_{\bullet} - 1/n) \times \ln \left(\frac{y_i/y_{\bullet}}{1/n} \right) = \frac{1}{2} \sum_{i=1}^n (1/n - y_i/y_{\bullet}) \times \ln \left(\frac{1/n}{y_i/y_{\bullet}} \right).$$

Like TI and MLD, STI remains nonnegative, and it equals zero when no disparity exists. Thus, STI is a semi-metric measure between two probability distributions [46,47]: STI satisfies all the properties of a distance measure except for the triangle inequality. Also, as with the indices TI and MLD, STI is population-weighted, **it makes it possible** to ascertain the relative inequity in an individual's shouldering the burden of disease relative to the rest of the population, regardless of the reference group, and it is sensitive to health differences **further from the average health status of the population** [48]. For a detailed review of the properties and limitations of entropy-based and other health disparity measures, we refer the interested reader to [34] and [43].

Decomposability

An important property of entropy-based measures such as TI and MLD is that they are *decomposable* into the sum of two independent components: a so-called *within-group* component and a *between-group* component [41,49,50]. Indeed, for a sample population with m groups, let y_{ij} be the value of dt for individual i in group j . Let n_j be the size of group j . Let $y_{\bullet j}$ denote the aggregate value of dt over group j , and $y_{\bullet\bullet}$ the aggregate over the sample population. Then $y_{\bullet j}/y_{\bullet\bullet}$ is the relative share of dt for group j , while n_j/n is the population share of group j . Note that, within group j , individual i contributes a relative share $y_{ij}/y_{\bullet j}$ to the group aggregate, and that its within-group population share is $1/n_j$. For STI, the decomposition $STI = STI_W + STI_B$ of STI into the sum of a within-group component STI_W and a between-group component STI_B is obtained as follows:

$$2 \times STI_W = TI_W + MLD_W = \sum_{j=1}^m \sum_{i=1}^{n_j} (y_{ij}/y_{\bullet\bullet} - 1/n) \times \ln \left(\frac{y_{ij}/y_{\bullet j}}{1/n_j} \right), \quad (4)$$

$$2 \times STI_B = TI_B + MLD_B = \sum_{j=1}^m (y_{\bullet j}/y_{\bullet\bullet} - n_j/n) \times \ln \left(\frac{y_{\bullet j}/y_{\bullet\bullet}}{n_j/n} \right). \quad (5)$$

Further, a “self-similar” property of TI and MLD [41,49,50] results from their characteristic of perfect decomposability, which allows for the separation of inequality into successively nested between and within components, so long as the population is partitioned exhaustively into mutually exclusive groups. Thus, multiple predictors of inequality may be considered simultaneously, just as they can be in multiple regression and ANOVA.

Note that, unlike the indices TI and MLD, where the respective within-group components TI_W and MLD_W are weighted-averages of within-group indices TI_j and MLD_j , namely

$$TI_W = \sum_{j=1}^m (y_{\bullet j}/y_{\bullet\bullet}) \times TI_j, \quad MLD_W = \sum_{j=1}^m (n_j/n) \times MLD_j,$$

the within-group component STI_W (eq. 4) is no longer a weighted-average of the indices STI_j . Nonetheless, as with TI and MLD, one can use the proportion STI_B/STI to ascertain how much inequality is explained by the grouping variable [41,42,49,50,59]. This is akin to the statistic $R^2 = SS_B/SS_{Total}$ in the decomposition $SS_{Total} = SS_W + SS_B$ of the total sum-of-squares. However, in the context of measuring health disparity (or economic inequality), there are important issues with benchmarking the between-group component against the overall index [60]. Indeed, as pointed out in [60], attempting to explain 100 percent of total inequality is not realistic, since $STI_B = STI$ only under two very unlikely scenarios: either each group is a singleton, or it is otherwise completely homogeneous, not allowing for any individual-level variability (since STI_W would be 0). Instead, the authors in [60] compare the between-group component STI_B to its maximum value over all comparable partitions of the population (in number and relative sizes of groups).

Measuring Disparities in Disease Prevalence

The above discussion has focused on assessing disparities in disease severity, where a particular individual-level outcome variable measures the severity of disease in those individuals with the disease, e.g. the number of untreated primary carious lesions (dt) amongst children suffering from untreated primary tooth decay. However, while the overall indices TI, MLD and STI are no longer defined for an individual-level disease indicator variable, their respective between-group “components” TI_B , MLD_B and STI_B are well-defined and are *bona fide* measures of disparity for disease prevalence between groups, since the ratios of aggregate counts $y_{\bullet j}/y_{\bullet\bullet}$ are well-defined, as seen next.

In this respect, as a stand-alone index, the between-group component STI_B has unique features that are not shared with TI_B or MLD_B , as we show next:

- i. For assessing inequalities in *disease severity*, using individual-level nonnegative data, STI_B is akin to between-group sum-of-squares (SS_B) in an analysis-of-variance decomposition;
- ii. For assessing inequalities in *disease prevalence*, using individual-level binary data, an approximation of STI_B is akin to Pearson’s chi-square test of independence in the contingency table of an individual’s disease status by socio-demographic group.

