A Novel Image Smoothing Filter Using Membership Function

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This paper presents a new class of image noise smoothing algorithms utilizing the membership information of the neighboring pixels. The basic idea of this method is to compute the smoothed output using neighboring pixels from the same cluster to avoid image blurring. A fuzzy c-means algorithm is first applied to the image to separate the image pixels into a certain number of clusters. A membership function is defined as the probability that a pixel belongs to a cluster. The proposed method uses this membership function as a weight to calculate the weighted sum of the pixel values from its neighboring pixels. The results of the application of this algorithm to various images show that it can smooth images with edge enhancement. The smoothness of the resultant images can be controlled by the cluster number and window size.

KEY WORDS: Membership function, fuzzy c-means, noise smoothing

INTRODUCTION

Effort has been devoted to the removal of noise in a digital image.¹ The purpose of image smoothing is to suppress noise while retaining object boundaries. Blurring occurs when the wrong population was introduced into the smoothing process. To avoid blurring the object boundary, it is better to calculate the smoothed output using pixels from the same cluster. Several order statistics filters²⁻⁵ perform noise smoothing based on this idea. Sigma filter⁶ uses local statistics to do the separation; pixels within three times of standard deviation are considered to be of the same cluster. Davis and Rosenfeld⁷ proposed pixels with gray levels fall within K ($K=6$ for a 3×3 window) nearest neighborhood to be from the same object. Tomita and Tsuji⁸ selected the most homogeneous neighborhood around each

pixel as the same cluster. These filters divide the pixels of a window into two groups based on fixed criteria. These criteria cannot represent all condition. It would be more flexible to use weights to represent the probability that a pixel might be from a specific cluster.

In this paper, we propose a new method that utilizes the membership function for noise smoothing. The membership function can be obtained from the results of a typical fuzzy c-means (FCM) clustering.^{9,10} The proposed algorithm uses this membership function to calculate the weighted average of the pixel values from its neighboring pixels to yield the smoothed output. In the following, we first introduce the conventional FCM to obtain membership function and the use of this function for image smoothing. We then describe the method used to incorporate the spatial information into FCM to improve noise suppression. Finally, the filter is applied to various images and the results are compared to regular averaging and median filters.

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METHODS

FCM

The FCM algorithm assigns pixels to each category by using fuzzy memberships. Let $\mathbf{X}=(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$ denote an image with N pixels to be partitioned into c clusters. For the j th sample $\mathbf{x}_j$ and the i th cluster center $\mathbf{v}_i$, there is a membership function $u_{ij} (\in [0,1])$ indicating the degree that the sample $\mathbf{x}_j$ belongs to the cluster center $\mathbf{v}_i$. FCM algorithm is an iterative optimization that aims to determine cluster centers $\mathbf{v}_i$ by minimizing the cost function defined as follows:

$$J = \sum_{j=1}^N \sum_{i=1}^c u_{ij}^m \|\mathbf{x}_j - \mathbf{v}_i\|^2 \quad (1)$$

subject to

$$\sum_{i=1}^c u_{ij} = 1 \quad \forall j = 1, \dots, N \quad (2)$$

where $\|\cdot\|$ is a norm metric. The fuzziness factor m is used to adjust the weighting effect of membership values. The fuzziness of the resulting partition increases with m . In this study, $2 \leq m \leq 3$ is used. The cost function is minimized when pixels close to the centroid of their clusters are assigned with high membership values, and low membership values are assigned to pixels with data far from the centroid. The membership function represents the probability that a pixel belongs to a specific cluster. In FCM algorithm, the probability is dependent solely on the distance between the pixel and each individual cluster center in the feature domain. Applying derivative to Eq. 1, one can derive the computation formulas of u_{ij} and $\mathbf{v}_i$ as:

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{\|\mathbf{x}_j - \mathbf{v}_i\|}{\|\mathbf{x}_j - \mathbf{v}_k\|} \right)^{\frac{2}{m-1}}} \quad m \neq 1 \quad (3)$$

$$\mathbf{v}_i = \frac{\sum_{j=1}^N u_{ij}^m \mathbf{x}_j}{\sum_{j=1}^N u_{ij}^m} \quad (4)$$

Starting with an initial guess for each cluster center, the FCM will converge to a solution for $\mathbf{v}_i$

representing the local minimum of the cost function. The iteration is stopped when the maximum difference between two cluster centers at two successive iterations is less than a threshold ($=0.02$).

