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Analysis of diffracted image patterns from volume holographic imaging systems and applications to image processing

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Diffracted image patterns from volume holograms that are used in volume holographic imaging systems (VHIS) are investigated. It is shown, that in VHIS, prior information about the shape and spectral properties of the diffracted patterns is important not only to determine the curvature and field of view of the image, but also for image registration and noise removal. A new methodology to study numerically and analytically the dependence of VHIS diffraction patterns with the hologram construction parameters and the read-out wavelength is described. Modeling and experimental results demonstrate that in most cases VHIS diffracted shapes can be accurately represented by hyperbolas.

1. Introduction

Highly selective spherical volume holograms are key elements in multimode multiplex spectroscopy [1-2] and 3D imaging applications [3-10]. For spectroscopy applications, volume holograms can be used as a spectral diversity filters (SDFs) [1] to map the spectrum of a diffuse source on a detector array using a spatially encoded pattern [2]. For the 3D imaging volume holograms are used to image simultaneously different depths of a sample on a CCD camera [3-10].

Volume holographic imaging systems (VHIS) can use a single wavelength laser or broadband sources such as LEDs. Using broadband source, a new technique denominated Spatial-Spectral VHIS (S²-VHIS) has the advantage of eliminating the need of any mechanical scanning [3-10]. Therefore, video rate images from different depths of a sample can be obtained.

A crescent shape, is a common shape produced by the transmitted or diffracted light from planar or curved volume hologram optical element (VHOE) under certain illumination conditions. It can be classified as a dark crescent when they are observed in the transmission beam or bright crescent when observed in the diffracted beam. For S²-VHIS the crescent shape is important since it provides information about the curvature of the field of view (FOV) and the spectral content of the image (4D imaging) [3-10]. Moreover, for in-vivo biomedical imaging, prior information about the crescent shape is essential for image registration and noise removal.

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Previous work has utilized the Born approximation to predict the crescent shape [1,11]. However, this method assumes a weak interaction between the incident and diffracted light [7]. In the case of thick volume holograms, where the incident light is strongly depleted, another approach is needed in the optical design process.

Kogelnik's two-beam approximated coupled-wave analysis (ACWA) [12-13] and rigorous coupled-wave analysis (RCWA) [14-15] are well suited to analyze these strong and highly selective gratings. In fact, these have been used extensively to analytically model the diffraction effects of uniform gratings formed by two spatially planar waves. Among these methods, RCWA provides the best accuracy, at the expense of significantly increasing computation time. In case of thick volume holograms where one diffracted order is dominant, a modified ACWA which includes the effects of arbitrary conical incidence provides accurate and fast results. This capability coupled with short computation time allows to be implemented in ray-tracing packages such as Zemax® [10].

In this paper, we develop a model to quantify the properties of crescent shaped images formed in volume holographic imaging systems. This model, based on ACWA, provides more accurate results than models based on the Born approximation. In most of the cases, the crescent shapes used in VHIS are not circular or elliptical but hyperbolic. These results are confirmed with experiments. An important application of this result is to pre-determine the image field to set post-processing boundaries. We also describe this process in the paper.

The remainder of this paper is divided in five sections. In the second Section the basic operation of VHIS is described. The third section describes the ACWA and the relationship between the crescent curvature and the Bragg detuning condition. In Section fourth experimental validation is presented. The last Section is devoted to discussion and summary.

2. Basic operation of a S²-VHIS

The layout depicted in Figure 1 serves as an example of the S²-VHIS operation. A more detailed description of the S²-VHIS operation can be found in several references [3-10]. The basic configuration of a S²-VHIS consists of a 4F system with a holographic element placed in the Fourier plane. The holographic optical element is composed of thick angle-multiplexed planar and spherical wave gratings with high angular and spectral selectivity. For each plane in the object space, a mapping is produced to the image space by two properties of volume holograms [12-17] the spatial degeneracy and the spectral-angular dispersion. Due to the first property, the wavefront of a point source at any position along the y-axis (cylinders in Fig. 1) satisfies the Bragg phase matching condition of the VHOE and therefore is diffracted to the collector lens. This diffraction is responsible for the y-axis field of view. Current prototypes have achieved a y-field of view of ~1 mm using an objective lens with NA=0.55, focal length of 3.6 mm and relays lenses as indicated in [6-10]. This y-field of view value corresponds to a full width half maximum (FWHM) angular FOV of 8°.