Between-Group STI as an Index of Between-Group Inequality

As with the between-group components of TI and MLD, the between-group component STI_B of STI can be used in and of itself to assess differences in disease prevalence (or severity) between groups. Using eq. (5), the index STI_B can be re-written as

$$STI_B = \frac{1}{2} \sum_{j=1}^m p_j (r_j - 1) \ln r_j, \quad (6)$$

where $r_j = \bar{y}_j / \bar{y}_\bullet$ is the ratio of the disease prevalence (or the average severity of the disease) in group j to that in the sample population, and $p_j = n_j / n$ is the relative size of group j in the sample population.

Interpretation of STI_B with numerical/continuous data

If, for each j , the ratio r_j is near 1, i.e. if the mean $\bar{y}_j$ for each group is close to the overall mean $\bar{y}_\bullet$, then STI_B is approximately equal to half the between-group sum-of-squares SS_B divided by the product of the sample size n and the square of the overall mean $\bar{y}_\bullet$. Indeed, if $r_j \approx 1$ for all j , then $\ln r_j \approx r_j - 1$, and

$$STI_B \approx \frac{1}{2} \sum_{j=1}^m p_j (r_j - 1)^2 = \frac{1}{2} \frac{SS_B}{n \bar{y}_\bullet^2}. \quad (7)$$

Recall that the between-group sum-of-squares in a one-way analysis of variance decomposition is given by $SS_B = \sum n_j (\bar{y}_j - \bar{y}_\bullet)^2$. As a function of the *differences* in the group means relative to the overall mean, SS_B is an *absolute* measure of health disparity, also known as “Between-Group Variance” (BGV) [34,51]. The between-group index STI_B is a function of *ratios* of group means relative to the overall mean, thus STI_B remains a *relative* measure of health disparity. Nonetheless, as seen in eq. (7), STI_B can be looked upon as a *quasi*-absolute measure when it is appropriately rescaled, in that it becomes an approximate function of differences in means. However, the distinct advantage of using STI_B is precisely when the mean $\bar{y}_j$ for a given group differs substantially from the overall mean $\bar{y}_\bullet$. Indeed, while the rescaled between-group sum-of-squares $SS_B / n \bar{y}_\bullet^2$ in eq. (7) is sensitive only to large values of $r_j > 1$, the between-group index STI_B is also sensitive to *small* values of $r_j < 1$, as shown in Figure 1. The latter is a desirable property for a measure of health disparity, as one group’s high average disease severity is rendered all the more inequitable when contrasted to another’s low average disease severity [48].

Interpretation of STI_B with individual-level binary data

For individual-level binary response data, such as presence/absence of untreated caries, an approximation of the between-group index STI_B is akin to Pearson’s chi-square statistic for testing independence in a $2 \times m$ contingency table of an individual’s disease status by socio-demographic group, and is also related to the “G-test” (a likelihood-ratio test) [61]. Indeed, Pearson’s chi-square is given by $\chi^2 = \sum (o - e)^2 / e$, where o is the observed cell count, e is the expected cell count under the null hypothesis of independence, and the sum is over the $2m$ cells. Straightforward algebra shows that

$$\chi^2 = \frac{\bar{y}_\bullet}{1 - \bar{y}_\bullet} n \sum_{j=1}^m p_j (r_j - 1)^2,$$

in a population of size n where the odds of untreated caries are $\bar{y}_\bullet / (1 - \bar{y}_\bullet)$. As previously, if $r_j \approx 1$ for all j , then $\ln r_j \approx r_j - 1$, and

$$STI_B \approx \frac{1}{2} \sum_{j=1}^m p_j (r_j - 1)^2 = \frac{1}{2} \frac{\chi^2}{n \times odds}. \quad (8)$$

Again, the distinct advantage of using STI_B is when a given group j has prevalence that differs substantially from the population prevalence. Indeed, while the rescaled chi-square statistic $\chi^2/(n \times odds)$ in eq. (8) is sensitive only to large values of $r_j > 1$, the between-group index STI_B is also sensitive to *small* values of $r_j < 1$, cf. Figure 1.

Sampling Variance of STI_B

Although resampling methods such as the bootstrap [62,63] for inequity measures such as TI and MLD have come under criticism [64], these methods remain in common use for estimating their standard errors [34,35]. Recently, a central limit theorem for the overall TI was established under simple random sampling [65], and, further, the sampling variances for both the overall TI and the overall MLD were derived for complex survey data using Taylor series linearization [40]. Here, we show how to extend Taylor series linearization to the case of grouped complex survey data, in order to obtain the sampling variance of the between-group index STI_B . The sampling variance we thus obtain is readily comparable to the ones obtained via Rescaled Bootstrap [35] or Balanced Repeated Replication [66–68]. Therefore, only the sampling variances using Taylor series linearization are presented in this paper.