Smoothing

The purpose of image smoothing is to suppress noise while retaining the boundary of the object. The simplest way of smoothing is to calculate the average from all neighboring pixels. Smoothing will cause blurring because that neighboring pixels might come from different clusters. To avoid blurring on the object boundary, it is best to calculate the output using pixels from the same cluster. We view that each pixel is made of several tissues. The percentage fraction of each tissue in a pixel is equivalent to the probability that the pixel belongs to that tissue and is just the membership function of a pixel to the tissue. The proposed method smoothes images by replacing the pixel under consideration with the weighted average (using membership function as weight) of the pixel values of its neighboring pixels. It can be expressed as

$$f'_j = \left(\sum_{i=1}^c u_{ij} \sum_{\mathbf{x}_k \in \text{NB}(\mathbf{x}_j)} u_{ik} f_k \right) / \left(\sum_{i=1}^c u_{ij} \sum_{\mathbf{x}_k \in \text{NB}(\mathbf{x}_j)} u_{ik} \right) \quad (5)$$

where f_k is the pixel value of k th pixel and $\text{NB}(\mathbf{x}_j)$ represents the neighboring pixels of $\mathbf{x}_j$ in the spatial domain. Note that $\sum_{\mathbf{x}_k \in \text{NB}(\mathbf{x}_j)} u_{ik} f_k$ denotes all the contribution from the i th tissue in the neighborhood, and the numerator is the weighted sum of all tissues. The denominator in Eq. 5 is a normalization factor.

If the pixel under consideration (said $\mathbf{x}_j$) is noisy, the membership function after clustering is very different from its neighboring pixels. If $\mathbf{x}_j$ is misclassified into cluster m while its neighboring pixels are not, i.e. $u_{mj} \geq u_{ij}$, $i \neq m$ and $u_{mj} \geq u_{mk}$, $k \neq j$. The denominator in Eq. 5 can be rearranged as $\left(u_{mj} f_j + \sum_{\mathbf{x}_k \in \text{NB}(\mathbf{x}_j)} u_{mk} f_k \right) u_{mj} + \sum_{i=1, i \neq m}^c \left(u_{ij} f_j + \sum_{\mathbf{x}_k \in \text{NB}(\mathbf{x}_j)} u_{ik} f_k \right) u_{ij}$. It can be seen that the major contribution to the weight of f_j is $u_{mj} \times u_{mj}$, which is significantly larger than the others. It means the output of pixel $\mathbf{x}_j$ is mainly from its original pixel value, and the smoothing effect is small. To improve the smoothing

Fig. 1. The pixel (marked by $\times$) under consideration is (a) on a sharp edge, (b) on a thin line, and (c) on a shallow edge. Neighboring pixels inside the 5×5 window centered on that pixel is used to generate the smoothed output.

effect, the central pixel is excluded from the averaging. The other technique is to utilize the spatial contextual information to improve the clustering results for the noisy pixels. In the following, we describe the methodology that incorporates the spatial weighting into FCM algorithm.

Spatial Weighting

Conventional FCM technique does not take into consideration any spatial information.^{11–14} Thus, the membership functions may exhibit sensitivity to noise in the resulted image.¹¹ One can smooth

Fig. 2. The stripe pattern images. (a) Original, (b) noise corrupted, (c) mean, (d) median, (e) sFCM ($c = 5$, $m = 2$), and (f) sFCM ($c = 5$, $m = 3$). Measurements made on the 8-pixel width stripes (marked by ABCD) were used for smoothing evaluation.

