Localizing intracavitary brachytherapy applicators from cone-beam CT x-ray projections via a novel iterative forward projection matching algorithm

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(Received 24 August 2010; revised 3 January 2011; accepted for publication 4 January 2011; published 31 January 2011)

Purpose: To present a novel method for reconstructing the 3D pose (position and orientation) of radio-opaque applicators of known but arbitrary shape from a small set of 2D x-ray projections in support of intraoperative brachytherapy planning.

Methods: The generalized iterative forward projection matching (gIFPM) algorithm finds the six degree-of-freedom pose of an arbitrary rigid object by minimizing the sum-of-squared-intensity differences (SSQD) between the computed and experimentally acquired autosegmented projection of the objects. Starting with an initial estimate of the object's pose, gIFPM iteratively refines the pose parameters (3D position and three Euler angles) until the SSQD converges. The object, here specialized to a Fletcher–Weeks intracavitary brachytherapy (ICB) applicator, is represented by a fine mesh of discrete points derived from complex combinatorial geometric models of the actual applicators. Three pairs of computed and measured projection images with known imaging geometry are used. Projection images of an intrauterine tandem and colpostats were acquired from an ACUTY cone-beam CT digital simulator. An image postprocessing step was performed to create blurred binary applicators only images. To quantify gIFPM accuracy, the reconstructed 3D pose of the applicator model was forward projected and overlaid with the measured images and empirically calculated the nearest-neighbor applicator positional difference for each image pair.

Results: In the numerical simulations, the tandem and colpostats positions (x, y, z) and orientations (α, β, γ) were estimated with accuracies of 0.6 mm and 2° , respectively. For experimentally acquired images of actual applicators, the residual 2D registration error was less than 1.8 mm for each image pair, corresponding to about 1 mm positioning accuracy at isocenter, with a total computation time of less than 1.5 min on a 1 GHz processor.

Conclusions: This work describes a novel, accurate, fast, and completely automatic method to localize radio-opaque applicators of arbitrary shape from measured 2D x-ray projections. The results demonstrate ~ 1 mm accuracy while compared against the measured applicator projections. No lateral film is needed. By localizing the applicator internal structure as well as radioactive sources, the effect of intra-applicator and interapplicator attenuation can be included in the resultant dose calculations. Further validation tests using clinically acquired tandem and colpostats images will be performed for the accurate and robust applicator/sources localization in ICB patients. © 2011 American Association of Physicists in Medicine. [DOI: [10.1118/1.3544661](https://doi.org/10.1118/1.3544661)]

Key words: intracavitary brachytherapy applicators, localization, generalized iterative forward projection matching, cone-beam CT, x-ray projections

I. INTRODUCTION

Despite more than a century of routine clinical use, intracavitary brachytherapy (ICB) treatment planning for gynecological malignancies, the six-decade old practice of using orthogonal 2D radiographs to localize the radioactive sources in the patient,^{1–3} continues to be widely used. Because this process involves manual drawing of the source positions on films or digital images, it is time-consuming and may be error-prone. Tandem sources are difficult to visualize in 30% of cases for larger patients (thickness greater than 38 cm) due to poor lateral radiograph quality, while the colpostat edges are almost always obscured in the lateral view because of the

pelvic bone and the overlap of the two colpostats.⁴ Recently, the American Brachytherapy Society (ABS) recommended⁵ the use of three-dimensional (3D) computed tomography (CT) or magnetic resonance (MR) image data sets for both applicator localization and reconstruction of 3D anatomy of the patient. These images could be used for contouring. However, this practice has not been widely applied in the gynecologic ICB treatment because the image-based dosimetry has not been widely adapted for routine community practice.⁶ Even when MR or CT is available for postinsertion treatment evaluation, it is often not available intraoperatively, so that 2D planar x-ray imaging maybe the only “on-line” imaging modalities available for intraoperative optimi-

zation during the applicator insertion process. Another problem with conventional 2D localization practices is that they focus only on estimating source tips and ends. Brachytherapy applicators have complex internal structures,^{7–10} the pose (position and orientation) of which relative to the patient's anatomy need to be determined for more accurate dose evaluation either via Monte Carlo simulation^{10–14} or by utilizing AcurosTM (Transpire, Inc., Gig Harbor, WA), which solves the Boltzmann transport equation using a discrete ordinates algorithm^{15–17} to account for dose perturbations due to tissue heterogeneities. Monte Carlo studies have shown that applicator shielding reduces doses up to 25% when the dose distributions from the bilateral colpostats and intrauterine tandem are included.^{10,11,13}

The problem of localizing applicators is closely related to the problem of metal streaking artifacts. The widely used shielded vaginal applicators^{18,19} are known to cause severe streaking artifacts, limiting the value of CT imaging for segmenting critical organs at risk relative to the implanted applicators. Mitigating metal streaking artifacts by restoring the missing sinogram information by interpolation^{20–23} or by using prior metal-free CT images²⁴ have met with only limited success. A more general and principled approach is an iterative statistical CT image-reconstruction solutions²⁵ which support artifact-free CT imaging of the soft tissues near implanted foreign metal bodies, provided that an accurate *a priori* model of the metal object, including its pose, shape, and attenuation map, are available.^{26–28} The iterative forward projection matching (IFPM) approach^{29–31} contributes to this solution by using high spatial resolution projections rather than streak-limited CT images for applicator localization and by making accurate applicator pose estimates available as an input to the iterative reconstruction algorithm.