Taylor Series Linearization Method

Using the notation in Biewen and Jenkins [40], we define the sufficient statistics:

$$U_{0j} = \sum_{s=1}^S \sum_{c=1}^{C_s} \sum_{i=1}^{n_{cs}} \delta_{ij} w_{ics}, \quad (9)$$

$$U_{1j} = \sum_{s=1}^S \sum_{c=1}^{C_s} \sum_{i=1}^{n_{cs}} \delta_{ij} w_{ics} y_{ics}, \quad (10)$$

$$T_{0j} = \sum_{s=1}^S \sum_{c=1}^{C_s} \sum_{i=1}^{n_{cs}} \delta_{ij} w_{ics} \ln y_{ics}, \quad (11)$$

$$T_{1j} = \sum_{s=1}^S \sum_{c=1}^{C_s} \sum_{i=1}^{n_{cs}} \delta_{ij} w_{ics} y_{ics} \ln y_{ics}. \quad (12)$$

In eqns. (9–12): S is the number of strata; C_s is the number of PSU's in stratum s ($C_s = 2$ for all s , except for one stratum in NHANES 1999–2004 where $C_s = 3$); n_{cs} is the number of sample observations in the PSU-stratum pair (c, s) ; w_{ics} is the sampling weight for sample observation i in the PSU-stratum pair (c, s) ; y_{ics} is the number of decayed teeth for sample observation i in the PSU-stratum pair (c, s) ; $\delta_{ij} = 1$ when observation i belongs to group j and $\delta_{ij} = 0$ otherwise; and j ranges from 1 to m , where m is the number of groups in the

population (e.g., $m=2$ for gender, $m=4$ for race/ethnicity in NHANES III). Biewen and Jenkins [40] express the TI and the MLD for the j th group as functions of the statistics U_{0j} , U_{1j} , T_{0j} , T_{1j} in eqns. (9–12), namely:

$$TI_j = \frac{T_{1j}}{U_{1j}} - \ln\left(\frac{U_{1j}}{U_{0j}}\right), \quad MLD_j = \ln\left(\frac{U_{1j}}{U_{0j}}\right) - \frac{T_{0j}}{U_{0j}}. \quad (13)$$

By taking the average of TI_j and MLD_j in eq. (13), we see that the STI_j becomes:

$$STI_j = \frac{1}{2} \left(\frac{T_{1j}}{U_{1j}} - \frac{T_{0j}}{U_{0j}} \right).$$

Further, Biewen and Jenkins [40] provide a Taylor series linearization of the sampling variance of the TI and the MLD by explicitly computing their partial derivatives with respect to each of the sufficient statistics U_{0j} , U_{1j} , T_{0j} , T_{1j} in eqns. (9–12). However, Biewen and Jenkins [40] only deal with ungrouped data (i.e., case $m = 1$ in the above expressions). Here, we show how to extend the Taylor series linearization method to grouped data (case $m > 1$), whence the decomposition $STI = STI_W + STI_B$ holds. For the sake of brevity, we focus the discussion here on the between-group component STI_B . The derivation of the sampling variance of the within-group component STI_W — and, in effect, for the components TI_B and TI_W of TI , and the components MLD_B and MLD_W of MLD — proceeds similarly and is implemented in the companion R code to this paper (cf. *Statistical Analysis* section below).

Using the statistics in eqns. (9–12) in the expression of STI_B in eq. (6), we have

$$p_j = U_{0j}/U_{0\bullet}, \quad p_j r_j = U_{1j}/U_{1\bullet}, \quad (14)$$

where $U_{0\bullet} = \sum_{j=1}^m U_{0j}$ and $U_{1\bullet} = \sum_{j=1}^m U_{1j}$, and where r_j is the ratio $\bar{y}_j/\bar{y}_\bullet$ of the within-group sample mean $\bar{y}_j = U_{1j}/U_{0j}$ to the overall sample mean $\bar{y}_\bullet = U_{1\bullet}/U_{0\bullet}$. Thus, the partial derivatives of STI_B with respect to each one of the statistics in eqns. (9–12), are:

$$\frac{\partial STI_B}{\partial U_{0j}} = \frac{1}{2} \frac{1}{U_{0\bullet}} [\ln(1/r_j) - MLD_B - (r_j - 1)], \quad (15)$$

$$\frac{\partial STI_B}{\partial U_{1j}} = \frac{1}{2} \frac{1}{U_{1\bullet}} [\ln r_j - TI_B - (1/r_j - 1)], \quad (16)$$

