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Reduced Circular Sinusoidal Cone-beam CT for Industrial Applications

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Abstract

Cone-beam computed tomography (CBCT) plays an important role in industrial, nondestructive testing applications not to mention in medical applications. Circular scanning configuration is widely used for its mechanical simplicity and for readily available and efficient reconstruction algorithms based on the Feldkamp algorithm. However, due to the lack of data sufficiency, circular CBCT does not guarantee image accuracy, and is not free from image artifacts related to the cone-angle and axial variation of the object anatomy. Therefore, non-circular scanning geometries such as helix or circular sinusoid, which provide sufficient data for exact image reconstruction within the scanned volume, are favorable when accurate images free of cone-angle artifacts are desired. Analytic algorithms have been developed for image reconstruction from cone-beam data acquired with a full-scan (or, equivalently, a closed) circular sinusoidal trajectory. In many applications such as industrial inspection of a product where a high throughput is important, it is desirable to have a short scan time. In this work, we propose an innovative imaging approach in which a reduced-scan (or, equivalently, an open) circular sinusoidal trajectory is used for acquiring data sufficient for exact 3D image reconstruction. A filtered backprojection (FBP) algorithm based on Park-Noo's reconstruction formula is applied for image reconstruction in reduced-scan circular sinusoidal scans. We have conducted numerical studies to demonstrate the reduced-scan approach and to validate the FBP reconstruction algorithm in the proposed approach. The proposed scanning method can contribute to increasing the throughput of a scanner, while improving the image quality compared to a conventional circular scan.

Keywords

reduced-scan; circular sinusoidal trajectory; cone-beam; computed tomography

1 Introduction

With the advanced development of large-area, digital detectors and high-energy X-ray generators, CBCT has been increasingly used for industrial, nondestructive testing applications such as visualization of inner and outer structures of electronic components, packaging, and interconnection technologies or crack-detection of automobile parts [1,2,3]. The CBCT has advantages of substantially shorter acquisition time and isotropic spatial resolution of the reconstructed images over the fan-beam CT which has been the dominant scanning modality in the industrial CT. The gain in scanning speed of the CBCT is particularly useful when a high throughput is required. Most of the industrial CBCT systems are employing a circular scan for its mechanical simplicity and for readily available, efficient reconstruction algorithms based on the Feldkamp algorithm. However, due to the lack of data sufficiency, circular CBCT does not guarantee image accuracy, and is not free from image artifacts related to the cone-angle and axial variation of the object anatomy.

To avoid these data-insufficiency problems in circular CBCT, many non-circular scanning geometries have been investigated that include helix, circular sinusoid, circle-plus-line, two-arcs, and so on [4,5,6,7,8,9]. Non-circular trajectories may provide sufficient data for accurate volumetric image reconstruction within the scanned volume, and analytic reconstruction algorithms for these trajectories have been significantly advanced during the last decade. Following the developments of Katsevich's and chord-based algorithms for exact image reconstruction in helical cone-beam CT [10,11,12], image-reconstruction algorithms have also been developed for cone-beam CT with general source trajectories [4,5,6,7,8,9]. The chord-based algorithms, such as the backprojection-filtration (BPF) algorithm, are directly applicable to a wide, general class of trajectories, which includes virtually all of the previously studied source trajectories of potentially practical interest as special cases [5,7]. Moreover, they have a unique property in that they can reconstruct images within regions of interest (ROIs) from projection data containing transverse truncations.

For a given continuous source trajectory, a chord is defined as a straight line segment connecting two points on the trajectory. The chord-based algorithms reconstruct an image within the ROI through reconstructing images on individual chords that completely cover the ROI. Therefore, the application of the chord-based algorithms relies upon the availability of chords intersecting the ROI. Conversely, some regions, although having sufficient data for accurate image reconstruction in principle, may not be reconstructible simply because of the lack of chords passing through the regions. Recently, a filtered backprojection (FBP) formula, which is referred to as Pack-Noo's formula in this work, has been proposed for possibly reconstructing images within such chordless regions [13,14,15,16], and we have successfully developed an algorithm based on it for a reverse helical trajectory [17].

As has been emphasized, shortening data acquisition time is critically important in industrial CT applications. In this regard, a circular short-scan, which collects cone-beam data from only 180° plus fan angle, can reduce acquisition time substantially compared to a full-scan. As an analogy, we propose a reduced circular-sinusoidal scan for more accurate image reconstruction compared to a circular scan and for faster data acquisition compared to a full, circular-sinusoidal scan in this paper. Here, we define a reduced circular-sinusoidal scan as a circular sinusoidal scan that collects cone-beam data from less than 360° . Image reconstruction algorithms have been developed, both in BPF and FBP frameworks, for a full-scan circular sinusoidal trajectory. However, a reduced circular sinusoidal scan has not been dealt with in any of the frameworks. In this work, we apply our recently developed FBP algorithm that is based on Pack-Noo's formula to reconstructing accurate images from a reduced circular sinusoidal CBCT. Throughout this work, for convenience, we will use the phrase of a closed circular sinusoidal trajectory interchangeably with a full-scan circular sinusoidal trajectory, and an open circular sinusoidal trajectory with a reduced-scan circular sinusoidal trajectory [18]. We believe a reduced circular sinusoidal cone-beam CT can reduce the scanning time thereby potentially contributing to increased throughput of the industrial scanning system while allowing an accurate image reconstruction.

2 Image Reconstruction for Full-scan Circular Sinusoidal Trajectories

In this section, we explain the circular sinusoidal trajectories and the existing methods for full-scan circular sinusoidal trajectories.

2.1 Circular sinusoidal trajectory

In a fixed-coordinate system, a circular sinusoidal source trajectory $\vec{r}_0(\lambda)$ of order m can be expressed mathematically as

$$\vec{r}_0(\lambda) = (R \cos \lambda, R \sin \lambda, h \cos m \lambda), \quad (1)$$

where λ denotes the rotation angle of the source point, R the distance from the source to the rotation axis, and h represents the amplitude of the source's oscillation along z direction. A circular sinusoidal trajectory has a period of $2\pi/m$, and, as shown in Fig. 1a, a closed circular sinusoidal trajectory is formed when the rotation angle λ covers 2π , which we refer to as a full-scan circular sinusoidal trajectory. In contrast, an open circular sinusoidal trajectory is formed, as shown in Fig. 1b, when the scanning angular range is smaller than 2π , which we refer to as a reduced-scan circular sinusoidal trajectory. Note that, when $m = 2$, the circular sinusoidal trajectory becomes the standard saddle trajectory [15,19] and that, when h becomes zero, a circular sinusoidal trajectory becomes the 2D circular trajectory.