We pursued generalized iterative forward projection matching (gIFPM) as a part of a larger project to develop intraoperative CT-based applicator insertion and dynamic dose planning capability in our dedicated brachytherapy procedure room, where outpatient interstitial and intracavitary insertions are performed. To this end, an ACUITY intraoperative 3D cone-beam CT (CBCT) imaging system (Varian Medical Systems, Palo Alto, CA) was installed to support more accurate imaging of the patient's soft-tissue anatomy relative to the implanted applicators, so that applicator insertion could be optimized and planned without moving or repositioning the patient. However, we found that artifact-free imaging was compromised in many cases by applicators, retractors, and other metal objects in the scan field of view. In addition, sometimes a full 360° scan rotation was precluded because of the patient's lithotomy position. The IFPM algorithm^{22–31} was adapted to the problem of accurately estimating the pose of metal objects with complex geometric shapes from CBCT projection data as a step toward implementing CT image-reconstruction algorithms known to be more tolerant of missing or incomplete data.²⁶

Because the spatial resolution of the applicators in the 2D image planes is superior to CT, reconstructing applicators from radiographic images can address both improving the generality and accuracy of conventional 2D applicator local-

FIG. 1. (a) Close-up photograph of the Fletcher–Weeks CT-compatible afterloadable colpostats and one of the tandems used in this study. (b) Computed CBCT projections of the 3D tandem and right colpostat models, where the image background represent uniform elliptical water cylinder. The image intensity values represent an arbitrary integer number assigned to each material in the model.

ization and CT artifact mitigation. To these ends, we have further generalized the IFPM (gIFPM) algorithm^{29–31} from five degrees-of-freedom needed to reconstruct permanent implant seed centroids and orientations to the six degrees-of-freedom problem of reconstructing the 3D pose of arbitrary-shaped objects from a few measured 2D x-ray projections. The applicator model is a fine mesh of discrete points derived from complex combinatorial geometric (CCG) modeling⁹ of the actual applicators. Each applicator model has three translational and three rotational degree-of-freedom. The parametrized 3D applicator model is then forward projected onto the 2D detector planes in the CBCT projection geometry and its pose adjusted iteratively until the computed and measured images match. In this approach, each applicator model moves independent of the others in the 3D space. Proof-of-concept is demonstrated by applying gIFPM both to synthetic data sets and experimentally acquired ACUITY images of actual applicator configurations.

II. MATERIALS AND METHODS

II.A. Example shielded applicator system

A low dose-rate manual afterloading system, consisting of ¹³⁷Cs sources loaded into Weeks⁸ CT-compatible Fletcher–Suit applicator, was used in this study [see Fig. 1(a)]. The applicators consist of thin-walled aluminum intrauterine tandems and vaginal colpostats. A volume based CCG modeling code⁹ was used to obtain a geometric model of each applicator component, i.e., mathematical representation of the ap-

plicator including internal structure and outer surface, in a known pose. Given a completed geometric model, the code package supports point and segment classification as well as ray tracing through the applicator system. Figure 1(b) shows computed radiographic projections of each component derived from its associated model in the CBCT projection geometry. The CCG modeling code consistently defines topological concepts such as region boundary, interior, and exterior and describes complex regions as set-theoretic unions, intersections, and differences of simple convex regions, i.e., mathematically defined geometric objects. Such complex modeling of the applicator geometry is essential to obtain partial transmission through the applicator components and to obtain accurate Monte Carlo dose distributions.

II.B. Generalized IFPM algorithm and initial estimates

The IFPM algorithm^{29–31} was further generalized to accurately localize ICB applicators of known metal body shape and known materials but unknown locations and orientations inside the patient's 3D anatomy using only a few CBCT projection images.

The generalized IFPM algorithm requires six pose parameters to fully describe each of the N applicators $k=1, \dots, N$ (typically $N=3$ for tandem and colpostats) in the world coordinate system (WCS), which takes as its origin the isocenter of the imaging system. The x axis is left-right, the y axis is anterior-posterior, and the z axis is superior-inferior direction for a patient in supine position with feet pointing away from the gantry stand. For the k th applicator, $(t_x, t_y, t_z, \alpha, \beta, \gamma)$, where $\mathbf{t}_k=(t_x, t_y, t_z)_k$ denotes the WCS coordinates of origin of the k th applicator's local coordinate system and the three equal-spaced angles $\Theta_k=(\alpha, \beta, \gamma)_k$ describe its orientation of this local coordinate system in WCS relative to CBCT isocenter around the x , y , and z axes, respectively. Each applicator is described by a fine mesh of Q_k

points: $\{\mathbf{r}\}_k=\{\mathbf{r}_{ik}|\mathbf{r}_{ik}\in A_k, i=1, \dots, Q_k\}$ derived from the CCG applicator model when $\Theta_k=0$ and $\mathbf{t}_k=0$ described in Sec. II A. Typically, a mesh of $1\times 1\times 1\text{ mm}^3$ was used. A CBCT projection-geometry model was made by specifying the translation and rotation matrices for each of the M imaging viewpoint (typically, AUCITY gantry angles). The flat panel detector WCS orientation of each isocentric projection is defined by the angles (φ, ψ, η) , which describe rotations of the detector image plane around the x , y , and z axes, respectively. The detector model is parametrized by describing its magnification, image center, image size, and pixel resolution for each image viewpoint. The source-to-isocenter and isocenter-to-detector distances are denoted by the symbols S and D , respectively.