$$\frac{\partial STI_B}{\partial T_{0j}} = 0, \quad (17)$$

$$\frac{\partial STI_B}{\partial T_{1j}} = 0. \quad (18)$$

For the last step in Taylor series linearization, Biewen and Jenkins [40] suggest introducing an artificial variable σ_{ijcs} that represents the variance contribution from each sample observation. The variable σ_{ijcs} is obtained by taking the dot product of the vector of partial derivatives (eqns. 15–18) with the vector of summands in the sufficient statistics U_{0j} , U_{1j} , T_{0j} , T_{1j} (eqns. 9–12):

$$\sigma_{ijcs} = \delta_{ij} w_{ics} \times \left(\frac{\partial STI_B}{\partial U_{0j}} + \frac{\partial STI_B}{\partial U_{1j}} y_{ics} + \frac{\partial STI_B}{\partial T_{0j}} \ln y_{ics} + \frac{\partial STI_B}{\partial T_{1j}} y_{ics} \ln y_{ics} \right)$$

An estimate of the sample variance of STI_B is given by the sampling variance of the total statistic

$$\sum_{i=1}^{n_{cs}} \sum_{j=1}^m \sigma_{ijcs}$$

The latter is readily available, e.g. using the command for survey estimation of variances of totals ('svytotal') in the R package 'survey' [69].

Statistical Analysis

For each survey (NHANES III and NHANES 1999–2004), prevalence of untreated primary and permanent tooth decay, mean number of primary and permanent teeth with carious lesions among those with untreated tooth decay, and group-specific as well as between-group STIs and their design-based standard errors for selected socio-demographic characteristics are presented for children and adolescents.

Statistical differences were assessed for group-specific and between-group STIs, as well as over time, using simple t-tests, with a Bonferroni adjustment for multiple comparisons when necessary. These differences were consistent with results from t-tests, ANOVA, Pearson and Cochran-Mantel-Haenszel chi-square tests obtained from SUDAAN for means and proportions comparisons. Data management procedures were carried out in SAS [70]. The statistical analyses were conducted in R [71], using the 'survey' package [69,72], and in SUDAAN [73], which both take into account the complex sampling design. R code for computing the indices TI, MLD and STI, their group-specific and between-group components in grouped-data, and their design-based standard errors, can be accessed at <http://biostat.hunter.cuny.edu/talih>. This code can be used in R or in SAS through the SAS/IML® Studio 3.2.

4. RESULTS

In general, the overall prevalence of untreated primary and permanent tooth decay for children and adolescents is nearly unchanged between NHANES III (22.8% and 20.5%) and NHANES 1999–2004 (23.4 % and 19.4%; p-values > 0.05; Table 1). However, children and adolescents of Mexican-American ethnicity, those who are foreign-born, those where the survey respondent's education was less than high school, those with low income and low PIR exhibit the highest prevalence of untreated caries regardless of the survey (all p-values for chi-square tests of independence < 0.05).

Although no significant differences were observed between surveys for children and adolescents in the mean number of decayed primary and permanent teeth in the overall population and among those with untreated tooth decay, there were differences within socio-

demographic characteristics in NHANES III and in NHANES 1999–2004 (Table 1). For instance, there were significant differences between income and PIR groups for mean number of untreated carious lesions among children with dental caries in both NHANES III and NHANES 1999–2004.

Table 2 shows the between-group STI for the prevalence of untreated tooth decay as well as the mean number of primary carious lesions among those with untreated tooth decay in children in NHANES III and NHANES 1999–2004. Significant values of the between-group STI range from 9.3% (PIR) to 1.3% (country of birth) in NHANES III while for NHANES 1999–2004 the range was from 8.7% (PIR) to 2.2% (race/ethnicity). The between-group STI was not significant for gender in either survey while country of birth was not significant in NHANES 1999–2004. These results parallel the results presented in Table 1 using the chi-square tests of independence. For the mean number of primary teeth with dental caries, the overall STI was 29.3% and 29.5% in NHANES III and NHANES 1999–2004 indicating not only inequalities among those with dental caries in each survey but also concurring with the results in Table 1, which indicated no change in inequalities during the time period between the two surveys. In both surveys, income and poverty-income ratio (PIR) explain a significant portion of the overall inequality, with the proportion of the overall STI corresponding to its between-group component ranging from 2.3% for income in NHANES 1999–2004 to 4.2% for PIR in NHANES III. While statistically significant, these results suggest that the larger contribution to inequalities in both the prevalence and severity of untreated tooth decay remains attributed to the (unexplained) within-group component of the STI. Finally, the pairwise comparisons of STI between surveys for the prevalence of untreated tooth decay and mean number of primary teeth with dental caries were not statistically significant.

For adolescents, the between-group STI for the prevalence of permanent untreated tooth decay was significant for comparison by race/ethnicity, survey respondent's education, income and PIR, with values ranging from 5.9% for race/ethnicity to 10.0% for PIR in NHANES III and 2.8% for race/ethnicity to 10.5% for PIR in NHANES 1999–2004 (Table 3). Country of birth had a significant 1.7% between-group STI in NHANES 1999–2004. The overall STI for the mean number of decayed permanent teeth in NHANES III was 30.7% while for NHANES 1999–2004 it was 28.5%. There were no significant contributions to the overall STI due to the between-group component for any characteristic in NHANES III or NHANES 1999–2004, indicating that contributions to overall STI come exclusively from the (unexplained) within-group component. Finally, as before, the pairwise comparisons of STI between surveys for the prevalence of untreated tooth decay and mean number of permanent teeth with dental caries were not statistically significant.