The cone-beam projection of an object function $f(\vec{x})$ at rotation angle λ can be mathematically expressed as

$$g(\lambda, \hat{\theta}) = \int_0^\infty dt f(\vec{r}_0(\lambda) + t\hat{\theta}), \quad (2)$$

where $\hat{\theta} \in S^2$, and S^2 is a set of all unit vectors over a sphere in 3D space.

2.2 Pack-Noo's formula

Pack-Noo's formula provides a basis for deriving FBP algorithms of image reconstruction in a general source trajectory [13,14]. For a continuous segment of a trajectory $\vec{r}_0(\lambda)$ that begins at λ^- and ends at λ^+ , we divide it into N adjacent sub-segment trajectories of which the i th sub-segment trajectory begins at λ_i and ends at λ_{i+1} , where $i = 1, 2, \dots, N$; $\lambda_1 = \lambda^-$, and $\lambda_{N+1} = \lambda^+$. Therefore, $[\lambda^-, \lambda^+] = [\lambda_1, \lambda_2] \cup [\lambda_2, \lambda_3] \dots \cup [\lambda_1, \lambda_{i+1}] \cup \dots \cup [\lambda_N, \lambda_{N+1}]$. For each of the sub-segment trajectories, Pack-Noo's formula provides a key reconstruction operator, which is given by

$$\mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1}) = -\frac{1}{2\pi^2} \int_{\lambda_i}^{\lambda_{i+1}} d\lambda \frac{1}{\|\vec{x} - \vec{r}_0(\lambda)\|} g_F(\lambda, \vec{x}, \hat{e}_i), \quad (3)$$

where $g_F(\lambda, \vec{x}, \hat{e}_i)$ represents the filtered projection data, given by

$$g_F(\lambda, \vec{x}, \hat{e}_i) = \int_{-\pi}^{\pi} d\gamma \frac{1}{\sin \gamma} \frac{\partial g(\lambda, \hat{\theta}(\lambda, \vec{x}, \hat{e}_i, \gamma))}{\partial \lambda}, \quad (4)$$

$$\hat{\theta}(\lambda, \vec{x}, \hat{e}_i, \gamma) = \cos \gamma \hat{\alpha}(\lambda, \vec{x}) + \sin \gamma \hat{\beta}(\lambda, \vec{x}, \hat{e}_i), \quad (5)$$

$$\hat{\alpha}(\lambda, \vec{x}) = \frac{\vec{x} - \vec{r}_0(\lambda)}{\|\vec{x} - \vec{r}_0(\lambda)\|}, \quad (6)$$

and

$$\widehat{\beta}(\lambda, \vec{x}, \widehat{e}_i) = \frac{\widehat{e}_i - (\widehat{e}_i \cdot \widehat{\alpha}(\lambda, \vec{x}))\widehat{\alpha}(\lambda, \vec{x})}{\|\widehat{e}_i - (\widehat{e}_i \cdot \widehat{\alpha}(\lambda, \vec{x}))\widehat{\alpha}(\lambda, \vec{x})\|}. \quad (7)$$

The integration over γ in Eq. (4) indicates a filtering process over the data derivative, while the filtering direction is determined by a pre-selected unit vector $\widehat{e}_i \in S^2$ for a subsegment trajectory specified by $[\lambda_i, \lambda_{i+1}]$. As discussed below, a specific form of the FBP reconstruction algorithm depends critically upon the selection of $\widehat{e}_i \in S^2$, which we refer to as the filtering unit vector.

For a given sub-segment trajectory $[\lambda_i, \lambda_{i+1}]$ and a selected unit vector $\widehat{e}_i$, the operator $\mathcal{K}(\vec{x}, \widehat{e}_i, \lambda_i, \lambda_{i+1})$, as indicated in Eq. (3), can be computed from cone-beam projections. Let $(R''f)(\widehat{\omega}, \vec{x} \cdot \widehat{\omega})$ denote the second order derivative of the 3D Radon transform of the object function $f(\vec{x})$. It has been proved that Pack-Noo's formula [14] establishes a relationship with $(R''f)(\widehat{\omega}, \vec{x} \cdot \widehat{\omega})$ as follows:

$$\mathcal{K}(\vec{x}, \widehat{e}_i, \lambda_i, \lambda_{i+1}) = -\frac{1}{8\pi^2} \int_{S^2} d\widehat{\omega} (R''f)(\widehat{\omega}, \vec{x} \cdot \widehat{\omega}) \sigma(\vec{x}, \widehat{\omega}, \widehat{e}_i, \lambda_i, \lambda_{i+1}), \quad (8)$$

where

$$\sigma(\vec{x}, \widehat{\omega}, \widehat{e}_i, \lambda_i, \lambda_{i+1}) = \frac{1}{2} \text{sgn}(\widehat{\omega} \cdot \widehat{e}_i) [\text{sgn}(\widehat{\omega} \cdot \widehat{\alpha}(\lambda_i, \vec{x})) - \text{sgn}(\widehat{\omega} \cdot \widehat{\alpha}(\lambda_{i+1}, \vec{x}))], \quad (9)$$

and sgn denotes the signum function. Therefore, for the entire trajectory over $\lambda \in [\lambda^-, \lambda^+]$, we have

$$\sum_{i=1}^N \mathcal{K}(\vec{x}, \widehat{e}_i, \lambda_i, \lambda_{i+1}) = -\frac{1}{8\pi^2} \int_{S^2} d\widehat{\omega} (R''f)(\widehat{\omega}, \vec{x} \cdot \widehat{\omega}) \overline{\sigma}(\vec{x}, \widehat{\omega}, \lambda^-, \lambda^+), \quad (10)$$

where

$$\overline{\sigma}(\vec{x}, \widehat{\omega}, \lambda^-, \lambda^+) = \sum_{i=1}^N \sigma(\vec{x}, \widehat{\omega}, \widehat{e}_i, \lambda_i, \lambda_{i+1}). \quad (11)$$