(a) Smoothing+Blurring

(b) Smoothing

(c) Blurring

Fig 3. The CV measured for (a) smoothing + blurring (b) smoothing, and (c) blurring effects ($c = 5$ for sFCM and $w = 5$ for all filters).

Fig 4. The profiles drawn across the stripe pattern images.

the image before applying FCM to compensate for this sensitivity. However, smoothing filters may result in a loss of important image details.

One of the important characteristics of an image is that there exist very high correlations between neighboring pixels. The spatial information is an important indication about the closeness between two pixels. Nearby pixels are more likely to be from the same cluster than distant pixels. In other words, these neighboring pixels possess similar feature values and there is a great probability that they belong to the same cluster.

To exploit the spatial information, a spatial function defined as

$$h_{ij} = \sum_{\mathbf{x}_k \in \text{NB}(\mathbf{x}_j)} u_{ik} \quad (6)$$

will be incorporated in the calculation of membership function. A 5×5 square window centered on the pixel under consideration is used throughout this work. Just like membership function, the spatial function h_{ij} represents the probability that pixel $\mathbf{x}_j$ belongs to i th cluster. The spatial function of a pixel for a cluster is large if the majority of its neighborhood belongs to the same cluster.

The spatial function can be incorporated into membership function via¹⁴

$$u'_{ij} = u_{ij}^p h_{ij}^q / \sum_{k=1}^c u_{kj}^p h_{kj}^q \quad (7)$$

where p and q are the parameters to control the relative importance of u and h . In this study, $p=q=1$ are employed. In a homogenous region,

the spatial functions simply fortify the original membership and the clustering result remains unchanged. However, for a noisy pixel, this formula reduces the weighting of a noisy cluster by the labels of its neighboring pixels. As a result, misclassified pixels from noisy regions or spurious blobs can be easily corrected.

The spatial FCM (sFCM) clustering is a two-pass process at each iteration. The first pass is the same as in standard FCM to calculate the membership function in the spectral domain. In the second pass, the membership information of each pixel is mapped to the spatial domain and the spatial function is computed from that. The FCM iteration is preceded with the new membership that is incorporating with the spatial function. After clustering, the weighted average from the neighborhood of each pixel can be calculated to generate the smoothed output.

MATERIALS

A synthesized stripe image was used for testing these filters. The image is comprised of three groups of stripes on top of a uniform background. The gray level of the background is set to be 30. There are four stripes in each group and their gray levels are 60, 110, 160, and 210, respectively. The stripes in each group have the same width and the widths of the three groups are 2, 4, and 8 pixels, respectively. The stripe image was corrupted by a random noise such that the signal-to-noise ratio (SNR) is equal to 10. In addition, a 128×128 positron emission tomography (PET) image reconstruction from a synthesized water phantom is also used for the test. The phantom is an elliptical column shape containing four cylindrical sources with the same concentrations of activity. The background activity of the phantom was 15% that

(a) Smoothing (sFCM5)

(b) Blurring (sFCM5)

Fig 5. The (a) smoothing CV and (b) blurring CV for various cluster numbers with sFCM ($m = 2$, $w = 5$).

of the hot cylinders. A cold spot with null activity was located inside the largest cylinder. A clinical magnetic resonance imaging (MRI) image with size of 512×512 and corrupted with noise of SNR = 10 is also used for the testing.