5. DISCUSSION

Although their oral health has generally improved over the years, tooth decay remains the most common chronic disease in US children and adolescents. Among the oral health objectives of Healthy People 2010 is to reduce the proportion of children and adolescents with untreated dental decay in their primary and permanent teeth (Objectives 21–2a and 21–2c) [2]. However, evidence suggests that there has been no change in the prevalence of untreated primary and permanent tooth decay in children and adolescents between 1988–1994 (the baseline data used for Healthy People 2010) and 1999–2004 [14]. This finding was also true for the mean number of teeth with dental caries in the primary and permanent dentition (i.e., the dental caries component of the dft and DMFT indices for children and adolescents). Our analysis concurs with these findings, with both the overall STI and its between-group component STI_B exhibiting no significant change between NHANES III and NHANES 1999–2004 for children and adolescents. Moreover, our findings were

consistently observed across race/ethnicity, country of birth, survey respondent's education, income and PIR.

We find that poverty-income ratio (PIR) remains the single characteristic with the highest between-group STI for the prevalence of untreated primary tooth decay in children in both surveys. A further decomposition of STI [41,49,50] by socio-demographic characteristics of children within each PIR level — which is equivalent to examining the interaction with PIR in a conventional logistic regression — reveals that only race/ethnicity had significant values of between-group STI among children with low PIR in NHANES III and those with high PIR in NHANES 1999–2004 (p-values < 0.01; data not shown). This finding is corroborated with logistic regression fitted in SUDAAN. We find that Mexican-American (OR:1.98) children in low PIR for NHANES III and non-Hispanic black (OR:2.72) and Mexican-American (OR:3.87) in high PIR for NHANES 1999–2004 exhibit greater odds than non-Hispanic white children. This underscores that 1) race/ethnicity remains a significant determinant of dental caries in US children; and 2) the relationship between socioeconomic indicators and race/ethnicity is complex as it relates to issues of health disparities in the US [8,10,74]. However, the same analysis, using permanent teeth for adolescents, yields no significant between-group STI values for any covariate in either survey.

Building on work by Harper et al. [34] and Cheng et al. [35], we used the STI and obtained consistent results both with binary and with continuous data, as well as over time. As discussed at length in this paper, STI shares many of the well-established properties of TI and MLD. However, STI is symmetric, therefore it avoids the aforementioned value judgment in the choice of null distribution that TI and MLD suffer from. In addition, as shown above, the between-group component STI_B of STI has unique features that are not shared with TI or MLD: (i) for assessing inequalities in *disease severity*, using individual-level nonnegative data, STI_B is akin to between-group sum-of-squares (SS_B) in an analysis-of-variance decomposition; and, (ii) for assessing inequalities in *disease prevalence*, using individual-level binary data, an approximation of STI_B is akin to Pearson's chi-square test of independence in the contingency table of an individual's disease status by socio-demographic group. Moreover, STI can be looked upon both as a relative and, to some extent, a quasi-absolute measure, an issue of current debate regarding the best measure to document health disparities [36,37]. Nonetheless, in examining disparities in dental health amongst US children and adolescents, the relatively small numerical values of STI_B , even while statistically significant (cf. Tables 2 and 3), underscore the small contribution of the groups within each socio-demographic characteristic, suggesting that much of the overall inequality in the population remains unexplained by the selected socio-demographic variables.

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Figure 1.

Comparing the behavior of the function $r \mapsto (r-1)^2$ (**dashed line**) to that of $r \mapsto (r-1) \ln r$ (**solid line**) for values of r near one. While the former is sensitive only to large values of $r > 1$, the latter is also sensitive to *small* values of $r < 1$. A value of $r_j = 1$ implies that the group mean or prevalence for group j is equal to the overall mean or prevalence.

Prevalence (%) of untreated tooth decay, overall mean number of decayed primary (dt) and permanent (DT) teeth,^a and conditional mean number of primary and permanent teeth with carious lesions among those with untreated tooth decay for selected socio-demographic characteristics in US children (2–11 years of age) and adolescents (12–19 years of age):The NHANES III and NHANES 1999–2004.*

