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Constraining the Initial Phase in Water-Fat Separation

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Abstract

An algorithm is described for use in chemical shift based water-fat separation to constrain the phase of both species to be equal at an echo-time of zero. This constraint is physically reasonable since the initial phase should be a property of the excitation pulse and receiver coil only. The advantages of phase-constrained water-fat separation, namely improved noise performance and/or reduced data requirements (fewer echos), are demonstrated in simulations and experiments.

Keywords

Water-Fat Separation; Dixon; Chemical Shift; Phase

Introduction

Numerical modeling of the magnetic resonance (MR) signal variation with echo-time has become a common approach for separating water and fat (1,2). The use of numerical modeling overcomes many of the limitations of earlier algebraic techniques, such as in-phase and out-of-phase imaging, that rely upon specific echo-times, a simplified fat spectrum and negligible T2* decay (3,4).

The most commonly used models remain oversimplified compared to the analysis used in spectroscopy. In spectroscopy there are typically thousands of acquired data points (echo-times) and thus analyses are amenable to curve-fitting many tens of free parameters. Additionally, often only a single voxel is considered and user interaction is expected to help initialize parameters and adjust fitting ranges. In comparison, chemical shift based water-fat separation imaging methods acquire very few echos and operate upon tens of thousands of voxels, so the possibility of user interaction is limited.

To improve the reliability of automated curve-fitting in imaging, prior knowledge and physical constraints must be employed. For example, the frequencies and relative peak areas are fixed and presumed known in accordance with previously measured values, which

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allows the five or more peaks in fat to be modeled as a single parameter (5,6). Such strategies are referred to as spectral modeling. Furthermore, all peaks in the same voxel are assumed to be Lorentzian with identical linewidth (i.e. same T2* decay). Changes in the magnetic field (B_0) from voxel to voxel are constrained to vary slowly over the image, which is both physically reasonable and helpful to improve robustness.

Current state-of-the-art models contain up to six unknowns per voxel, which comprise linear terms (complex water and fat) and nonlinear terms (field inhomogeneity and T2*). The unknowns are estimated by iterative search over a 6-dimensional parameter space to minimize the sum of squares difference between the model and the measured data points, subject to constraints. Such constraints include imposing smoothness of the field and/or T2* maps, which may be enforced by applying a low-pass filter on each iteration (1), by penalizing non-smooth estimates (2) or by a parameterization in terms of smooth basis functions (7). There is substantial difficulty in ensuring iterations converge to the global solution and no approach is clearly superior at the current time.

An important innovation in improving reliability has been to use separable least squares to obtain the linear terms at virtually no computational cost (8). This reduces the dimension of the parameter space to the nonlinear terms only and is now widely used in MR analyses (1,2,9,10). According to separable least squares, the linear terms are obtained at each iteration of the nonlinear search by solving a linear equation

$$\mathbf{A}\mathbf{x}=\mathbf{b} \quad (1)$$

where $\mathbf{A}$ is a complex $m \times 2$ matrix representing the echo-time variation of water and fat, m is number of echos, $\mathbf{x}$ is a complex 2-vector representing water and fat and $\mathbf{b}$ is a complex m -vector of acquired data points at different echo-times (corrected for the effects of field inhomogeneity and/or T2*). As a concrete example, three echo-times are chosen from Ref ¹¹ ($5\pi/6$, $9\pi/6$ and $13\pi/6$) and the liver fat spectrum from Ref 12, which leads to the following matrix.

$$\mathbf{A} = \begin{bmatrix} 1.000 & -0.657+0.478i \\ 1.000 & -0.011 - 0.837i \\ 1.000 & 0.726+0.293i \end{bmatrix} \quad (2)$$

For most echo-time choices the matrix $\mathbf{A}^H\mathbf{A}$ is non-singular and the linear least squares solution to Eq 1 can be easily obtained by Moore-Penrose pseudoinverse. The condition number depends on the echo-times and careful choices can help to reduce noise amplification in the final water and fat images (11,13).