RESULTS AND DISCUSSION

Characteristics of the sFCM Smoothing Method

In this section, we describe the characteristics in the image smoothing of the proposed filter. The performances of the sFCM smoothing can be characterized by its ability in (1) noise smoothing, (2) thin lines preservation, and (3) edge enhancement. When the pixel under consideration is at the edge as shown in Figure 1a, the membership function of this pixel (belonging to black cluster) is large. The output of the sFCM filter is weighted largely from the black cells and leastly from the white cells. As a result, a smoothing result is generated from the average of the black cells with very small influence from the white cells. When the pixel is on a thin line (Fig. 1b), the membership function of this pixel that belong to the

black cells is large and its output is obtained mainly from the mean values of the black cells on the line. As a result, the line is preserved. For an edge with a gentle slope (Fig. 1c), the result of this filter depends on the value of m . As noted in Eq. 3, the probability that a pixel assigned to a cluster is an inverse function of m . For example when m is very small, u_{ij} is strongly affected by the difference between x_j and v_i . As a consequence, the filter classifies the gray pixel into either a black cluster or a white cluster, depending on which cluster is closer to the gray pixel. In either case, the edge contrast is enhanced because of the removal of the gray pixel. With large m , the membership function is not influenced so much by the difference between the cluster and pixel and the smooth output will be the mixture of the two clusters. The edge is blurred in this case.

Noise Smoothing

Figure 2a, b shows the original and noise corrupted stripe images and Figure 2c–e the images smoothed by filters (window size = 5×5) of averaging, median, and sFCM, respectively. In general, all filters generate smooth images. The image produced by sFCM filter has significant

Fig 6. The (a) smoothing CV and (b) blurring CV for various window sizes (w) of the sFCM filter ($c = 5$, $m = 2$).

edge preservation while the images by both mean and median filters show edge blurring artifacts. For sFCM filter, if a pixel is contaminated by noise, the membership function of this noisy pixel is corrected by the spatial function in the neighborhood. The major contribution to the output will be the average from the pixel values of the same tissue type and no blurring artifacts will be generated.

Figure 3a shows the coefficients of variation (CV=standard deviation/mean) measured on an area of 8×8 on the stripes of 8 pixels in width. The four stripes to be measured are marked as A, B, C, and D in Figure 1. The CV value represents overall uniformity in the distribution of the gray level inside the window. The higher this value is, the more heterogeneous is the gray level distribution. This value is the result of two effects: edge blurring and unsmoothed noise. To separate these two effects, we measured the CV on the same image using a 4×8 area with the exclusion of

2 pixels in width (window size of filter=5) on each side of the stripes. This smaller area can be thought to be free from edge blurring and its CV measures the smoothness of the noise. The results are plotted in Figure 3b. It can be seen that the smoothing effects generated by sFCM filter with $m=3$ and mean filter are better than by median filter and sFCM filter with $m=2$.

Thin Lines Preservation

The properties of thin lines preservation of all filters are demonstrated in Figure 2. The averaging filter smears out all lines as expected. Thin lines with a width smaller than one half of the window size are removed by the median filter. For both sFCM filtering ($m=2$ and 3), no matter what the window size is, pixels on the thin lines can always be identified as the same cluster with the pixel under consideration. As a result, smoothing

Fig 7. The PET images. (a) Original, (b) mean, (c) median, (d) sFCM ($c = 5$, $m = 2$), (e) sFCM ($c = 5$, $m = 3$), and (f) sFCM ($c = 3$, $m = 2$).

Fig 8. The profiles drawn across the center of the PET images.

is achieved by averaging from these neighboring pixels and thin lines are preserved.

Edge Blurring

The difference between the overall CV (measured on the 8×8 region) and smoothing CV (4×8) is because of the effects of edge blurring. If there are no blurring effects, the noise level between the two regions is basically the same and the difference should be small. Figure 3c plots the CV because of edge blurring of these filters. As

predicted, the mean filter shows the highest blurring artifacts. The blurring artifact of sFCM filter increases with the increasing fuzziness factor m . When $m=2$, it has the least blurring artifacts and for $m=3$, its degree of blurring artifacts is slightly larger than median filter but smaller than mean filter.