The FOV does not follow a straight line indicated by the axis of each cylinder, but a crescent shape as shown in Fig 3 and 4. The curvature of the diffractive pattern is not produced by the curvature of the hologram as could be misinterpreted from [1-3]. Both planar and curved hologram fringes result in curved diffraction shapes due to the spatial degeneracy property of the hologram. The diffractive shape, which resembles a crescent pattern, depends on the construction parameters of the hologram and the reconstruction wavelength. An accurate knowledge of this shape is required to correctly determine the space-spectral mapping of this type of imaging system.

The spectral angular dispersion produces the field of view along the x-axis. This FOV is represented by several cylinders along the x-axis. Each cylinder has different gray level intensity which indicates that it is being imaged by a different wavelength. The FOV depends on the spectral bandwidth of the read-out source utilized. Current prototypes utilize LEDs with FWHM bandwidth ranging from 20 to 50 nm, producing FOV along the dispersive axis of approximately 150 μm which correspond to a FWHM angular FOV < 2.5°.

Multiplexing several gratings into the same volume can be used to map axial points at multiple depths in the object space to the image or camera plane as explained in [3-10].

3. ACWA Modeling Techniques for Volume Holograms

Modeling techniques to evaluate the performance volume holograms include the Born approximation, approximate coupled wave analysis [12,13] and rigorous couple wave analysis [13-14]. The Born approximation in Ref. [3-5,8-9,11], can provide simplified analytical models for volume holographic gratings. However, this method requires weak interaction between the incident beam and diffracted beam. This is not the case for the holograms used in the S²-VHIS.

The modified RCWA [14-15] provides the most accurate analysis for the diffraction phenomena for gratings with arbitrary reconstruction conditions. However, RCWA is a numerical technique that is computationally intensive and difficult to implement in component design algorithms.

An approximate coupled wave algorithm [12-13] can provide reasonable accuracy and fast estimation of grating diffraction efficiency. Moreover, the analytical expression of ACWA provides a more intuitive description of the diffraction phenomena than RCWA or the Born approximation, which is helpful in the investigation on the shape of the diffractive pattern.

3.1 Approximated coupled wave analysis: general model

In ACWA there are two fundamental parameters that describe the diffraction process: the strength and the direction of the diffracted beam. The direction of the diffracted vector inside the grating medium can be obtained from [12-13]

$$\begin{aligned}\vec{u}^{(d)} &= (u_x^{(i)} - K_x)\vec{x} + (u_y^{(i)} - K_y)\vec{y} + u_z^{(d)}\vec{z} \\ \vec{u}^{(d)} &= u_x^{(d)}\vec{x} + u_y^{(d)}\vec{y} + u_z^{(d)}\vec{z},\end{aligned}\quad (1)$$

where $\vec{u}^{(i)}$ $\vec{u}^{(d)}$ are the incident and diffracted propagation vectors inside the medium. The z direction component is determined using,

$$u_z^{(d)} = \sqrt{k_2^2 - |u_x^{(d)}|^2 - |u_y^{(d)}|^2}, \quad (2)$$

where $k_2 = \frac{2\pi n_2}{\lambda}$, λ is the free-space wavelength, n_2 is the refractive index of the medium, and the grating vector is given by,

$$\vec{K} = K_x\vec{x} + K_y\vec{y} + K_z\vec{z} = \frac{2\pi n_2}{\lambda_c} [\vec{r} - \vec{s}], \quad (3)$$

where λ_c is the grating construction wavelength, $\vec{r}$ and $\vec{s}$ are the construction reference and signal beam vectors inside the recording medium. The period of the grating can be

computed using $\Lambda = \frac{2\pi}{|\vec{K}|}$. Utilizing polar coordinate notation the diffracted vector is given by,

$$\vec{u}^{(d)} = k_2 [\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta], \quad (4)$$

where θ and ϕ are the zenith and azimuth angle in the holographic medium. In air, the zenith angle is obtained using the Snell's law $\theta' = \sin^{-1}(n_2 \sin \theta)$.