The mesh of discrete points $\{\mathbf{r}\}_k$ need be derived from the CCG applicator model only once. For each candidate pose $(\mathbf{t}_k, \Theta_k)$, we obtain the rotated and translated set of mesh points

$$\begin{aligned} \{\mathbf{r}'|(\mathbf{t}_k, \Theta_k)\}_k &= \{\mathbf{r}'_{ik}|\mathbf{r}'_{ik} \\ &= [\mathbf{R}_{\alpha, \beta, \gamma}(\Theta_k)]\mathbf{r}_{ik} + \mathbf{t}_k, \mathbf{r}_{ik} \in A_k, i=1, \dots, Q_k\}, \end{aligned} \quad (1)$$

where, the rotation matrix $[\mathbf{R}_{\alpha, \beta, \gamma}(\Theta_k)]$ is derived in the Appendix. Each point $\mathbf{r}'_{ik} \in \{\mathbf{r}'|(\mathbf{t}_k, \Theta_k)\}_k$ is projected onto the detector plane defined by gantry angle η , giving rise to a 2D coordinate position $P_\eta(\mathbf{r}'_{ik})=(u'_{ik}, v'_{ik})_\eta$ where $P_\eta(\mathbf{r})$ denotes the continuous coordinates of the point $\mathbf{r}$ in 3D space projected into the detector plane for a gantry angle of η . Then, we obtain the computed binary image of the N applicators $I_0(u, v|\{\mathbf{r}(\mathbf{t}_k, \Theta_k)\}_k, \eta)$, corresponding to the initial set of pose estimates $\{\mathbf{r}(\mathbf{t}_k, \Theta_k)\}_k$ by setting to unity all those detector pixels (u, v) that contain one or more projected points $(u'_{ik}, v'_{ik})_\eta$ from one of the applicator mesh projections and zero elsewhere. More explicitly, the discretized applicator image pixels intensity is given by

$$I_0(u, v|\{\mathbf{r}_k, \Theta_k\}, \eta) = \begin{cases} 1 & \text{if } \exists i, k \text{ such that for } \mathbf{r}_{ik} \in \{\mathbf{r}|(\mathbf{t}_k, \Theta_k)\}_k, P_\eta(\mathbf{r}_{ik}) \in (u, v) \\ 0 & \text{if not} \end{cases}, \quad (2)$$

where (u, v) denotes the set of 2D coordinates of the detector pixels represented by u and v . The binary mask representation of the projected applicator is then blurred by convolving it with 2D Gaussian blurring function with a standard deviation σ . For a set of N applicators projected from the world frame, then the total computed image is

$$\begin{aligned} I_c(u, v|\{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) &= \sum_{u'_{ik}, v'_{ik}} I_0(u'_{ik}, v'_{ik}|\mathbf{r}_k, \Theta_k, \eta) \\ &\quad \times \exp[-(u - u'_{ik})^2/2\sigma^2 \\ &\quad - (v - v'_{ik})^2/2\sigma^2], \end{aligned} \quad (3)$$

where (u'_{ik}, v'_{ik}) denote the spatial coordinates of the image pixel center indexed by u and v . The main purpose of the Gaussian blurring is to create a continuous-value grayscale image to which a gradient-driven iterative search process can be applied. In the absence of any blurring on the images, large areas of the intensity map would have zero intensity, providing no gradient to guide the similarity minimization search. The blurring creates a “source attractive” potential well around each applicator with tails extending beyond the applicator footprint, causing computed applicator images to be pulled toward measured applicator images and accelerating the convergence of the iterative minimization search.

The similarity metric sum-of-squared-intensity differences (SSQD) is the total of the pixel-by-pixel sum of the squared-intensity differences between the computed $I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta)$ and the experimentally acquired (measured) blurred $I_m(u, v | \sigma, \eta)$ applicator image pairs, summed over all M projections and pixels therein, i.e.,

$$\text{SSQD}(\{\mathbf{r}_k, \Theta_k\} | \sigma, \eta) = \sum_{\eta} \sum_{u, v} [I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) - I_m(u, v | \sigma, \eta)]^2. \quad (4)$$

The applicator pose parameters $\{\mathbf{r}_k, \Theta_k\}$ are iteratively updated by simultaneously adjusting the poses of all applicators, computing updated images $I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta)$, and reevaluating the objective function [Eq. (4)]. The pose updates are calculated from the first derivatives of SSQD with respect to each degree-of-freedom. For example, the derivative with respect to the x -coordinate of the k th applicator was computed as follows:

$$\partial(\text{SSQD})/\partial x_k = 2 \sum_{\eta} \left(\sum_{u, v} [I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) - I_m(u, v | \sigma, \eta)] \partial I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) / \partial x_k \right). \quad (5)$$

Because the image grayscale intensities are represented entirely by the Gaussian blurring function, each grayscale partial derivative, such as $\partial I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) / \partial x_k$, was calculated analytically for each applicator pixel (u, v) . The final derivative with respect to each degree-of-freedom for updating $\{\mathbf{r}_k, \Theta_k\}$ was found by averaging $\partial I_c(u, v) / \partial x_k$ over all pixels (u, v) contained within the shadow of the k th applicator model. Similarly, we computed the first derivatives of SSQD with respect to all other spatial and angular coordinates. Detailed derivations of the gradient calculation can be found in the Appendix.