Horizontal profiles for images in Figure 2 are plotted in Figure 4. The edge-preserving properties of these filters are demonstrated in the profiles. It can be seen that the sFCM filtering with $m=2$ smoothes images with edge enhance-

ment and thin line preservation. For sFCM filtering with $m=3$, the edge blurring effect is quite obvious but it preserves thin lines better than a median filter.

Effect of Cluster Number

The probability of membership of a pixel in a cluster depends on the total number of clusters. When the cluster number is small, the number of neighboring pixels from the same cluster as the

pixel under consideration is large and the smoothing effects are enhanced. However, the chance of mixing various tissues also increases and this results in an increase in the blurring artifact in the output image. On the other hand, when a large cluster number is employed, pixels from the same tissue maybe misclassified into different clusters. The number of pixels in the neighborhood that are categorized as the same cluster as the pixel under consideration will be small and subsequently, there is not enough population to generate a

Fig 9. The MRI images. (a) Original, (b) sFCM ($c = 5, m = 2$), (c) sFCM ($c = 5, m = 3$), and (d) sFCM ($c = 3, m = 2$).

smoothed image. The effect of cluster number on the sFCM is demonstrated in Figure 5. It can be seen that smoothing is better for a smaller cluster number ($c=3$) and the blurring effect is smaller for larger cluster number ($c=7$).

Effect of Window Size

The smoothing effect increases with large window size as more pixels are included in the smoothing calculation. It can be seen in Figure 6a that the smoothing is better when the window size is increased. In general, the increasing window size will cause blurring because of the inclusion of more irrelevant pixels. For the sFCM algorithm, the contribution of each pixel to the averaging output is dependent upon the similarity of its membership function with the pixel under consideration. The membership functions of pixels from different clusters are small and their contribution to the weighted averaging is insignificant. As a result, the blurring effect is less affected by the window size for the sFCM filters. Figure 6b shows that there is no significant blurring artifact as window size increases.

PET Image

Figure 7 compares the results of applying various smoothing filters to a PET image. The mean filter shows the most blurred image. The smallest hot spot in the median filtered image becomes dimmed because of the edge blurring. The sFCM techniques with $m=2$ (Figure 7d, f) enhance the edge at all the hot spot regions. It can be seen that the effect of contrast enhancement is reduced when the cluster number is changed from 5 to 3. Note that the size of the cold spot looks smaller for these two sFCM techniques. This is because of the removal of blurred edge of the cold spot. For sFCM with $m=3$, the edge blurring is obvious as in both mean and median filtered images. Figure 8 plots the profile across the center of the PET image. The profiles show that the smallest spots are badly blurred in median, mean, and sFCM ($m=3$) filtered images while they are quite visible in both sFCM filtered images. It is noted that the smoothing effect on the background area (inside the phantom) in the sFCM filter with cluster number equal to 3 is as good as the mean

filter. However, the edge is much more blurred than the sFCM with $c=5$.

MR IMAGE

Figure 9 shows the result of applying sFCM filter of various parameters on MR image. All these filters smooth noise. For small m ($=2$), the sFCM filter enhances the edge to a degree that it looks unnatural. The use of larger m ($=3$) does not show this artifact. The fuzziness factor can be used to control the degree of edge enhancement.

CONCLUSION

In this paper, we presented a new type of smoothing filter called sFCM filter. Image smoothing of this filter is performed by the weighted averages from the neighboring pixels of the same tissue. The membership function is used to represent the fractional weight that a pixel belongs to a certain tissue. The membership function can be obtained from the conventional FCM technique. Besides, the spatial information of neighboring pixels was incorporated into the membership function to improve noise suppression. Among the filters compared, the proposed sFCM filter is best in its overall performance. The characteristics of the sFCM filter are as follows: (1) it utilizes the spatial information to improve the smoothing effects, (2) it can be used to enhance edge contrast, (3) it preserves thin lines, and (4) the degree of smoothing effect can be controlled by the cluster number.

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