For a transmission grating, the diffraction efficiency (DE) for the TE wave is given by [12-13,16],

$$DE = \frac{\sin^2(\sqrt{\nu^2 + \xi^2})}{1 + \frac{\xi^2}{\nu^2}}, \quad (5)$$

where ν is a parameter that determines the maximum diffraction efficiency of the grating for a specific reconstruction wavelength and angle, and ξ indicates the variation of the reconstruction parameters from the Bragg condition. For a lossless transmission grating, these parameter functions are given by,

$$\nu = \frac{\pi \Delta n t_H}{\lambda \sqrt{c_r c_s}}, \quad (6)$$

$$\vartheta = \frac{2(\vec{u}^{(i)} \cdot \vec{K}) - |K|^2}{2k_2}, \quad (7)$$

$$\xi = \frac{\vartheta t_H}{2c_s}, \quad (8)$$

where Δn and t_H are the index modulation and the thickness of the hologram respectively, ϑ is the detuning parameter and c_r and c_s are the obliquity factors described in [12].

3.2 ACWA: analytical model to predict the crescent shape

The ACWA model described in Section 3.1 can be used to numerically model the complete operation of the VHIS. However, in order to get a good insight of the dependence of crescent shape image properties with wavelength and hologram parameters an analytical model is required.

Here an analytical model from the ACWA equations was developed to provide an accurate description of these conical diffraction phenomena for the FOV used in VHIS. In the layout shown in Fig. 2, the trace produced by the beam diffracted by the HOE can be described using,

$$\begin{aligned} R &= d \tan \theta' \\ x &= R \cos \phi, \\ y &= R \sin \phi \end{aligned} \quad (9)$$

where d is the distance between the HOE and the screen, θ' , ϕ are the diffracted angles in air described in (4) and,

$$\tan \theta' = n_2 \tan \theta \frac{\cos \theta}{\cos \theta'}, \quad (10)$$

For highly selective volume holograms such as the ones used in VHIS light from the object to image space propagate in well defined light paths that satisfy the Bragg condition (7). For the diffracted ray on Bragg condition, $\vec{u}^{(d)} = \vec{u}^{(i)} - \vec{K}$, the detuning equation is equal to zero when,

$$2(\vec{u}^{(d)} \cdot \vec{K}) = -|\vec{K}|^2. \quad (11)$$

For simplicity and without loss of generality it is assumed here that $K_y=0$. This can be achieved selecting a proper coordinate system. In this case, the $\vec{K}$ vector is given by,

$$\vec{K} = |\vec{K}| [\sin \gamma, 0, \cos \gamma], \quad (12)$$

where $\gamma = \tan^{-1} \left(\frac{K_x}{K_z} \right)$. Using (4) and (12) in (11), the following expression is obtained,

$$\sin \gamma \sin \theta \cos \phi + \cos \gamma \cos \theta + q = 0, \quad (13)$$

where

$$q = \frac{|\vec{K}|}{2k_2} = \frac{\lambda}{2n_2\Lambda}. \quad (14)$$

Dividing (13) by $\cos \theta$,

$$\sin \gamma \tan \theta \cos \phi + \cos \gamma = \frac{-q}{\cos \theta}. \quad (15)$$

Multiplying previous equation by $n_2 \frac{\cos \theta}{\cos \theta'}$ and using (10) in (15),

$$\sin \gamma \tan \theta' \cos \phi + n_2 \frac{\cos \theta}{\cos \theta'} \cos \gamma = -n_2 \frac{q}{\cos \theta'} = -n_2 q \sqrt{1 + \tan^2 \theta'} \quad (16)$$

Squaring both sides of (16) and after some algebraic manipulation, the following equation is obtained,

$$\left(\sin^2 \gamma \cos^2 \phi - (n_2 q)^2\right) \tan^2 \theta' + n_2 \frac{\cos \theta}{\cos \theta'} \sin 2\gamma \cos \phi \tan \theta' + \left(n_2 \frac{\cos \theta}{\cos \theta'} \cos \gamma\right)^2 - (n_2 q)^2 = 0. \quad (17)$$

This equation relates the direction of the diffracted vector in terms of the azimuth and zenith angle in the medium with the hologram construction parameters and reconstructing wavelength (γ and q parameters). As described in Section 2 the S²-VHIS has a small angular

FOV ($|\theta| < 10^\circ$). In this angular range, $\frac{\cos \theta}{\cos \theta'} \approx 1$ with error $< 2\%$.