After computing the gradients, the free parameters were updated using a nonlinear gradient search method²⁹⁻³¹ that combines steepest descent gradient search with a parabolic approximation of the SSQD surface around the global minimum. The process iteratively refines the 3D applicator pose parameters until the closest match between the computed and measured projections is achieved. The computed and measured images should have the same imaging geometry, image size, and pixel resolution. Each three pairs of computed and measured image data sets with corresponding imaging parameters are required for one reconstruction process. In this way, the voxelized 3D geometric model of arbitrary shape was integrated into the forward projection matching method for computing the 2D projection images of the 3D ICB applicators iteratively.

II.C. Validation via simulated applicator models

Numerical simulation studies were performed to demonstrate the feasibility of our approach. The 3D geometry of the Fletcher–Weeks tandem and colpostats was modeled as described in Sec. II A, producing a fine mesh of points to represent the applicator.

Two different types of experiments were performed: (a) Applicator reconstruction using intrauterine tandem only and (b) applicator components reconstruction using the entire applicator system consisting of bilateral colpostats plus tandem. In these simulation studies, the true applicator system pose consisted of a tandem placed in the middle of the field of view and bilateral colpostats placed on either side of the tandem with equivalent transaxial bisecting planes and centers 25, 35, and 45 mm apart. This is the true pose of the applicator configuration that we wished to determine. Using these applicator configurations, synthetic measured projection images were computed using the CBCT projection geometry at different gantry angles within $\pm 30^\circ$. The projected applicators on the imaging plane were masked to create the binary bitmap images. In 3D space, each applicator component was shifted by a displacement of ± 2.5 mm in each coordinate axis and rotated $\pm 8^\circ$ around each rotation axis. This was our initial estimate. Both the true/synthetic measured and computed images were blurred using 2D Gaussian blurring function ($\sigma = 3.0$ – 4.0 mm) with a known intensity distribution. The optimal value of blurring width σ was obtained from trial and error for each run. The source-to-detector distance was 150 cm and the source-to-object distance was 100 cm. The images were 288×288 pixels square and had a resolution of 0.388 mm/pixel. Several trials were performed by varying colpostat separation distance from 25 to 45 mm.

II.D. Validation via physical (actual) applicators

To experimentally validate this algorithm, the applicators (tandem and colpostats) were imaged on a Varian ACUTY imaging system that is used for performing image-guided brachytherapy insertions in our dedicated brachytherapy suite. Projection images of the actual applicators were acquired in different gantry angle positions within $\pm 30^\circ$ using the Varian 4030CB imager. The detector is 40 cm $\times$ 30 cm with a 1024 $\times$ 768 image size and a resolution of 0.388 mm/pixel. The image postprocessing involved (a) cropping the images to 576 $\times$ 576 pixels square; (b) normalizing the image intensity by finding its maximum and minimum values in the image; (c) morphological top-hat filtering to suppress the background; and (d) automatic thresholding using the 3 standard deviation value of the pixel intensity histogram to create binary applicator images in each projection in order to separate the applicators from the background. This process resulted in binary bitmap images with zero intensity in the background and intensity one over the area of each projected applicator model. The binary images were then convolved with a 2D Gaussian blurring function chosen from $\sigma = 3.0$ – 4.0 mm from trial and error for each run as described earlier. An example case of image postprocessing is shown in Fig. 2. Three different experiments were performed by keeping the colpostats separation at 30, 40, and 50 mm, respectively.

Initial estimates of the applicator model were obtained by perturbing the known 3D applicator components, as described earlier. To make the computed projection images, the

FIG. 2. An example illustrating postprocessing of experimentally acquired ACUTY projection images. (a) Raw projection image, (b) top-hat filtered image, (c) binary image, and (d) blurred grayscale image used as an input to the generalized IFPM algorithm.

perturbed applicators' configuration was rotated and translated to each imaging viewpoint and then projected on the (u, v) detector planes. For each candidate set of poses $\{\mathbf{r}_k, \boldsymbol{\Theta}_k\}$, computed applicator images were evaluated as described in Sec. II B using the same CCG geometric model.

II.E. Assessment of the applicator reconstruction/registration error

For the simulation study, the actual known pose of the applicators was used to benchmark gIFPM. First, the accuracy of each trial was quantified by calculating the difference between all estimated and the true 3D pose parameters of the each applicator component. In addition, agreement was qualitatively assessed by calculating the difference between the measured and final computed images for each gantry angle. In the second approach, the residual 2D registration error was computed by reprojecting the applicator pose at convergence onto the 2D image planes, overlaying the computed and measured applicator projections, and calculating the nearest-neighbor positional difference between the measured and computed applicator positions in each image plane. In this approach, for all computed applicator components, we empirically calculated the applicator center location (center of mass) in each 2D image plane and compared to those obtained from the measured applicator components images at convergence. The center of mass position of the entire tandem was used. For the bilateral colpostats, 2D images were further processed to isolate each computed and measured source holder from the colpostat handle at convergence in each image plane. We then empirically calculated the center of mass coordinates of the each colpostat source position.