It can be observed that, if the weighting function

$$\overline{\sigma}(\vec{x}, \widehat{\omega}, \lambda^-, \lambda^+) = 1, \quad (12)$$

the right-hand side of Eq. (10) yields exactly the inverse 3D Radon transform of the object function. Therefore, under the condition in Eq. (12), the object function $f(\vec{x})$ is given by

$$f(\vec{x}) = \sum_{i=1}^N \mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1}). \quad (13)$$

2.3 The FBP algorithm for a full circular sinusoidal scan

Using Pack-Noo's formula, an FBP algorithm has been derived for a full-scan circular sinusoidal trajectory [20,15], which is briefly summarized below.

Consider a plane Π intersecting a full-scan circular sinusoidal trajectory at $N (\geq 3)$ different points $\vec{r}_0(\lambda_i)$, where $i = 1, \dots, N$; $-\pi < \lambda_1 < \lambda_2 \dots < \lambda_N \leq \pi$; and $\lambda_{N+1} = \lambda_1 + 2\pi$. A convex polygon Ω_Π is formed by N chords that connect these intersecting points. For side i of this polygon, there exists a continuous sub-segment trajectory $[\lambda_i, \lambda_{i+1}]$.

It has been shown [20,15] that, for any point $\vec{x}$ within the polygon Ω_Π ,

$$\overline{\sigma}(\vec{x}, \hat{e}_i, \lambda_i^-, \lambda_i^+) = 2 \quad (14)$$

when the filtering unit vectors $\hat{e}_i$ are chosen along the directions of the corresponding chords, which also serve as the sides of the polygon, i.e.,

$$\hat{e}_i = \frac{\vec{r}_0(\lambda_{i+1}) - \vec{r}_0(\lambda_i)}{\|\vec{r}_0(\lambda_{i+1}) - \vec{r}_0(\lambda_i)\|}, \quad (15)$$

where $i = 1, 2, \dots, N$. Consequently an exact reconstruction within the polygon can be obtained by

$$f(\vec{x}) = \frac{1}{2} \sum_{i=1}^N \mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1}). \quad (16)$$

For example, for a full-scan circular sinusoidal trajectory of order $m = 3$ shown in Fig. 2, one can choose a set of six-side polygons parallel to the x - y plane at different $z \in [-h, h]$, where h is the amplitude of the source motion along z -axis. Each of polygons intersects the full-scan circular sinusoidal trajectory at six different points $\vec{r}_0(\lambda_i)$, $i = 1, \dots, 6$.

3 Image Reconstruction for Reduced-Scan Circular Sinusoidal Trajectories

We derive below an FBP algorithm for image reconstruction from data acquired with a reduced-scan circular sinusoidal trajectory.

3.1 Computation of κ for a reduced-scan

Without loss of generality, a reduced-scan circular sinusoidal trajectory is obtained by elimination of a sub-segment trajectory $[\lambda_N, \lambda_{N+1}]$ from the corresponding full-scan circular sinusoidal trajectory. We display in Fig. 2b, a reduced-scan circular sinusoidal trajectory of order $m = 3$ obtained from the full-scan circular sinusoidal trajectory in Fig. 2a by removal of the sub-segment trajectory $[\lambda_6, \lambda_7]$.

Using the $N - 1$ available sub-segment trajectories in the reduced-scan circular sinusoidal trajectory, one can compute a function

$$\bar{f}(\vec{x}) = \frac{1}{2} \sum_{i=1}^{N-1} \mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1}), \quad (17)$$

within the selected polygons Ω_{Π} . Obviously, $\bar{f}(\vec{x})$ is not the object function $f(\vec{x})$ because it misses the contribution of the last term, $\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_N, \lambda_{N+1})$, in Eq. (16). Therefore, the key for obtaining an exact reconstruction is to compute the last term from the data available only over the $N - 1$ sub-segment trajectories.

For a given reconstruction point $\vec{x}$ and the filtering unit vector $\hat{e}_i$, it can be observed that $\mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1})$ in Eq. (3) depends only upon the starting and ending points, λ_i and λ_{i+1} , of the sub-segment trajectory [5], that it remains unchanged when the shape and length of the sub-segment trajectories vary, and thus that it satisfies

$$\mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1}) = -\mathcal{K}(\vec{x}, \hat{e}_i, \lambda_{i+1}, \lambda_i).$$

Therefore, the last term $\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_N, \lambda_{N+1})$ can also be rewritten as

$$\begin{aligned} \mathcal{K}(\vec{x}, \hat{e}_N, \lambda_N, \lambda_{N+1}) &= -\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_{N+1}, \lambda_N) \\ &= -\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_1, \lambda_N). \end{aligned} \quad (18)$$

In the last step of Eq. (18) we have invoked the periodicity property of a circular sinusoidal trajectory. The term $\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_N, \lambda_{N+1})$, as shown in in Eq. (18), can be obtained from $\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_1, \lambda_N)$, which can be computed from data available over the $N - 1$ sub-segment trajectories. Using Eq. (18) in Eq. (16), one obtains the object function as

$$\begin{aligned} f(\vec{x}) &= \bar{f}(\vec{x}) + \frac{1}{2} \mathcal{K}(\vec{x}, \hat{e}_N, \lambda_N, \lambda_{N+1}) \\ &= \frac{1}{2} \left[\sum_{i=1}^{N-1} \mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1}) - \mathcal{K}(\vec{x}, \hat{e}_N, \lambda_1, \lambda_N) \right]. \end{aligned} \quad (19)$$

Each term in Eq. (19) can be obtained from data available over the $N - 1$ subsegment trajectories. As such, for a reduced-scan circular sinusoidal trajectory, Eq. (19) provides a recipe for exactly reconstructing the object function. It can be observed that the data over the $N - 1$ sub-segments have been filtered twice: the data are filtered first along the direction $\hat{e}_i$ to obtain the terms $\mathcal{K}(\vec{x}, \hat{e}_i, \lambda_i, \lambda_{i+1})$ for $i = 1, \dots, N - 1$; and they are filtered then along the direction $\hat{e}_N$ to obtain $\mathcal{K}(\vec{x}, \hat{e}_N, \lambda_1, \lambda_N)$, the last term in Eq. (19). In the following section, we will give the details about the implementation of data filtering.