Using this approximation and replacing (9) in (17) produces,

$$x^2 \left(\frac{\sin^2 \gamma - (n_2 q)^2}{d^2} \right) - \left(\frac{n_2 q}{d} \right)^2 y^2 + n_2 \frac{\sin 2\gamma}{d} x + (n_2 \cos \gamma)^2 - (n_2 q)^2 = 0. \quad (18)$$

Equation (18) describes the diffracted rays trajectories projected on the screen as shown in Fig. 2. Therefore the crescent shape image in terms of wavelength and hologram construction parameters (γ and q parameters) can be estimated.

In the Cartesian coordinate system, the graph of a quadratic equation in two variables (x, y) as shown in (18) is a conic section. The conic discriminant classification described in [18] can be utilized to determine the type of conic produced by the diffraction of the volume hologram. For the conic equation derived in (18), the possible curves are:

$$\begin{aligned} \text{Ellipse} &: \sin^2 \gamma - (n_2 q)^2 < 0 \\ \text{Parabola} &: \sin^2 \gamma - (n_2 q)^2 = 0 \\ \text{Hyperbola} &: \sin^2 \gamma - (n_2 q)^2 > 0 \end{aligned} \quad (19)$$

4. Analysis of the crescent shapes and experimental validation

Previous studies of the diffractive patterns have shown somewhat ambiguous results in estimating the shape of the crescent. For example, the shape has been attributed to the curvature of the hologram fringes [1,3], or it has been assumed exclusively circular or elliptical [1,11]. In some cases this ambiguity can be attributed to the utilization of the paraxial approximation, the effect of other optical components in the imaging system, and the utilization of a reduced FOV.

In order to minimize errors, our initial modeling and experiments use a simple setup indicated in Fig. 2. This setup minimizes the effects of undesired aberrations caused by other optical elements other than the hologram. In this setup, the beams are co-axially aligned before passing through a microscope objective (NA=0.55) which then focuses the beams onto a hologram. This setup permits to fill completely the objective aperture producing a cone of light incident on the hologram larger than the one produced in the imaging system shown in Fig. 1. This allows for a greater portion of the angular bandwidth of the volume grating to be illuminated, therefore more of the resulting diffraction pattern can be observed. The expansion of the FOV minimizes the error in the estimation of the diffractive shape.

Fig. 3 shows bright crescents produced by two beams incident on a hologram using this setup. The light sources producing these images are the 488nm Argon laser and a 632.8 nm HeNe.

The best fit for the measured traces follows a curve given by hyperbolic equation given by

$$Ax^2 - By^2 = C. \quad (20)$$

For the HeNe wavelength the curve parameters were $[A,B,C]=[1, 1.08, 8900]$, whereas for the 488 nm beam the curve parameters were $[A,B,C]=[1, 0.445, 3900]$.

The curve parameters were obtained by minimizing the error between fitting and measured curves (both shown in Fig. 3). In order to simplify comparison, the fitting parameters are normalized by A. More details of this computation are shown in Appendix I.

To model the crescent of this VHOE, the ACWA (described in Section 2) was implemented as a custom dynamic link library (dll) in Zemax™. Results of our general numerical model are shown in Fig 4. In that figure the dotted lines represent the curve fitted to the hyperbolas obtained from previous measurements (Fig. 3). Comparison of modeled and measured traces shows very good accuracy (error < 3%).

However, the numerical model alone does not explain the causes for the obtained hyperbolic shapes. Neither, does it provide a good insight of the dependence of this diffractive pattern on the reconstruction wavelength and the hologram construction parameters. To pre-determine the image field and apply fast image processing techniques to VHIS it is important to understand and predict the diffractive image properties.