For the physical applicator study, there were no ground truth coordinates. The applicator registration error was computed in terms of residual 2D radial difference between computed and measured applicator images at convergence in each image plane as described above.

III. RESULTS

III.A. Simulated applicator models

Several experiments were performed to test the accuracy and robustness of this algorithm, including different initial estimates of the digital tandem. The error in positional and angular pose components was found to be (0.32, 0.46, -0.37) mm and (1.03, 0.87, 1.16)°, respectively. Each test

FIG. 3. An illustration of the iterative convergence process for a simulated implant consisting of tandem and bilateral colpostats for a 25 mm colpostat separation (see convergence rate graph in Fig. 4). (a) Initial estimate of the applicator configuration, (b) computed images at convergence, (c) the true/synthetic measured images, and (d) difference between (b) and (c), where the rows represents different gantry angles. The horizontal line in the third row shows the qualitative comparison of an initial estimate in part (a) with computed and measured images in part (b) and (c), respectively. The generalized IFPM algorithm was able to reproduce each applicator pose, as well as overlapping components.

trial converged in eight to ten iterations with a total computation time of less than 40 s on a 1 GHz processor.

Figure 3 shows an example of the iterative matching process for the simulated full ICB applicator system, i.e., intrauterine tandem and bilateral colpostats. The three projections have gantry angles of 0° and $\pm 30^\circ$. The small difference in Fig. 3(d) shows very good agreement between the measured and computed images at convergence, including reproducing overlapping applicator components [see part (b)]. For the qualitative estimate of the forward projection matching, the reference line drawn in the third row of the Fig. 3 shows the initial estimate of the applicators pose compared with respect to computed and measured images in parts (b) and (c), respectively. In Fig. 4, we show the cost function convergence as a function of iteration number for all three simulated example cases. All test trials converged in 10–12 iterations (Gaussian width, $\sigma = 3.8$ mm) with a total computation time of about 1.5 min on a 1 GHz processor. The comparison is summarized for all three cases in Table I. In all trials, the differences in errors were less than 0.7 mm and 2.1° for each of the positional and angular coordinates, respectively. Figure 5 shows reconstructed 3D view overlaid with the true/measured tandem and colpostats.

A number of experiments were performed using different gantry angle combinations as well as different initial starting configurations of the applicator models in the course of this study. Theoretically, one should expect SSQD=0 at the convergence, i.e., all computed applicators images exactly match with those measured. However, from Fig. 4, for the combined-applicators geometries, it is evident that the SSQD does not exactly converge to zero (i.e., less than 3% difference between measured and computed applicators images),

FIG. 4. The similarity metric score vs iteration number for the generalized IFPM algorithm for the three simulated full ICB applicator configurations.

showing less than optimal convergence (i.e., trapping in local minima). That means if the initial estimate of the applicators configuration is far apart from the measured configuration, there is a chance of less than optimal matching. The finite width of the detector pixel size may also affect the convergence rate of gIFPM algorithm (i.e., the larger the pixel size, the less than optimal the convergence).

III.B. Validation test with measured images

Several experiments were performed using projection images of the actual intrauterine tandem and bilateral colpostats acquired from the ACUTY digital simulator to validate this algorithm. An example of the reconstructed applicators projected onto the imaging planes is presented in Fig. 6 when using colpostat separation of 40 mm. In this case, the residual 2D registration errors in computed vs measured appli-

FIG. 5. Point-by-point overlay of the reconstructed applicators with the true/synthetic measured applicators, demonstrating near coincidence achieved by the 3D/2D gIFPM registration and applicator reconstruction as a unified process.

cators image at each gantry angle were less than 1 mm for the intrauterine tandem and about 1.5 mm for the bilateral colpostats. This corresponds to positioning accuracy of about 0.67 mm for the intrauterine tandem and 1 mm for the bilateral colpostats in the 3D space. Table II shows the residual 2D registration error between the measured and computed applicator components in each image plane. In all cases, the majority of the 2D radial difference of the applicator registration error was about 1.5 mm or less and no error exceeded 2.0 mm in the detector plane. These preliminary results indicate good agreement between measured and computed applicators images.

IV. DISCUSSION

A novel IFPM algorithm has been successfully extended to the more complex six degrees-of-freedom problem of automatically reconstructing the 3D pose of radio-opaque non-symmetric objects (e.g., ICB applicators) from a few mea-

TABLE I. Accuracy of gIFPM reconstructed poses for three simulated full applicator system configurations. The difference for the each applicator component position and orientation coordinates is reported.