$$\mathbf{x}_{lin} = (\mathbf{A}^H\mathbf{A})^{-1} \mathbf{A}^H\mathbf{b} \quad (3)$$

With this linear formulation, water and fat are modeled as independent complex variables having their own amplitude and phase. However this is an overly general model for the water-fat separation problem. In many imaging sequences water and fat are known to have the same phase at certain echo-times, which gives rise to the familiar in-phase/out-of-phase behavior. E.g. images obtained at 3.0 T using a spoiled gradient recalled (SPGR) echo sequence are considered in-phase at 0.0, 2.3, 4.6... ms.

However because of the complexity due to spectral modeling and J -coupling, water and fat are only truly in-phase at an echo-time of zero. At this point the *initial phase* is the same for water and fat and is a property of the receiver coil and radiofrequency (RF) pulse (4,14). While this is certainly true for low flip angle and symmetric RF pulses, the initial phase may exhibit a dependence on the off-resonance frequency with some non-standard pulses (15). Phase accrual during the RF pulse is predictable from the chemical shift and may be included in the model but is not considered further in the present study.

Applying a constraint that the phases should be equal at a certain echo-time provides a better model of the MR physics and has advantages due to the fact that fewer parameters need to be estimated (i.e. two amplitudes and one phase rather than two amplitudes and two phases). Previous studies that have employed a phase constraint have reported advantages over the linear approach, such as reduced data requirements (16) and reduction of noise errors (17). The main disadvantage has been that the linear formulation of Eq 3 cannot be used and so the common phase term must be included as a nonlinear parameter. This increases the dimension of the nonlinear parameter space, which in turn reduces the speed and reliability, and offsets some of the advantages of the phase-constraint.

The present study presents a new, direct method for estimating the initial phase that does not require curve-fitting and therefore enables fast and reliable calculation of this parameter. Comparisons between the linear and phase-constrained methods have been performed on simulated and acquired data to measure the noise properties and to demonstrate the feasibility of water-fat separation using small numbers of echos.

Methods

Water and fat signals were estimated either by linear least squares (Eq 3) or by phase-constrained least squares using the formula given in Eq 4 (see Appendix)

$$\mathbf{x}_{pc} = \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} \text{Re}(\mathbf{A}^H \mathbf{b} e^{-i\phi}) e^{i\phi} \quad (4)$$

where initial phase $\phi = \frac{1}{2} \angle (\mathbf{A}^H \mathbf{b})^T \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} (\mathbf{A}^H \mathbf{b})$. Two key differences between Eq 3 and 4 are the elimination of $\text{Im}(\mathbf{A}^H \mathbf{b} e^{-i\phi})$ and the presence of $\text{Re}(\mathbf{A}^H \mathbf{A})^{-1}$ rather than $(\mathbf{A}^H \mathbf{A})^{-1}$. The loss of the imaginary noise and smaller condition number (e.g. 1.524 and 1.530, respectively for the matrix in Eq 2) indicate that the phase-constrained estimator should exhibit lower noise error than the linear.

Simulations

Simulated data were created using matrix $\mathbf{A}$ from Eq 2 (corresponding to three echo-times: 0.96, 1.73 and 2.49 ms at 3.0 T), water + fat = 1 and initial phase = 0. Gaussian random noise with standard deviation 0.0707 was added to the real and imaginary parts such that the amplitude in-phase image had a signal-to-noise ratio (SNR) of 10. Water and fat were estimated using Eq 3 and Eq 4 and means and standard deviations were computed in up to 10^8 trials.