The analytical model described in Section 3.2 provides a criterion to determine the type of conic produced by the diffraction based on the hologram construction parameters and reconstruction wavelength.

In this model, the degree of tilt of the grating fringes is given by γ as shown in (12). For the gratings utilized in S²-VHIS, it is desirable to keep $\gamma > 80^\circ$. The use of smaller angles increases the tilt of the fringes and affects negatively the resolution and FOV of the imaging systems.

Replacing $\gamma = 80^\circ$ in (13) it is found that hyperbolic shapes are produced when,

$$q = \frac{\lambda}{2\Lambda n_2} \leq \frac{\sin 80^\circ}{n_2} \leq 0.66. \quad (21)$$

Also, the diffracted angle in the medium has to be lower than the total internal reflection angle of the hologram medium $q < \sin 90^\circ / n_2$. These limits indicate that for almost all of the operational spectral range of VHIS, the x²-term coefficient in (18) is positive. Therefore, using (19) it can be found that in the majority of the cases using S²-VHIS diffracted shapes are hyperbolas. The asymptotes slopes of the hyperbolas obtained from (18) are given by,

$$\pm \sqrt{\frac{\sin^2 \gamma - (n_2 q)^2}{(n_2 q)^2}}. \quad (22)$$

For the gratings diffraction patterns shown in Fig 3 and 4, $\gamma=90^\circ$ the asymptotes slopes are 1.49 for 488 nm and 0.96 for 633 nm. Theoretical and experimental values agree with error < 3% .

In cases where the HOE has large tilt in the K vector, such as cases close to the 90-degree geometry VHIS, γ is very small and it is possible to obtain other shapes such as a circle, ellipse or parabola. However, those grating construction geometries are not very useful for low aberration and high resolution VHIS.

For simplicity, the equations derived in this section were obtained assuming a camera aligned normal to the HOE (Fig 2). However, we have derived the equation for other cases such as the one shown in [11] in which the camera is aligned normal to the in-plane diffracted vector (zero components in the y-axis). In general, this case involves a rotation along the y axis to match the normal of the camera surface to the in-plane diffracted vector. Although this rotation affects the way the curvature of the diffracted pattern is projected in the camera, the discriminant classification shown in (19) still holds for the hyperbolic shape produced by holograms used in S^2 -VHIS . In general, for a given curve defined by $f(x,y)$, the different orientation of the camera will produce a curve given by $f(ax+by)$, where a and b are parameters related with the coordinate system rotation [19] . This means that an ellipse may be transformed in a circle or vice versa. However, there is not a transformation from the hyperbola to an ellipse or circle by simply rotating the camera around the y-axis.

The importance of pre-determining the crescent shape information in S^2 -VHIS can be appreciated in Fig. 5. This figure shows the image of a tissue sample at two different depths separated by $\sim 50 \mu\text{m}$. The images were obtained using an LED source with center wavelength of 633 nm and a two-grating multiplexed hologram as described in [7]. In part a) of this figure two depth images are shown without segmentation or any other image processing technique. Part b) shows the segmented images using the hyperbolic shape pattern and spatial filtering optimized for each depth image. The segmentation is necessary to determine the boundaries of each depth section, and to select the best filter for each image. A prior-knowledge of the segmentation area and shape significantly reduce the image processing time. This also improves the SNR since spurious noise out of the segmented regions is avoided.

For 4D imaging, each hyperbolic line can be related to a specific wavelength of the illumination source. Therefore, an accurate knowledge of the diffractive shape at each position of the FOV allows partial recovery of spectral information.

It can be seen in this figure, that both planar and curved HOE produced hyperbolic traces. The hyperbolic shape is related to the construction parameters of the grating as described in the preceding sections of this paper. An example showing the derivation of the hyperbolic traces fitting parameters from the analytical model is shown in Appendix I.