3.2 Data filtering

The explicit form of the FBP algorithm derived in Sec. 3.1 depends upon the detector geometry, and a flat-panel detector is assumed in this work.

3.2.1 Determination of filtering lines—We first introduce a rotation-coordinate system $\{u, v, w\}$ in which its origin is on the source $\vec{r}_0(\lambda)$, as shown in Fig. 3, and its u - w plane and v -axis are parallel to the x - y plane and to the z -axis, respectively, of the fixed-coordinate system $\{x, y, z\}$. The unit vectors of the rotation-coordinate system at a scanning angle λ can be expressed in the fixed-coordinate system as

$$\begin{aligned}\widehat{e}_u(\lambda) &= (-\sin\lambda, \cos\lambda, 0) \\ \widehat{e}_v(\lambda) &= (0, 0, 1) \\ \widehat{e}_w(\lambda) &= (\cos\lambda, \sin\lambda, 0).\end{aligned}\quad (20)$$

Let the flat-panel detector be placed within a plane at $w = -D$ that is parallel to the u - v plane. We introduce a 2D-detector-coordinate system $\{u_d, v_d\}$ on the plane in which the u_d - and v_d -axes are parallel to the u - and v -axes, respectively, and its origin is at the projection, along $\widehat{e}_w$, on the detector plane, of the source point $\vec{r}_0(\lambda)$. For a point $\vec{x}_d$ on the detector plane, the relationship between its fixed coordinates (x_d, y_d, z_d) and 2D-detector coordinates (u_d, v_d) can be obtained as

$$\begin{aligned}x_d &= -u_d \sin\lambda + (-D+R) \cos\lambda \\ y_d &= u_d \cos\lambda + (-D+R) \sin\lambda \\ z_d &= v_d + h \cos m\lambda.\end{aligned}\quad (21)$$

As discussed in Sec. 3.1, for a given polygon, filtering unit vectors $\hat{e}$ are determined to be tangential to the sides of the polygon. For simplicity, the polygon planes considered in this work are parallel to the x - y plane, and so is a filtering unit vector, which can thus be expressed as

$$\widehat{e} = (\cos\phi, \sin\phi, 0), \quad (22)$$

where ϕ denotes the angle between the filtering unit vector $\hat{e}$ and the x -axis. From the filtering unit vector $\hat{e}$ and a given point $\vec{x}_0$, one can form a plane Γ with a normal vector given by

$$\widehat{n}_\Gamma = \widehat{e} \times \widehat{a}(\lambda, \vec{x}_0). \quad (23)$$

The intersection of the plane Γ with the detector plane forms a line specified by

$$\widehat{n}_\Gamma \cdot (\vec{x}_d - \vec{r}_0(\lambda)) = 0, \quad (24)$$

and it can be written in terms of the 2D-detector coordinates as

$$u_d = C v_d + u_c, \quad (25)$$

where the slope C and the intercept u_c are given by

$$C = \frac{R \sin(\lambda - \phi) + x_0 \sin \phi - y_0 \cos \phi}{(h \cos m \lambda - z_0) \cos(\lambda - \phi)} \quad \text{and} \quad u_c = D \tan(\lambda - \phi), \quad (26)$$

and (x_0, y_0, z_0) are the fixed coordinates of the point $\vec{x}_0$. As discussed below, Eq. (25) represents a filtering line on the detector for all points within the plane Γ .

For a selected filtering unit vector $\hat{e}$ and a given view λ , the slope C (see Eq. (26)) of the filtering line is determined by $\vec{x}_0$. Therefore, points that are not contained in plane Γ form different filtering lines with different slopes. However, as Eq. (26) indicates, u_c is independent of $\vec{x}_0$. Therefore, for the selected filtering unit vector $\hat{e}$ and a given view λ , all the filtering lines with different slopes, determined by different $\vec{x}_0$, intersect each other at point $(u_d, v_d) = (u_c, 0)$, i.e., a point on the u_d -axis, on the detector plane, which is also the projection, along $\hat{e}$, of the source point onto the detector. For convenience, the point $(u_d, v_d) = (u_c, 0)$ on the detector is referred to as the intersecting point of the filtering lines.

We display in Fig. 4 several example filtering lines on detector planes determined for different selections of $\hat{e}$ and λ for a circular sinusoidal trajectory with $m = 3$. The upper row

of Fig. 4 shows the filtering lines determined by use of $\hat{e} = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right)$ and $\lambda = -\frac{5\pi}{6}$ (a), $-\frac{2\pi}{3}$ (b), and $-\frac{\pi}{2}$ (c), whereas the lower row of Fig. 4 shows the filtering lines determined by use of $\hat{e} = (0, -1, 0)$ and $\lambda = -\frac{5\pi}{6}$ (a), $-\frac{2\pi}{3}$ (b), and $-\frac{\pi}{2}$ (c), respectively. One can also determine the intersecting points of the filtering lines in these cases. For the three cases in the upper row of Fig. 4, we have $(u_c, 0) = (\sqrt{3}D, 0)$ (a), $(\infty, 0)$ (b), and $(-\sqrt{3}D, 0)$ (c), whereas, for the three cases in the lower row of Fig. 4, we have $(u_c, 0) = (-\sqrt{3}D, 0)$ (a), $(-\frac{\sqrt{3}}{3}D, 0)$ (b), and $(0, 0)$ (c), respectively.