Separation between the colpostats (mm)	Image viewpoint used (deg)	Applicator components	gIFPM vs true applicator pose					
			Difference in applicator position (mm)			Difference in applicator orientation (deg)		
			Δx	Δy	Δz	$\Delta \alpha$	$\Delta \beta$	$\Delta \gamma$
25	0, -30, +30	Tandem	0.41	0.59	-0.38	-0.98	0.78	1.86
		R. colpostats	0.18	-0.56	0.48	2.03	1.71	-0.89
		L. colpostats	-0.38	-0.48	0.61	-0.85	2.04	1.95
35	0, +20, -20	Tandem	0.15	-0.37	0.46	0.89	0.95	-0.86
		R. colpostats	-0.32	-0.15	0.36	1.56	-0.74	1.62
		L. colpostats	-0.28	0.54	-0.51	-0.87	1.42	0.73
45	0, -30, +30	Tandem	-0.18	0.27	0.34	-0.86	-0.64	-0.72
		R. colpostats	0.14	0.18	-0.35	0.46	0.58	0.65
		L. colpostats	-0.20	0.34	0.52	0.87	1.06	-0.56

FIG. 6. Superposition of experimentally acquired binary images (white) with automatically reconstructed applicators positions (black) projected onto the detector planes. (a) 0° gantry angle, (b) -30° gantry angle, and (c) $+30^\circ$ gantry angle, respectively, when using 40 mm colpostat separation. The applicator registration error was less than 1 mm for the intrauterine tandem and about 1.5 mm for the bilateral colpostats on each image plane.

sured 2D projections. Unlike conventional approach, gIFPM is a fully automated process that does not require lateral films, thereby avoiding common image quality issues afflicting manual orthogonal film analysis. The gIFPM algorithm is applicable to a wide range of ICB applicator geometries without the need to modify the general method to accommodate individual geometries. With further testing and integration into the clinical workflow, our gIFPM has the potential to support real-time, robust, and unsupervised reconstruction of implanted object pose for the purpose of supporting real-time intraoperative ICB geometry evaluation or as an input to CT image-reconstruction algorithms that are robust in the face of missing or nonlinear projection data.

Several methods are available for automatically localizing ICB sources and applicators from 3D CT studies,^{8,12,32} including interstitial catheter reconstruction.³³ An approach developed by Lerma and Williamson¹² in which a 3D rigid model of the external applicator shape is rigidly registered to the corresponding surface manually contoured in the reconstructed CT images. For typical ICB applicator orientations, they demonstrated that the localization accuracy was about 1/3 of the slice thickness. Because 2D projection image resolution is superior to that of 3D CBCT, gIFPM has the potential to reconstruct 3D applicator poses more accurately

than the 3D CT method. Another advantage of gIFPM over the 3D CBCT method is that the latter requires acquisition and reconstruction of the entire 3D image, which currently requires 2–4 min for the Varian ACUITY system to acquire the data and reconstruct images (although it cannot provide useful soft-tissue image information with the metal applicators in the patient scan region) in addition to the time required to reposition of the patient (so that a 180° orbit sinogram can be acquired) and to manually segment and threshold the applicators. In contrast, gIFPM takes ~ 1 min to acquire three projections within 30° gantry rotation (avoiding patient repositioning) plus minimal time (~ 1 min in our algorithm that has yet to be optimized for fast execution) for applicator localization.

Another 2D radiographic approach for localizing ICB applicators developed by Li *et al.*³⁴ is based on identifying several corresponding landmark points on the 2D projections, allowing the object pose to be reconstructed. gIFPM can be viewed as a generalization of this process that does not require manual identification of landmark points and uses all information available in the projected applicator footprint. Because of the excellent soft-tissue contrast, MRI-based applicator localization^{35–37} has the potential to substantially improve ICB optimality. A few MR-compatible ICB applicators

TABLE II. The residual 2D registration error between computed and measured applicator projection in terms of 2D radial difference in each image plane. The center of mass position of the entire tandem is reported, whereas the center of mass of the source position is empirically calculated for the right and left colpostats.

Separation between the colpostats (mm)	Applicator components	gIFPM vs measured applicator position: 2D residual error (mm)		
		Gantry= 0°	Gantry= -30°	Gantry= $+30^\circ$
30	Tandem	0.88	1.12	1.16
	R. colpostats	0.93	1.88	1.57
	L. colpostats	1.25	1.75	1.93
40	Tandem	0.67	0.89	0.78
	R. colpostats	1.14	1.46	1.81
	L. colpostats	0.96	1.67	1.66
50	Tandem	0.58	0.91	0.75
	R. colpostats	0.87	1.46	1.42
	L. colpostats	0.72	1.58	1.54

are available in the market and currently used in a few institutions;³⁸ however, very few institutions have open MR imaging capability available in the procedure room, effectively limiting its use to postinsertion plan evaluation at the present time.

ICB applicators containing high Z structural (steel or brass) or shielding materials (tungsten alloys) cause severe metal streaking artifacts^{19–21} that have challenged the 3D CT method since its inception. Such artifacts reduce the accuracy of applicator body delineation as well as difficulty in visualization of the soft-tissue organ boundaries and introduce large error in the dose calculations, clearly hindering integration of CT into the planning process. The streaking artifacts are sufficient to defeat deformable image registration, commonly used for image-guided procedures. In the context of intraoperative CT-based or CBCT-based dose planning, the situation is compounded by the additional metal objects (stirrups, retractors, etc.) in the field of view and the fact that patient position limits the circular orbit over which projections can be collected. Our six parameter model exploits 3D pose of the each applicator component using few CBCT projections sinogram. In the future, we plan to integrate the gIFPM applicator pose-search process into the CT image-reconstruction process to mitigate streaking artifacts. The gIFPM pose-search can be used to extract the metal object boundaries from the sinogram region, enabling restoration of the missing sinogram data by interpolation or from preoperative metal-free projections²⁴ or as an input to statistical image-reconstruction algorithms^{25–28} that have been shown to effectively suppress the metal streaking artifacts.