5. Conclusion

Light focused on a planar or curved fringe hologram diffract light into crescent shaped images. Prior knowledge of the crescent shape allows selecting the image regions for post processing to improve the image contrast. Previous studies used the Born approximation to predict the shape and width of the crescent image. However, this model is not accurate for the gratings strength parameters and selective characteristics of gratings used in volume holographic imaging systems.

In this paper, we have studied the diffractive patterns of a hologram using approximate couple wave analysis. It has been shown that ACWA accurately predict the diffracted beam

shapes for planar and spherical gratings to within 3%. The model also explains the image shape dependence on wavelength and holographic grating construction parameters. Results of our analysis demonstrate that for the gratings used in S²-VHIS, the diffracted shape is a hyperbola and not a circles or ellipse as previously assumed.

The techniques shown in this paper to pre-determine the diffraction pattern, greatly decrease the time it takes to segment and process the acquired images as described in previous section.

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Appendix I

I.1 Example showing the derivation of the fitting parameters from the analytical model

For an unslanted planar grating ($\gamma=90^\circ$), the K vector can be obtained using (3),

$$|\vec{K}| = 2 \frac{2\pi}{\lambda_c} \sin(\theta_i)$$

where θ_i is the construction angle in air and λ_c the construction wavelength For this example, using $\theta_i = 34^\circ$ and $\lambda_c = 488 \text{ nm}$, $|\vec{K}| = 14.39 \text{ } \mu\text{m}^{-1}$. The period of the grating is

given by, $\Lambda = \frac{2\pi}{|\vec{K}|} = 0.436 \text{ } \mu\text{m}$ Using a reconstruction wavelength of $\lambda = 632 \text{ nm}$, a medium with refractive index $n_2 = 1.5$ the q parameter is computed using (14),

$$q = \frac{\lambda}{2n_2\Lambda} = 0.482$$

For a setup configuration similar to the one shown in Fig. 2, the distance between the hologram and the screen is $d \sim 91 \text{ mm}$. Using (18) the coefficients of the conic curve are obtained as follows:

$$\begin{aligned} A &= \frac{\sin^2 \gamma - (n_2 q)^2}{d^2} = 5.78 \text{ E} - 5 \text{ } \text{mm}^{-2} \\ B &= \left(\frac{n_2 q}{d}\right)^2 = 6.37 \text{ E} - 5 \text{ } \text{mm}^{-2} \\ C &= (n_2 q)^2 = 0.524 \end{aligned}$$

These results are divided by A. The normalized coefficients that are then

$$A=1, \quad B=1.1$$

After the operation, the parameter C is proportional to d^2 . For the utilized setup $C = 9072 \text{ mm}^2$.

The normalized A and B parameters depends only on the grating properties and not in the distance between the hologram and screen. The slopes of the hyperbola asymptotes can be

obtained from these normalized parameters ($\frac{1}{\sqrt{B}}$). The slope facilitates comparison between the diffractive shapes produced at different wavelengths.

Using the same procedure for another reconstruction wavelength ($\lambda=488$ nm) the values of [A,B,C] are [1, 0.454, 3742 mm²].

Comparison between the analytical model (eq. 18) and the numerical implementation of the full ACWA model (described in Section 3.1) produce errors < 0.5% for gratings with high angular selectivity (< 0.04°) which are commonly used in VHIS.

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Fig. 1. Basic layout of S2-VHIS showing the Objective lens (L_0), the collector lens (L_c) and the hologram (HOE).

Fig. 2.

Experiment layout used to measure crescent shape of the diffracted beam. The Z-axis is normal to the hologram and screen. The angles, θ_i and θ' , represent the incident and diffracted beam angles respectively. The distance from the hologram to the screen is denoted by d and the radius of curvature of the resulting pattern denoted by R .

Fig. 3. Measured diffractive patterns from a VHOE using two wavelengths. Curve fitting to a hyperbolic shape are indicated in dotted lines. Construction parameters of the VHOE indicated in Section 2

Fig. 4. Modeled diffractive traces compared with curve fitting of Fig. 1.

Fig. 5. Sample images: A) Before segmentation and without any imaging processing. B) After segmentation and spatial filtering. In both cases, images from holograms with curved fringes ($z=-50\mu\text{m}$) are on the left side.