3.2.2 Computation of the filtered data—It can be observed that the FBP formula in Eq. (3) involves the backprojection (i.e., the integration over λ) of filtered data (i.e., Eq. (4)). As described above, for image reconstruction $\vec{x}_0$, a filtering line in Eq. (25) on the detector plane can be determined at projection view λ . Using the approach described in the Appendix of Ref. [21], the filtered data in Eq. (4) can be expressed in terms of the 2D-detector coordinates (u_d, v_d) . Specifically, when the determined filtering line is not parallel to the v_d -axis of the detector plane, Eq. (4) can be written as

$$g_F(\lambda, u_d, v_d) = - \int_{u_l}^{u_u} du'_d \frac{A}{A'} h_H(u_d - u'_d) \frac{\partial g(\lambda, u'_d, v'_d)}{\partial \lambda}, \quad (27)$$

where $h_H(u)$ is the kernel of Hilbert transform, $A = \sqrt{D^2 + u_d^2 + v_d^2}$, and $A' = \sqrt{D^2 + u_d'^2 + v_d'^2}$. Whereas, when the determined filtering line is not parallel to the u_d -axis of the detector plane, Eq. (4) can be written as

$$g_F(\lambda, u_d, v_d) = - \int_{v_l}^{v_u} dv'_d \frac{A}{A'} h_H(v_d - v'_d) \frac{\partial g(\lambda, u'_d, v'_d)}{\partial \lambda}, \quad (28)$$

where (u_d, v_d) indicates the projection point of $\vec{x}$ along $\hat{\alpha}(\lambda, \vec{x}_0)$ on the detector plane, and (u'_d, v'_d) denote points on the filtering line specified by Eq. (25). It should be pointed out that (u_d, v_d) is also on the filtering line. The lower and upper limits, i.e., u_l and u_u in Eq. (27), and v_l and v_u in Eq. (28), depending upon the reconstruction point $\vec{x}_0$, are determined below.

It can be observed in Eq. (4) that the lower and upper limits of the integration over γ are $-\pi$ and π . For the cases under study, when the integration variable is changed from γ in Eq. (4) to u'_d in Eq. (27), or to v'_d in Eq. (28), the corresponding lower and upper limits can be

determined by those of γ and by $\frac{du'_d}{d\gamma}$ or $\frac{dv'_d}{d\gamma}$.

One can relate u'_d or v'_d to γ through

$$u'_d = -D \frac{\widehat{\theta} \cdot \widehat{e}_u}{\widehat{\theta} \cdot \widehat{e}_w} \quad \text{or} \quad v'_d = -D \frac{\widehat{\theta} \cdot \widehat{e}_v}{\widehat{\theta} \cdot \widehat{e}_w}, \quad (29)$$

where $\widehat{\theta}$ is given by Eq. (5). Using Eq. (29), it can be shown that

$$\frac{du'_d}{d\gamma} = D \frac{(u_d - u_c) \cos(\lambda - \phi)}{\sqrt{u_d^2 + v_d^2 + D^2} \|(\widehat{e} - (\widehat{e} \cdot \widehat{\alpha})\widehat{\alpha})\| (\widehat{\theta} \cdot \widehat{e}_w)^2}, \quad (30)$$

and

$$\frac{dv'_d}{d\gamma} = D \frac{v_d \cos(\lambda - \phi)}{\sqrt{u_d^2 + v_d^2 + D^2} \|(\widehat{e} - (\widehat{e} \cdot \widehat{\alpha})\widehat{\alpha})\| (\widehat{\theta} \cdot \widehat{e}_w)^2}, \quad (31)$$

where u_c is given by Eq. (26), and $(u_c, 0)$ denote the coordinates of the point on detector plane at which all of the filtering lines intersect, as displayed in Fig. 4. For a selected filtering unit vector $\widehat{e}$ within the x - y plane and at a given view λ , it can readily be observed in

Eq. (30) that the sign of $\frac{du'_d}{d\gamma}$ changes when the sign of $(u_d - u_c) \cos(\lambda - \phi)$ changes.

Similarly, as indicated in Eq. (31), the sign of $\frac{dv'_d}{d\gamma}$ changes when the sign of $v_d \cos(\lambda - \phi)$ changes. Therefore, when $(u_d - u_c) \cos(\lambda - \phi) < 0$, we have $u_l = -\infty$ and $u_u = \infty$, whereas, when $(u_d - u_c) \cos(\lambda - \phi) > 0$, we have $u_l = \infty$ and $u_u = -\infty$. Similarly, when $v_d \cos(\lambda - \phi) < 0$, we have $v_l = -\infty$ and $v_u = \infty$, whereas when $v_d \cos(\lambda - \phi) > 0$, we have $v_l = \infty$ and $v_u = -\infty$. Combining these results, we can rewrite Eq. (27) as

$$g_F(\lambda, u_d, v_d) = \text{sgn}[(u_d - u_c) \cos(\lambda - \phi)] \int_{-\infty}^{\infty} du'_d \frac{A}{A'} h_H(u_d - u'_d) \frac{\partial g(\lambda, u'_d, v'_d)}{\partial \lambda}, \quad (32)$$

where $u_d \neq u_c$, and the filtering line is not parallel to the v_d -axis. Similarly, we can rewrite (28) as

$$g_F(\lambda, u_d, v_d) = \text{sgn}[v_d \cos(\lambda - \phi)] \int_{-\infty}^{\infty} dv'_d \frac{A}{A'} h_H(v_d - v'_d) \frac{\partial g(\lambda, u'_d, v'_d)}{\partial \lambda}, \quad (33)$$

where $v_d \neq 0$, and the filtering line is not parallel to the u_d -axis. Finally, considering the signum function in Eq. (32) or Eq. (33) as a function of λ , one can observe that the contribution of the filtered data from the intersecting point $(u_c, 0)$ of the filtering lines to the backprojection (i.e., the integration over λ) can be interpreted as 0, namely,

$$g_F(\lambda, u_c, 0) = 0. \quad (34)$$

In the numerical studies for a flat-panel detector below, Eqs (32), (33), and (34) form the recipe for computing the filtered data. When the intersecting points of the filtering lines are within the projection support of the object function (see (b) in the lower row of Fig. 4), the filtering data for two reconstructing points that share the same filtering line can have different signs, which are determined by the signum function. This may not occur in image reconstruction on polygon planes for a full-scan circular sinusoidal trajectory because the intersecting points in this case are generally outside the projection support of the object function on the detector plane. However, for a reduced-scan circular sinusoidal trajectory, the intersecting points of the filtering lines at some views are within the projection supports of the object function when the actual sub-segment trajectories are used as the replacement of the missing sub-segment trajectories. Moreover, the projection are required to be non-truncated to provide filtered data for these views.