The outcomes on these early test cases, while not comprehensive, suggest that gIFPM is accurate, fast, and robust process for localizing ICB applicators, in support of intraoperative brachytherapy planning and adaptive replanning. To further improve the accuracy of the applicator localization validation study, we are designing a precision-machined pelvic phantom containing ICB applicators of known pose to test gIFPM performance against a more rigorous ground truth. That would address the issue of quantifying the orientation estimation accuracy, which was not accurately measurable by the current residual method. Also, the dependence of convergence rate and accuracy on the initial estimate needs further investigation. Our current IFPM implementation is limited to estimating the 3D applicator pose. Incorporating geometric uncertainties such as gantry angle inaccuracy and detector displacement and orientation into the estimation model is an area of future development. The algorithm could also be extended to other types of objects, e.g., stirrups, retractors, and table supports, to describe flex or internal motion within the applicators, to include partial transmission objects, and to match unsegmented grayscale images. Use of grayscale image matching could avoid one of the major problems of segmenting and thresholding fully plastic applicators³² having low Z material components.

V. CONCLUSION

We have presented a new approach to brachytherapy applicator localization, gIFPM that we believe to be the first

fully automatic approach described in the literature for accurately recovering the orientation as well as location of radio-opaque objects of arbitrary geometry from three 2D x-ray projections. By localizing the applicator internal structure and the sources, the effect of intra-applicator and interapplicator attenuation can be included in the resultant Monte Carlo or other dose calculations. Based on both simulated and actual applicator models, the localization errors were less than 1.5 mm and 2° for orientation angles. By incorporating six degrees-of-freedom search capability the IFPM approach can be extended to pose estimation of any shape radio-opaque object that can be geometrically modeled. The gIFPM algorithm is fast and has the potential to support conventional 2D ICB planning; near real-time intraoperative evaluation of applicator insertion quality in the absence of useful intraoperative 3D imaging modalities; and to provide the needed input to more advanced CT image-reconstruction algorithms designed to mitigate metal streaking artifacts.

ACKNOWLEDGMENTS

This work was supported in part by grants from Varian Medical System and the National Institutes of Health (Grant No. P01 CA 116602). The authors would like to thank Virginia Gilbert of Virginia Commonwealth University (VCU) for her continuous support of collecting data. The authors are grateful to Dr. Dimitrios Lazos (Beth Israel Medical Center, NY) and Andrew Sampson (VCU) for their help with combinatorial geometric modeling the authors described.

APPENDIX: ANALYTIC GRADIENT OF THE SIMILARITY, SSQD WITH RESPECT TO SIX DEGREES-OF-FREEDOM OF THE APPLICATOR MODEL

For each of the i th coordinates of the k th applicator model in the rotated and translated CT frame,

$$\begin{aligned} \{\mathbf{r}' | (\mathbf{t}_k, \Theta_k)\}_k &= \{\mathbf{r}'_{ik} | \mathbf{r}'_{ik} \\ &= [\mathbf{R}_{\alpha, \beta, \gamma}(\Theta_k)] \mathbf{r}_{ik} + \mathbf{t}_k, \mathbf{r}_{ik} \in A_k, i = 1, \dots, Q_k\}. \end{aligned} \quad (\text{A1})$$

Rewriting Eq. (A1) more explicitly,

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}_k = [\mathbf{R}_{\alpha, \beta, \gamma}]_k \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{ik} + \mathbf{t}_k, \quad (\text{A2})$$

where

$$[\mathbf{R}_{\alpha,\beta,\gamma}]_k = \begin{bmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{bmatrix}_k = \begin{bmatrix} \cos \gamma \cos \beta & \cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha & \cos \gamma \sin \beta \cos \alpha + \sin \gamma \sin \alpha \\ \sin \gamma \cos \beta & \sin \gamma \sin \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \gamma \sin \beta \cos \alpha - \cos \gamma \sin \alpha \\ -\sin \beta & \cos \beta \sin \alpha & \cos \beta \cos \alpha \end{bmatrix}_k$$

is the complete rotation matrix. The complete rotation matrix was obtained by taking the product of the three rotation matrices defined in the world coordinate system for each image viewpoint. No translation is applied (i.e., $\mathbf{t}_k=0$). Equation (A2) can be written as

$$\begin{cases} x'_k = (R11.x_i)_k + (R12.y_i)_k + (R13.z_i)_k \\ y'_k = (R21.x_i)_k + (R22.y_i)_k + (R23.z_i)_k \\ z'_k = (R31.x_i)_k + (R32.y_i)_k + (R33.z_i)_k \end{cases} \quad (\text{A3})$$