Table 1

Demographic and SEP characteristics	Children: 2–11 years of age					Adolescents: 12–19 years of age						
	NHANES III (N=7,537)			NHANES 1999–2004 ^{b,c} (N=5,527)			NHANES III (N=3,140)			NHANES 1999–2004 (N=6,595)		
	% ^d	Overall Mean ^e dt	Conditional Mean/ ^f dt (for dt > 0)	%	Overall Mean dt	Conditional Mean dt (for dt > 0)	%	Overall Mean DT	Conditional Mean DT (for DT > 0)	%	Overall Mean DT	Conditional Mean DT (for DT > 0)
Overall	22.8 (1.01)	0.62 (0.04)	2.71 (0.10)	23.4 (1.23)	0.65 (0.05)	2.79 (0.10)	20.5 (1.23)	0.52 (0.05)	2.53 (0.10)	19.4 (1.06)	0.46 (0.04)	2.38 (0.12)
Gender												
Male	22.6 (1.09)	0.61 (0.05)	2.71 (0.14)	24.6 (1.67)	0.67 (0.06)	2.75 (0.14)	19.8 (1.33)	0.51 (0.05)	2.29 (0.20)	19.6 (1.24)	0.50 (0.05)	2.56 (0.21)
Female *	22.9 ^d (1.27)	0.62 ^e (0.05)	2.71 (0.12)	22.1 ^d (1.55)	0.63 ^f (0.05)	2.85 (0.11)	21.2 ^d (1.84)	0.52 ^e (0.08)	2.47 (0.29)	19.1 ^d (1.30)	0.42 ^f (0.04)	2.19 (0.10)
Race/Ethnicity												
NH White	18.1 (1.28)	0.44 (0.05)	2.46 (0.15)	19.8 (1.79)	0.55 (0.07)	2.76 (0.17)	15.4 (1.36)	0.37 (0.04)	2.37 (0.17)	15.9 (1.49)	0.41 (0.06)	2.58 (0.21)
NH Black	28.3 (1.54)	0.74 (0.06)	2.63 (0.10)	28.4 (1.57)	0.77 (0.07)	2.69 (0.12)	33.1 (2.13)	0.83 (0.05)	2.51 (0.10)	25.4 (1.39)	0.54 (0.04)	2.17 (0.10)
Mexican American	37.7 (1.86)	1.19 (0.09)	3.15 (0.16)	33.7 (1.85)	0.95 (0.06)	3.81 (0.11)	32.6 (1.87)	0.76 (0.06)	2.34 (0.09)	28.4 (1.48)	0.62 (0.05)	2.19 (0.08)
Other Hispanic	32.0 (4.32)	1.06 (0.18)	3.31 (0.35)	21.8 (3.39)	0.62 (0.12)	2.87 (0.20)	24.8 (4.23)	0.84 (0.29)	3.37 (0.95)	24.7 (3.06)	0.52 (0.08)	2.12 (0.21)
Other-Multiracial	-	-	-	26.3 (4.84)	0.87 (0.23)	3.31 (0.58)	-	-	-	18.0 (3.64)	0.39 (0.12)	2.19 (0.45)
Country of Birth												
US	21.9 (1.01)	0.57 (0.04)	2.59 (0.09)	23.0 (1.30)	0.64 (0.05)	2.78 (0.11)	19.2 (1.27)	0.45 (0.03)	2.35 (0.10)	18.1 (1.12)	0.44 (0.04)	2.41 (0.13)
Foreign	42.1 (5.48)	1.69 (0.37)	4.01 (0.63)	32.9 (4.18)	0.97 (0.16)	2.96 (0.25)	31.9 (4.38)	1.09 (0.30)	3.42 (0.71)	31.6 (2.34)	0.70 (0.08)	2.20 (0.17)
Education												
< 12 years	36.2 (1.85)	1.10 (0.09)	3.04 (0.16)	33.5 (2.27)	1.05 (0.12)	3.13 (0.21)	32.9 (1.77)	0.99 (0.14)	3.02 (0.35)	31.4 (1.82)	0.79 (0.08)	2.53 (0.19)
=12 years	24.5 (4.52)	0.66 (0.06)	2.69 (0.19)	26.8 (2.31)	0.72 (0.07)	2.67 (0.15)	21.8 (2.09)	0.44 (0.05)	2.02 (0.13)	20.3 (1.69)	0.49 (0.06)	2.44 (0.18)
>12 years	13.6 (1.31)	0.30 (0.04)	2.20 (0.19)	16.8 (1.41)	0.43 (0.05)	2.59 (0.15)	11.2 (1.59)	0.27 (0.04)	2.42 (0.27)	12.8 (1.12)	0.27 (0.04)	2.15 (0.16)
Missing	-	-	-	24.9 (4.06)	0.66 (0.14)	2.63 (0.25)	-	-	-	27.1 (4.09)	0.67 (0.14)	2.48 (0.34)
Family Income												
Low	34.4 (2.07)	1.09 (0.09)	3.18 (0.16)	31.1 (1.72)	0.96 (0.07)	3.08 (0.15)	31.0 (2.47)	0.72 (0.07)	2.31 (0.13)	26.7 (2.03)	0.64 (0.07)	2.40 (0.15)
Medium	27.8 (2.14)	0.72 (0.09)	2.60 (0.23)	29.0 (2.38)	0.84 (0.11)	2.91 (0.24)	29.8 (3.27)	0.80 (0.20)	2.67 (0.53)	26.0 (1.62)	0.68 (0.08)	2.61 (0.23)
High	15.8 (1.06)	0.36 (0.03)	2.26 (0.12)	14.0 (1.07)	0.32 (0.03)	2.27 (0.15)	13.7 (1.16)	0.33 (0.04)	2.41 (0.22)	10.3 (1.10)	0.21 (0.03)	2.05 (0.15)
Missing	24.4 (2.51)	0.80 (0.12)	3.30 (0.31)	22.5 (3.47)	0.57 (0.12)	2.53 (0.22)	19.1 (3.44)	0.70 (0.16)	3.64 (0.54)	25.4 (4.05)	0.62 (0.10)	2.46 (0.31)