4 Numerical Studies

We have conducted numerical studies, as summarized below, to investigate and evaluate the proposed reduced-scan approach and the FBP reconstruction algorithm. For the comparison, the image reconstructions for a full-scan circular sinusoidal trajectory and for a circular trajectory were performed as well.

4.1 Image reconstructions for reduced-scan and full-scan circular sinusoidal trajectories

In this section, we applied the proposed FBP algorithm to reconstructing 3D images from data acquired with a reduced-scan circular sinusoidal trajectory of order $m = 3$:

$$\vec{r}_0(\lambda) = (R \cos \lambda, R \sin \lambda, h \cos 3\lambda)^T,$$

where $R = 57.0$ cm and $h = 6.1$ cm. A 3D Shepp-Logan phantom was used to generate cone-beam data. This phantom contains several ellipsoids with different absorption properties, as shown in the top row of Fig. 6. A virtual detector plane was placed at the distance $S = 57.0$ cm from the source point, and it was assumed to have a square shape consisting of $1000 \times$

1000 detection elements of size $0.33 \times 0.33 \text{ mm}^2$. A full-scan data set containing cone-beam projections at 1000 views over $[-\pi, \pi]$ was generated. The data over the angular range

$\lambda \in [-\frac{7}{9}\pi, \frac{7}{9}\pi]$ were used as our reduced-scan cone-beam data.

Consider image reconstruction within a set of polygons sandwiched by polygons $\Omega_{\Pi 1}$ at $z = -3.05 \text{ cm}$ and $\Omega_{\Pi 2}$ at $z = 3.05 \text{ cm}$, as shown in Fig. 5. All of these polygons are parallel to the x - y plane, and each of them intersects the trajectory at six points, $\vec{r}_0(\lambda_i)$, $i = 1, 2, \dots, 6$, and one can show that

$$\begin{cases} \lambda_1 = \frac{1}{3}(-2\pi - \cos^{-1} \frac{z}{h}) \\ \lambda_2 = \frac{1}{3}(-2\pi + \cos^{-1} \frac{z}{h}) \\ \lambda_3 = -\frac{1}{3}\cos^{-1} \frac{z}{h} \\ \lambda_4 = \frac{1}{3}\cos^{-1} \frac{z}{h} \\ \lambda_5 = \frac{1}{3}(2\pi - \cos^{-1} \frac{z}{h}) \\ \lambda_6 = \frac{1}{3}(2\pi + \cos^{-1} \frac{z}{h}) \end{cases} \quad -3.05 \text{ cm} < z < 3.05 \text{ cm}$$

We used the FBP algorithm to reconstruct images from the reduced-scan data described above.

The reconstructed images within 2D slices at $x = 0$, $y = 0$, and $z = 0 \text{ cm}$, are displayed in the middle row of Fig. 6 for the reduced-scan circular sinusoidal trajectory. For comparison, we also display the reconstructed images and the corresponding profile for the full-scan trajectory in Figs. 6 and 7. These results demonstrate that the FBP algorithm for reduced-scan circular sinusoid trajectories can reconstruct images of which accuracy is comparable to that of images obtained from the full-scan circular sinusoidal cone-beam data.

4.2 Image reconstructions for reduced-scan circular sinusoidal trajectory and circular trajectory

We also conducted a study using a disk phantom, which has more variations than the Shepp-Logan phantom to the axial direction, for further demonstrating this proposed reconstruction approach and for comparing with the reconstruction in a circular trajectory. The disk phantom consists of a cylinder and a stacks of disks, as shown in the top row of Fig. 8. In order to provide the sufficient projection data for reconstructing the whole cylinder, we used a pitch $h = 12.2 \text{ cm}$. The other scanning parameters adopted in this study were identical to that for the Shepp-Logan phantom study. The reconstructed images within 2D slices at $x = 0$, $y = 0$, and $z = 0 \text{ cm}$, are displayed in Fig. 8 for the reduced-scan. For comparison, we also display the reconstructed images for the corresponding circular trajectory in the bottom row of Fig. 8. The line profiles along the axial direction are also displayed in Fig. 9. These results demonstrate that the FBP algorithm for reduced-scan circular sinusoid trajectories can yield the image without cone-beam artifacts which appears in the image reconstructed from the circular cone-beam projection data. Typical cone-beam image artifacts are observed in the images reconstructed from a circular scan: intensity drop, streaks, and image deformation.

4.3 Preliminary Noise Study

A preliminary noise study was also conducted in which the FBP algorithm was used for reconstructing images from noisy data. In this study, the Shepp-Logan phantom was used to generate the noiseless cone-beam projection data for the circular sinusoidal trajectory of order $m = 3$. The trajectory has a radius of $R = 28.5 \text{ cm}$ and a pitch of $h = 6.1 \text{ cm}$. In order to

reduce computational effort, we generated the full-scan data at only 300 projection views uniformly distributed over $[-\pi, \pi]$, and a detector consisting of 256×256 detection elements of size $1.3 \times 1.3 \text{ mm}^2$ was used. A portion of the full-scan data over the angular range

$[-\frac{3}{4}\pi, \frac{3}{4}\pi]$ was used to simulate the data acquired in a reduced scan. To investigate the noise properties of the proposed algorithm, 500 realizations of noisy data are generated. The realizations are obtained from the addition of uncorrelated Gaussian noise with a standard deviation of 0.13% of the maximum projection value. Each ensemble of projection data was used to generate an ensemble of reconstructed images. The image variance, which indicates the noise level, was obtained by taking the difference between the mean of the square voxel value and the square of the mean for all voxels in the reconstructed image ensemble.