MR Imaging

To demonstrate the use of the phase-constraint in separating water and fat in MR imaging, data were acquired on a 3.0T Signa HDx (GE Healthcare, Milwaukee, WI) using an 8-channel torso array. Four echos were acquired using a 3D SPGR sequence with flyback

gradients at TE 0.92, 1.65, 2.38 and 3.10 ms. Other parameters were TR 8.3 ms, flip angle 6°, matrix 256, slice 8 mm, bandwidth ± 166 kHz and Array Spatial Sensitivity Encoding Technique (ASSET) factor 2. Abdominal scanning was performed on a volunteer who was known to have no liver fat. The SNR measured in the right lobe of the liver on the amplitude in-phase image was 18.1. Real and imaginary images were taken offline for processing, as described below.

A water-fat separation method was implemented in MATLAB (The Mathworks, Natick MA) using the Gauss-Newton algorithm with separable least squares and prior knowledge of the fat spectrum (12). The field map was parameterized as a 3rd-order polynomial defining a surface: $f(x, y) = \beta_0 + \beta_1x + \beta_2y + \beta_3x^2 + \beta_4y^2 + \beta_5xy + \dots$ with 10 unknown coefficients (β_j). For simplicity T2* was neglected but may be included by allowing β_j to be complex (13). Parameters were initialized to zero and typical final values were ± 1 . Numerical derivatives used a stepsize of $\sqrt{\epsilon}$ times the current value, or $\sqrt{\epsilon}$ if the value was zero, where $\epsilon \sim 10^{-7}$ is machine precision (18). The number of iterations was fixed at 20 although convergence was usually complete after 5 iterations.

Results

Figure 1A plots the absolute error in the mean water and fat estimates from 10^8 trials (i.e. |mean value – true value|) as a function of the fat fraction. This measures estimator bias and should approach zero as the number of trials increases for an unbiased estimator. Table I gives the average error over all fat fractions for different numbers of trials. While the Gauss-Markov theorem assures that minimizing the least squares error results in an unbiased estimate for the linear model, there is no such guarantee for the nonlinear model (19). In the linear case the error is essentially random and decreases rapidly with number of trials. The phase-constrained case exhibits structure and a non-vanishing error that suggest a bias of approximately 0.01% with SNR 10. Doubling the SNR reduces bias by a factor of approximately 4.

Figure 1B plots the standard deviations of the water and fat estimates as a function of the fat fraction. Fat generally exhibits a higher standard deviation than water due to the interference between the different peaks, which results in lower signal at non-zero echo-times. The linear technique exhibits constant standard deviation over the entire range of fat fraction. The constrained technique varies substantially but is always equal to or less than the linear technique by up to 41% at the edges of the range and 18% on average.

Figure 2 shows separated water and fat images from reconstructions using two echos. There are large differences between the linear and phase-constrained methods, particularly in the fat image. A simple count of the number of unknowns versus data points confirms the linear method should not be expected to simultaneously estimate water, fat and field map from two echos: there are five unknowns (amplitude and phase of water and fat plus the field map) versus four data points (real and imaginary parts of two echos). Thus there are insufficient degrees of freedom to estimate all the parameters. As a consequence these images are essentially uncorrected for the field map. The phase-constraint removes one of the unknowns, so a meaningful solution can be obtained.

Figure 3 shows separated water and fat images using three echos. In this data set the number of data points is six (real and imaginary parts of three echos), hence the linear and phase-constrained methods are both able to obtain meaningful solutions. Accordingly, differences in the images are less conspicuous although the SNR should be higher in the phase-constrained images. To verify this, regions of interest containing 100 pixels were drawn in the right lobe of the liver on the (complex) water and fat images. Ratios of the standard

deviations (linear / phase constrained) in these regions were 1.02 for water and 1.50 for fat, which are close to the simulation values 1.00 and 1.41 at low fat fraction.