Demographic and SEP characteristics	Children: 2–11 years of age						Adolescents: 12–19 years of age					
	NHANES III (N=7,537)			NHANES 1999–2004 ^{b,c} (N=5,527)			NHANES III (N=3,140)			NHANES 1999–2004 (N=6,595)		
	% ^d	Overall Mean ^e dt	Conditional Mean/ ^{dt} (for dt > 0)	%	Overall Mean dt	Conditional Mean dt (for dt > 0)	%	Overall Mean DT	Conditional Mean DT (for DT > 0)	%	Overall Mean DT	Conditional Mean DT (for DT > 0)
Poverty-Income Ratio												
<=1.85	31.9 (1.55)	0.94 (0.07)	2.96 (0.14)	31.5 (1.73)	0.95 (0.07)	3.02 (0.12)	30.5 (2.32)	0.77 (0.10)	2.54 (0.22)	26.6 (1.50)	0.66 (0.06)	2.47 (0.16)
1.851–3.50	17.5 (1.35)	0.39 (0.05)	2.24 (0.17)	21.1 (1.66)	0.56 (0.07)	2.68 (0.24)	15.5 (1.80)	0.35 (0.06)	2.28 (0.25)	19.0 (1.95)	0.45 (0.06)	2.35 (0.18)
3.501–5.00	9.3 (1.75)	0.17 (0.03)	1.79 (0.21)	10.2 (1.18)	0.19 (0.03)	1.88 (0.18)	9.4 (1.55)	0.20 (0.05)	2.12 (0.35)	8.2 (1.10)	0.17 (0.03)	2.09 (0.31)
Missing	24.4 (2.51)	0.80 (0.12)	3.30 (0.31)	24.3 (3.40)	0.60 (0.10)	2.49 (0.18)	18.2 (3.31)	0.69 (0.16)	3.81 (0.55)	25.3 (2.85)	0.59 (0.07)	2.32 (0.24)

Notes for table 1:

* Superscripts on the row for Female in the table are for illustration: they also apply to the other rows.

^aPrevalence & means (standard errors) of untreated tooth decay for primary teeth for children aged 2–11 and for permanent teeth for adolescents aged 12–19.

^bAll p-values for prevalence comparisons were <0.01 with the exception of gender in both children (=0.40) and adolescents (=0.75).

^cAll p-values for mean untreated dental caries lesions in the overall and among those with untreated tooth decay were >0.05 for children and adolescents with the exception of income for those with untreated tooth decay in children (0.04) and race/ethnicity in the overall population for adolescents (=0.002) and income among adolescents with untreated decay (=0.04).

^dAll p-values for chi-square tests of independence are <0.01 with the exception of gender in NHANES III (=0.81 for children and =0.49 for adolescents) and in NHANES 1999–2004 (=0.25 for children and =0.72 for adolescents) and country of birth in NHANES 1999–2004 for children (=0.035).

^eAll p-values for ANOVA were <0.01 for children with exception of gender (=0.51) and country of birth (=0.014) in the overall population, and gender (=0.75), race/ethnicity (=0.06), country of birth (=0.02) and education (=0.02) in those with untreated tooth decay in NHANES III. For adolescents, all p-values for ANOVA were <0.01 with exception of gender (=0.91) and country of birth (=0.04) in the overall population, and gender (=0.76), race/ethnicity (=0.39), country of birth (=0.13) and education (=0.015) in those with untreated tooth decay in NHANES III.

^fP-values for ANOVA were <0.01 for children for race/ethnicity, country of birth, education, income and PIR in the overall population only in NHANES 1999–2004. For adolescents, p-values for ANOVA were <0.01 for country of birth, education, income and PIR in the overall population only in NHANES 1999–2004. P-values for ANOVA were <0.01 only for income and PIR for the mean number of teeth with dental caries in children with untreated tooth decay in NHANES 1999–2004.