In Fig. 10, we display one example of the reconstructed noisy images within the 2D slice at $z = 0 \text{ cm}$ for the reduced-scan circular sinusoidal trajectory of order $m = 3$. Using the images reconstructed from these noisy data sets for the reduced-scan circular sinusoidal trajectory, we computed empirical image variances within the field of view (FOV), which are shown in the top right of Fig. 10. Image variances along the middle horizontal lines and the middle vertical lines are also displayed in the bottom row of Fig. 10. It can be observed in Fig. 10 that image variances obtained from the reduced-scan data by use of the FBP algorithms are non-uniform. This is because that the spatially varying weighting factors (i.e., $1/|\vec{x} - \vec{r}_0(\lambda)|$) involved in the FBP algorithms can amplify data noise non-uniformly [22,23]. One can also observe, for a reduced scan in which the trajectory segment of the left side of the FOV is missing, the spatially varying weighting factors result in images with image-variance lower on the left side than that on the right side of the image.

5 Conclusions

Circular cone-beam CT is widely used in industrial nondestructive testing applications, and the FDK algorithm is used for image reconstruction there. In addition to the simplicity and convenience, circular cone-beam CT has been widely used in industrial nondestructive testing applications mainly for its ease of implementation and readily available reconstruction programs based on the Feldkamp algorithm. In addition to the computational efficiency, the Feldkamp algorithm allows a short scan for image reconstruction and therefore can contribute to increasing throughput of the scanners. However, the Feldkamp algorithm is essentially an approximate algorithm, and the image artifacts such as deformation, intensity drop, or streaks can result from a circular scan particularly when the cone-angle is large and/or axial variation of the object is large. Circular sinusoidal trajectory, in contrast to a circular trajectory, can provide enough cone-beam data for exact volumetric image reconstruction, and can be implemented relatively easily on an existing scanner by allowing the scanner bed translation. As an analogy of a short-scan to a full-scan in circular cone-beam CT, a reduced-scan circular sinusoidal cone-beam CT has been investigated in this work.

An FBP algorithm based on Pack-Noo's formula has been developed for a reduced-scan circular sinusoidal cone-beam CT, and we have successfully demonstrated that the FBP algorithm for a reduced-scan circular sinusoidal trajectory can yield reconstructions comparable to those obtained by use of the existing FBP algorithm for a full-scan circular sinusoidal trajectory. Preliminary noise propagation property of the algorithm has also been studied. The proposed reduced-scan approach can potentially contribute to increasing throughput of the scanner while preserving good image quality that is achievable by a full-scan circular sinusoidal cone-beam CT.

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References

1. Thierry R, Miceli A, Hofmann J, Flisch A, Sennhauser U. Hybrid simulation of scatter intensity in industrial cone-beam computed tomography. *Nucl Inst Meth A* 2009;598:611–619.
2. Nukaga J, Kitazawa S, Kamimura H. A new scanning technique for X-ray computed tomography. *NDT & E International* 2008;41:564–569.
3. Danczak M, Juergen-Wolter K. Cone-beam computed tomography as a new testing method for industrial application. *Int'l Spring Seminar on Electronics Technology* 2004:105–109.
4. Katsevich A. A general scheme for constructing inversion algorithms for cone beam CT. *Int J of Mathematics and Math Sciences* 2003;2003:1305–1321.
5. Pack JD, Noo F, Clackdoyle R. Cone-beam reconstruction using the backprojection of locally filtered projections. *IEEE Trans Med Imaging* 2005;24:2317–2336.
6. Ye Y, Wang G. Filtered backprojection formula for exact image reconstruction from cone-beam data along a general scanning curve. *Med Phys* 2005;32:42–48. [PubMed: 15719953]
7. Zou Y, Pan X, Sidky EY. Theory and algorithms for image reconstruction on chords and within region of interests. *Journal of the Optical Society of America A* 2005;22:2372–2384.
8. Katsevich A. Image reconstruction for the circle-and-arc trajectory. *Phys Med Biol* 2005;50:2249–2265. [PubMed: 15876665]
9. Zhuang T, Chen G. New families of exact fan-beam and cone-beam image reconstruction formulae via filtering the backprojection image of differentiated projection data along singly measured lines. *Invs Prob* 2006;22:991–1006.
10. Katsevich A. Theoretically exact FBP-type inversion algorithm for spiral CT. *SIAM J Appl Math* 2002;62:2012–2026.
11. Noo F, Pack J, Heuscher D. Exact helical reconstruction using native cone-beam geometries. *Phys Med Biol* 2003;48:3787–3818. [PubMed: 14703159]
12. Zou Y, Pan X. Exact image reconstruction on Pi-line from minimum data in helical cone-beam CT. *Phys Med Biol* 2004;49:941–959. [PubMed: 15104318]
13. Pack, JD.; Noo, F. Cone-beam reconstruction outside R-lines using the backprojection of 1-d filtered data. *Proceedings of the 2005 International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine; Salt Lake. 2005. p. 287-290.*
14. Pack JD, Noo F. Cone-beam reconstruction using 1d filtering along the projection of m-lines. *Inv Prob* 2005;21:1105–1120.
15. Yang H, Li M, Koizumi K, Kudo H. Exact cone beam reconstruction for a saddle trajectory. *Phys Med Biol* 2006;51:1157–1171. [PubMed: 16481685]
16. Yang H, Li M, Koizumi K, Kudo H. View-independent reconstruction algorithms for cone beam CT with general saddle trajectory. *Phys Med Biol* 2006;51:3865–3883. [PubMed: 16861786]
17. Cho S, Xia D, Pelizzari CA, Pan X. Exact reconstruction of volumetric images in reverse helical cone-beam CT. *Med Phys* 2008;35:3030–3040. [PubMed: 18697525]
18. Xia, D.; Cho, S.; Pan, X. Image reconstruction for a reduced scan in circular sinusoidal cone-beam CT. *Proceedings of the 2007 International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine; Lindau, Germany. 2007. p. 213-216.*
19. Pack JD, Noo F, Kudo H. Investigation of saddle trajectories for cardiac CT imaging in cone-beam geometry. *Phys Med Biol* 2004;49:2317–2336. [PubMed: 15248580]
20. Yang, H.; Li, M.; Koizumi, K.; Kudo, H. Application of Pack and Noo's cone-beam inversion formula to a wide class of trajectories. *IEEE Medical Imaging Conference Record; San Diego. 2006. p. M14-450.*