Also note the qualitative similarity in separation performance between the two echo phase-constrained and the three echo linear images (Figure 2, bottom row and Figure 3, top row), which suggests the constraint can substitute for one echo. The quantitative noise performance, as measured by ratios of the standard deviations (two echo phase-constrained / three echo linear) is mixed: 0.81 for water and 1.16 for fat. While decreasing the number of echos from 3 to 2 should decrease the SNR by a factor of $\sqrt{2/3} \approx 0.82$, the loss is mitigated by up to $\sqrt{2}$ by the phase-constraint, hence the factor $\sqrt{2/3} \cdot \sqrt{2} \approx 1.15$. This explains the results observed for water and fat at low fat fraction. Different fat fractions will exhibit the noise performance indicated in Figure 1B.

Discussion

This study describes a phase-constrained method for use in separating water and fat in chemical shift based imaging. Current models for water-fat separation use unconstrained linear fitting (1,2), which is numerically expedient but can allow unphysical solutions. This is because the addition of complex noise can cause water and fat to possess different phases even when they should be in-phase.

Employing a phase-constraint has advantages shown by previous authors: fewer echos are required (16) and random noise is reduced in the estimated parameters (17). Simulation and imaging results from this study confirm a reduction in noise standard deviation of up to $\sqrt{2}$ and also reveal a small estimator bias, which is not unusual in nonlinear least squares curve-fitting (19). The bias is substantially less than the standard deviation (on the order of 0.01% versus 10% at SNR 10) and may therefore be considered negligible in clinical imaging applications, where the SNR is typically 10 or higher.

The main disadvantage has been the use of iterative curve-fitting, which can be slow and unreliable without good initial estimates. This may be partly because the sum of squares error is periodic in ϕ rather than quadratic (see Figure 4), which can confound optimization algorithms. The proposed method (Eq 4) eliminates this problem by computing the initial phase from an analytic formula. Note this does not necessarily help with curve-fitting other nonlinear terms in the model, such as the field map, except by enabling the phase-constraint to be used without increasing the dimensionality of the nonlinear search.

Although Eq 4 is intended as a drop-in replacement for Eq 3, one area for caution is in the generalization to complex field maps. Numerical calculations indicate the Cauchy-Riemann equations apply in the linear case but do not apply in the phase-constrained case. Therefore derivatives required for the nonlinear search must be performed separately for the real and imaginary parts, which doubles the reconstruction time.

A limitation of this study is that the spatial smoothness of the initial phase has not been utilized. Since the initial phase arises from the coil and the RF pulse, it is physically reasonable that it should vary slowly over the image. Previously nonlinear curve-fitting of low-resolution images was used to impart spatial smoothness onto the estimated phase (16). The phase was then used as prior knowledge in the high-resolution image reconstructions. An alternate approach might be to use the low-resolution phase to phase-correct the high-resolution images and then constrain the estimated phase to be close to zero, as used in some partial Fourier methods (20,21).

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Appendix

The goal is to solve $\mathbf{Ax} = \mathbf{b}$ under the constraint that all elements of $\mathbf{x}$ have the same phase. A solution of the desired form is $\mathbf{x}_{real}e^{i\phi}$, with real vector $\mathbf{x}_{real}$ and real scalar ϕ .

$$\mathbf{Ax}_{real}e^{i\phi} = \mathbf{b} \quad (\text{A1})$$

For any value of ϕ the optimal least squares estimate of $\mathbf{x}_{real}$ may be obtained by equating $d(\mathbf{r}^H \mathbf{r})/d\mathbf{x}_{real}$ to zero, where residual $\mathbf{r} = \mathbf{b} - \mathbf{Ax}_{real}e^{i\phi}$.

$$\widehat{\mathbf{x}}_{real} = \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} \text{Re}(\mathbf{A}^H \mathbf{b} e^{-i\phi}) \quad (\text{A2})$$

This allows the residual to be expressed as a function of ϕ only.

$$\mathbf{r} = \mathbf{b} - \mathbf{A} \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} \text{Re}(\mathbf{A}^H \mathbf{b} e^{-i\phi}) e^{i\phi} \quad (\text{A3})$$