Rewriting Eq. (A2) more explicitly,

$$\begin{cases} x'_k = (\cos \gamma \cdot \cos \beta x_i)_k + (\cos \gamma \cdot \sin \beta \cdot \sin \alpha \cdot y_i - \sin \gamma \cdot \cos \alpha \cdot y_i)_k + (\cos \gamma \cdot \sin \beta \cdot \cos \alpha \cdot z_i + \sin \gamma \cdot \sin \alpha \cdot z_i)_k \\ y'_k = (\sin \gamma \cdot \cos \beta x_i)_k + (\sin \gamma \cdot \sin \beta \cdot \sin \alpha \cdot y_i + \cos \gamma \cdot \cos \alpha \cdot y_i)_k + (\sin \gamma \cdot \sin \beta \cdot \cos \alpha \cdot z_i - \cos \gamma \cdot \sin \alpha \cdot z_i)_k \\ z'_k = (-\sin \beta x_i)_k + (\cos \beta \cdot \sin \alpha \cdot y_i)_k + (\cos \beta \cdot \cos \alpha \cdot z_i)_k \end{cases} \quad (\text{A4})$$

The applicator models in the rotated CT frame project to the detector plane (u, v) are given by

$$(u'_{ik}, v'_{ik}) = M(z'_k)(x'_k, y'_k), \quad (\text{A5})$$

where $M_k = (S + D/S + z'_k)$ is the magnification factor, which is different for each point mesh; S and D are the source-to-isocenter and isocenter-to-detector distances, respectively. Since, the brachytherapy applicator has nonrotational symmetry (i.e., arbitrary shape) around the axis of rotation, we computed one derivative per point per applicator per degree-of-freedom and found the best derivative for that degree-of-freedom. From Eq. (3), the image grayscale gradient for x -degree-of-freedom was calculated as follows:

$$\begin{aligned} \partial I_c(u, v | \{\mathbf{r}_k, \Theta_k\} | \sigma, \eta) / \partial x_k &= \sum_{u'_{ik}, v'_{ik}} (I_0(u'_{ik}, v'_{ik} | \mathbf{r}_k, \Theta_k) / \sigma^2) \\ &\times [(u - u'_{ik}) \partial u'_{ik} / \partial x_{ik} \\ &+ (v - v'_{ik}) \partial v'_{ik} / \partial x_{ik}] \\ &\cdot g(u - u'_{ik}, v - v'_{ik} | \sigma), \end{aligned} \quad (\text{A6})$$

where, $g(u - u'_{ik}, v - v'_{ik} | \sigma) = \exp[-(u - u'_{ik})^2 / 2\sigma^2 - (v - v'_{ik})^2 / 2\sigma^2]$, η is the gantry angle, and Θ_k related to the original pose variables (α, β, γ)_k in the 3D space.

Finally, from Eqs. (A3) and (A5), using chain rule, we get

$$\begin{cases} \partial u'_{ik} / \partial x_{ik} = M_k [R11 - (R31)x'_k / (S + z'_k)] \\ \partial v'_{ik} / \partial x_{ik} = M_k [R21 - (R31)y'_k / (S + z'_k)] \end{cases} \quad (\text{A7})$$

and similarly for the y and z coordinates of each applicator model. The analytical gradient of the similarity SSQD with respect to α -angle coordinate for each applicator model was calculated from Eq. (4) as follows:

$$\begin{aligned} \partial(\text{SSQD}) / \partial \alpha_k &= 2 \sum_{\eta} \left(\sum_{u, v} [I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) \right. \\ &\quad \left. - I_m(u, v | \sigma, \eta)] \partial I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) / \partial \alpha_k \right). \end{aligned} \quad (\text{A8})$$

Again, from Eq. (3), we have computed the image grayscale gradient with respect to α -degree-of-freedom

$$\begin{aligned} \partial I_c(u, v | \{\mathbf{r}_k, \Theta_k\}, \sigma, \eta) / \partial \alpha_k &= \sum_{u'_{ik}, v'_{ik}} (I_0(u'_{ik}, v'_{ik} | \mathbf{r}_k, \Theta_k) / \sigma^2) \\ &\times [(u - u'_{ik}) \partial u'_{ik} / \partial \alpha_k \\ &+ (v - v'_{ik}) \partial v'_{ik} / \partial \alpha_k] \\ &\cdot g(u - u'_{ik}, v - v'_{ik} | \sigma). \end{aligned} \quad (\text{A9})$$

Finally, from Eqs. (A4) and (A5), using chain rule, we get

$$\begin{aligned} \partial v'_{ik} / \partial \alpha_k &= M_k \left\{ [\cos \gamma \sin \beta \cos \alpha + \sin \gamma \sin \alpha] y_{i,k} - [\cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha] z_{i,k} \right. \\ &\quad \left. - \frac{y'_k}{(S + z'_k)} \{ [\cos \beta \cos \alpha] y_{i,k} - [\cos \beta \sin \alpha] z_{i,k} \} \right\} \end{aligned}$$

$$\partial v'_{ik}/\partial \alpha_k = M_k \left\{ [\sin \gamma \sin \beta \cos \alpha - \sin \alpha \cos \gamma] y_{i,k} - [\sin \gamma \sin \beta \sin \alpha + \cos \gamma \cos \alpha] z_{i,k} \right. \\ \left. - \frac{y'_k}{(S + z'_k)} \{ [\cos \beta \cos \alpha] y_{i,k} - [\cos \beta \sin \alpha] z_{i,k} \} \right\}. \quad (\text{A10})$$

Similarly, we have computed analytical gradient of SSQD with respect to β and γ -angle coordinates of each applicator model and obtained the best derivatives.

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