21. Zou Y, Pan X. Image reconstruction on Pi-lines by use of filtered backprojection in helical cone-beam CT. *Phys Med Biol* 2004;49:2717–2731. [PubMed: 15272684]
22. Yu L, Pan X. Halfscan fan-beam computed tomography with improved noise and resolution properties. *Med Phys* 2003;30:2629–2637. [PubMed: 14596299]
23. Xia D, Yu L, Sidky EY, Zou Y, Zuo N, Pan X. Noise properties of chord-image reconstruction. *IEEE Trans Med Imaging* 2007;26:1328–1344. [PubMed: 17948724]

Figure 1. Circular sinusoidal trajectories of order $m = 3$. (a) A full-scan (or, equivalently, closed) circular sinusoidal trajectory, where $\lambda \in [-\pi, \pi]$. (b) A reduced-scan (or, equivalently, open) circular sinusoidal trajectory, where $\lambda \in [\lambda_a, \lambda_b]$, and $-\pi < \lambda_a < \lambda_b < \pi$.

Figure 2.

Illustration of the polygons and the associated filtering unit vectors $\hat{e}_i$ for a full-scan and reduced-scan circular sinusoidal trajectory of order $m = 3$. (a) A six-side polygon parallel to the x - y plane and intersecting the trajectory at six different points, $\vec{r}_0(\lambda_i)$, $i = 1, \dots, 6$. (b) A reduced-scan circular sinusoidal trajectory is obtained from the full-scan circular sinusoidal trajectory by removal of the sub-segment trajectory $[\lambda_6, \lambda_7]$. In both cases, the selected filtering unit vectors $\hat{e}_i$ are along the chords (i.e., sides of the polygons).

Figure 3.

Illustration of the projection and filtering geometries at view λ . A unit vector $\hat{a}(\lambda, \vec{x}_0)$ can be determined such that it points from the source point $\vec{r}_0(\lambda)$ to $\vec{x}_0$ on which the image to be reconstructed. Using $\hat{a}(\lambda, \vec{x}_0)$ and an appropriately selected filtering unit vector $\hat{e}$, a plane Γ is formed with a normal unit vector $\hat{n}_\Gamma = \hat{e} \times \hat{a}(\lambda, \vec{x}_0)$. The intersection of plane Γ and the detector forms a filtering line (i.e., the dash line) on the detector plane. Each point on the filtering line can be specified by an angular parameter γ . The u_d - and v_d -axes of the 2D-detector-coordinate system are parallel to u - and v -axes of the rotation-coordinate system, and its origin is at the projection of the source point $\vec{r}_0(\lambda)$ onto the detector.

Figure 4.

Filtering lines on detector planes at views $\lambda = -\frac{5}{6}\pi$ (a), $-\frac{2}{3}\pi$ (b), and $-\frac{\pi}{2}$ (c), respectively, for a circular sinusoidal trajectory of order $m = 3$ determined by use of filtering unit vectors

$\hat{e} = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right)$ (upper row) and $(0, -1, 0)$ (lower row). The intersecting points of the filtering lines are at $(u_c, v_c) = (\sqrt{3}D, 0)$ (a), $(\infty, 0)$ (b), and $(-\sqrt{3}D, 0)$ (c) in the upper row

and at $(u_c, v_c) = (-\sqrt{3}D, 0)$ (a), $\left(-\frac{\sqrt{3}}{3}D, 0\right)$ (b), and $(0, 0)$ (c) in the lower row, respectively. The regions enclosed by the circle indicate the projection supports of the object function. Depending upon the selection of filtering unit vector $\hat{e}$, the intersecting points of the filtering lines at some views λ can be within the projection support of the object function, as (b) in the lower row shows.

Figure 5.

Illustration of image reconstruction for a stack of polygons sandwiched between polygons $\Omega_{\Pi1}$ and $\Omega_{\Pi2}$ for a reduced-scan circular sinusoid trajectory of order $m = 3$. These polygons, including $\Omega_{\Pi1}$ and $\Omega_{\Pi2}$, are parallel to the x - y plane. The collection of 2D reconstructions within these polygons forms a 3D volume.

Figure 6.

Top row: true images within the 2D slices at (a) $x = 0$, (b) $y = 0$, and (c) $z = 0$ cm. Middle row: noiseless images within the 2D slices at (a) $x = 0$, (b) $y = 0$, and (c) $z = 0$ cm reconstructed for the reduced-scan circular sinusoidal trajectory of order $m = 3$. Bottom row: noiseless images within the 2D slices at (a) $x = 0$, (b) $y = 0$, and (c) $z = 0$ cm reconstructed for the full-scan circular sinusoidal trajectory of order $m = 3$. The display gray scale is [1.0, 1.04].

Figure 7. Profiles along the line indicated in Fig. 6 for the reduced-scan (left) and full-scan (right) circular sinusoidal trajectory of order $m = 3$. The solid and dashed lines represent the profiles of the reconstructed image and the true image, respectively.

Figure 8.

Top row: true images within the 2D slices at (a) $x = 0$, (b) $y = 0$, and (c) $z = 0$ cm. Middle row: noiseless images within the 2D slices at (a) $x = 0$, (b) $y = 0$, and (c) $z = 0$ cm reconstructed for the reduced-scan circular sinusoidal trajectory of order $m = 3$. Bottom row: noiseless images within the 2D slices at (a) $x = 0$, (b) $y = 0$, and (c) $z = 0$ cm reconstructed for the circular trajectory. The display gray scale is $[0.5, 1.5]$.

Figure 9. Profiles along the line indicated in Fig. 8 for the reduced-scan (left) and full-scan (right) circular sinusoidal trajectory of order $m = 3$. The solid and dashed lines represent the profiles of the reconstructed image and the true image, respectively.

Figure 10.

Top row: reconstructed image (left) and image variances (right) within the 2D slices at $z = 0$ cm obtained from the noisy projection data in a reduced scan. Bottom row: variance profiles on the middle horizontal line (left) and the middle vertical line (right) of the variance images.