All that remains to minimize $\mathbf{r}^H \mathbf{r}$ with respect to ϕ . Equating $d(\mathbf{r}^H \mathbf{r})/d\phi$ to zero yields

$$-2 \text{Im}(\mathbf{A}^H \mathbf{b} e^{-i\phi})^T \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} \text{Re}(\mathbf{A}^H \mathbf{b} e^{-i\phi}) = 0 \quad (\text{A4})$$

which is the imaginary part of $(\mathbf{A}^H \mathbf{b})^T \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} (\mathbf{A}^H \mathbf{b}) e^{-2i\phi}$. For the imaginary part of this expression to be zero the overall phase must be zero, which requires

$$\widehat{\phi} = \frac{1}{2} \angle (\mathbf{A}^H \mathbf{b})^T \text{Re}(\mathbf{A}^H \mathbf{A})^{-1} (\mathbf{A}^H \mathbf{b}). \quad (\text{A5})$$

To verify this equation, a numerical example is constructed using $\mathbf{A}$ from Eq 2 and $\mathbf{b} = [0.993 - 0.371i \ 1.172 + 0.004i \ 0.433 + 2.333i]^T$. The solution from Eq 4 is $[1.000 \ 1.000]^T e^{1.000i}$. Figure 4 plots $\mathbf{r}^H \mathbf{r}$ around this point, indicating it is minimized with respect to all parameters (minimum value 3.038). The linear solution from Eq 3 is $[1.053e^{0.648i} \ 1.130e^{1.506i}]^T$ (minimum value 2.075); the additional free parameter permits a lower error at the cost of producing a non-physical solution (phase terms not equal).

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Figure 1.

Simulation results are shown for the linear and phase-constrained methods in 10^8 trials with added noise. (A) shows the absolute errors in the means of the water and fat estimates (i.e. the estimator bias). Errors are of the order 0.01% of the mean values and decrease with number of trials (see Table I). (B) shows the standard deviations of the water and fat estimates. The phase-constrained method reduces uncertainty in the water and fat estimates by 18% on average and by up to 41% at edges of the range.

Figure 2.

Separated water and fat images from two gradient echos using (top row) the linear equation (Eq 3) and (bottom row) the phase-constrained equation (Eq 4). With only two echos the linear method is unable to achieve separation, as there are more unknowns (five) than data points (four). Arrows indicate erroneous signals in the subcutaneous fat and in the liver (this subject is known to have no liver fat). The phase-constrained method reduces the number of unknowns by one and hence the values can be estimated.

Figure 3. Separated water and fat images from three gradient echos using (top row) the linear equation (Eq 3) and (bottom row) the phase-constrained equation (Eq 4). With three echos there are minor differences in the images but overall the separation performance is comparable.

Figure 4.

Sum of squares error for the example given in the Appendix, for which the optimal phase-constrained solution is $x_1 = x_2 = \phi = 1.000$. Note that the sum of squares error with respect to ϕ is non-quadratic unless close to the optimal value (inset).

The mean of the absolute errors over all fat fractions for the linear and phase-constrained (pc) methods, corresponding to Figure 1A. The errors are small and decrease with number of trials, suggesting an asymptotic bias that is negligible in comparison to the mean values (of the order 1). Increasing the SNR by a factor of 2 reduces the errors proportionately for the linear method and by a larger factor for the phase-constrained method.

Table 1

No. Trials	SNR	Water (linear)	Fat (linear)	Water (pc)	Fat (pc)
10^6	10	4.57×10^{-5}	6.57×10^{-5}	6.08×10^{-5}	7.66×10^{-5}
10^7	10	1.66×10^{-5}	1.93×10^{-5}	4.35×10^{-5}	5.44×10^{-5}
10^8	10	0.54×10^{-5}	0.58×10^{-5}	4.03×10^{-5}	5.22×10^{-5}
10^8	20	0.28×10^{-5}	0.33×10^{-5}	1.00×10^{-5}	1.33×10^{-5}