Symmetrized Theil Index (STI) and 95% Confidence Intervals (CI) for disparities in the prevalence and severity of untreated primary tooth decay among US children (2–11 years of age) by selected socio-demographic characteristics: The NHANES III and NHANES 1999–2004

Table 2

Characteristics	NHANES III			NHANES 1999–2004		
	Prevalence of untreated primary tooth decay					
	STI ^{a,b,e}	95% CI ^{b,c}	% of total ^d	STI ^{a,b,e}	95% CI ^{b,c}	% of total ^d
Overall	-	-	-	-	-	-
Gender	0.00	-0.07, 0.07	-	0.14	-0.48, 0.76	-
Race/Ethnicity	4.15	2.05, 6.25	-	2.17	0.37, 3.97	-
Country of Birth	1.29	0.23, 2.35	-	0.30	-0.14, 0.74	-
Survey Respondent Education	7.71	4.32, 11.09	-	4.45	1.81, 7.09	-
Income	6.12	3.72, 8.52	-	6.36	4.32, 8.39	-
Poverty-Income Ratio	9.34	5.42, 13.26	-	8.66	5.49, 11.84	-
Mean number of untreated carious lesions for those with primary tooth decay						
	STI	95%CI	% of total	STI	95%CI	% of total
Overall	29.31	25.92, 32.70	-	29.47	26.08, 32.85	-
Gender	0.00	-0.08, 0.08	0.00	0.02	-0.08, 0.11	0.07
Race/Ethnicity	0.72	-0.22, 1.66	2.46	0.10	-0.26, 0.46	0.34
Country of Birth	0.90	-0.58, 2.37	3.07	0.00	-0.05, 0.05	0.00
Survey Respondent Education	0.77	-0.18, 1.73	2.63	0.36	-0.31, 1.03	1.22
Income	1.12	0.21, 2.03	3.82	0.69	-0.02, 1.41	2.34
Poverty-Income Ratio	1.24	0.01, 2.48	4.23	0.97	0.18, 1.76	3.29

^aWith the exception of the overall STI for the mean number of untreated carious lesions among children with primary tooth decay, the STI is the between-group STI for the characteristics presented;

^bSTI values multiplied by 100;

^cDesign-based confidence intervals are computed via Taylor series linearization (for the standard errors) and using the normal approximation (for the 95% confidence level);

^dThe proportion is the proportion of the overall STI corresponding to its between-group component;

^ePairwise comparisons of STI between surveys for prevalence of untreated tooth decay and mean number of primary teeth with dental caries for the overall and each variable indices are not statistically significant.

Symmetrized Theil Index (STI) for disparities in the prevalence and severity of untreated permanent tooth decay among US adolescents (12–19 years of age) by selected socio-demographic characteristics: The NHANES III and NHANES 1999–2004

Table 3

Characteristics	NHANES III			NHANES 1999–2004		
	Prevalence of untreated primary tooth decay					
	STI ^{a,b,e}	95% CI ^{b,c}	% of total ^d	STI ^{a,b,e}	95% CI ^{b,c}	% of total ^d
Overall	-	-	-	-	-	-
Gender	0.06	-0.27, 0.39	-	0.01	-0.08, 0.10	-
Race/Ethnicity	5.95	2.99, 8.92	-	2.84	0.57, 5.11	-
Country of Birth	1.43	-0.43, 3.28	-	1.70	0.47, 2.94	-
Survey Respondent Education	9.20	4.24, 14.15	-	7.01	4.10, 9.91	-
Income	7.88	4.17, 11.59	-	9.88	5.72, 14.05	-
Poverty-Income Ratio	9.99	4.71, 15.26	-	10.52	5.80, 15.24	-
Mean number of untreated carious lesions for those with primary tooth decay						
	STI	95%CI	% of total	STI	95%CI	% of total
Overall	30.74	23.72, 37.76	-	28.49	24.66, 32.33	-
Gender	0.03	-0.33, 0.39	0.10	0.30	-0.39, 0.99	1.05
Race/Ethnicity	0.72	-1.67, 3.10	2.34	0.38	-0.33, 1.09	1.33
Country of Birth	1.06	-1.37, 3.49	3.45	0.06	-0.13, 0.24	0.21
Survey Respondent Education	1.57	-0.54, 3.67	5.11	0.24	-0.29, 0.78	0.84
Income	0.17	-0.81, 1.15	0.55	0.41	-0.37, 1.19	1.44
Poverty-Income Ratio	0.20	-0.47, 0.87	0.65	0.15	-0.31, 0.62	0.53

^aWith the exception of the overall STI for the mean number of untreated carious lesions among adolescents with permanent toothdecay, the STI is the between-group STI for the characteristics presented;

^bSTI values multiplied by 100;

^cDesign-based confidence intervals are computed via Taylor series linearization (for the standard errors) and using the normal approximation (for the 95% confidence level);

^dThe proportion is the proportion of the overall STI corresponding to its between-group component;

^ePairwise comparisons of STI between surveys for prevalence of untreated tooth decay and mean number of permanent teeth with dental caries for the overall and each variable indices are not statistically significant.