CT scanner x-ray spectrum estimation from transmission measurements

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Purpose: In diagnostic CT imaging, multiple important applications depend on the knowledge of the x-ray spectrum, including Monte Carlo dose calculations and dual-energy material decomposition analysis. Due to the high photon flux involved, it is difficult to directly measure spectra from the x-ray tube of a CT scanner. One potential method for indirect measurement involves estimating the spectrum from transmission measurements. The expectation maximization (EM) method is an accurate and robust method to solve this problem. In this article, this method was evaluated in a commercial CT scanner.

Methods: Two step-wedges (polycarbonate and aluminum) were used to produce different attenuation levels. Transmission measurements were performed on the scanner and the measured data from the scanner were exported to an external computer to calculate the spectra. The EM method was applied to solve the equations that represent the attenuation processes of polychromatic x-ray photons. Estimated spectra were compared to the spectra simulated using a software provided by the manufacturer of the scanner. To test the accuracy of the spectra, a verification experiment was performed using a phantom containing different depths of water. The measured transmission data were compared to the transmission values calculated using the estimated spectra.

Results: Spectra of 80, 100, 120, and 140 kVp from a dual-source CT scanner were estimated. The estimated and simulated spectra were well matched. The differences of mean energies were less than 1 keV. In the verification experiment, the measured and calculated transmission values were in excellent agreement.

Conclusions: Spectrum estimation using transmission data and the EM method is a quantitatively accurate and robust technique to estimate the spectrum of a CT system. This method could benefit studies relying on accurate knowledge of the x-ray spectra from CT scanner. © 2011 American Association of Physicists in Medicine. [DOI: [10.1118/1.3547718](https://doi.org/10.1118/1.3547718)]

Key words: x-ray spectrum, transmission measurements, CT scanner

I. INTRODUCTION

Knowledge of the x-ray spectrum is important for several diagnostic x-ray imaging applications, including Monte Carlo dose calculations and dual-energy material decomposition analysis. Currently, interest in estimating patient dose from CT continues to grow, as do the clinical applications of dual-energy CT. Thus, methods of determining the x-ray spectrum for clinical CT scanners are needed. However, because of the high photon flux produced by CT x-ray tubes, directly measuring spectra from a CT scanner is difficult. Various methods have been developed to estimate CT spectra. These methods can be categorized into two groups: Model-based and measurement-based methods.

The model-based methods usually generate spectra from empirical or semiempirical physical models^{1–6} and the measurement-based methods reconstruct spectra from measured data, essentially making an indirect measurement of the spectra.^{7–16} The transmission method is one of the measurement-based methods. This method is relatively simple and has been implemented in a large range of x-ray energies.^{14–18} Transmission methods usually consist of two steps: (1) Measuring the transmission data for different thicknesses of materials and (2) reconstructing the spectra by solving the linear equations that represent the attenuation

processes of x-ray photons. In this study, an expectation maximization (EM) method was used to solve the equations. The theory and the simulation of the EM method for spectrum estimation were presented by Sidky *et al.*¹⁶

In this work, we focused on evaluating the performance of a transmission/EM method and discussing the practical issues related to applying it to a commercial CT scanner. In Sec. II, the mathematic model is briefly presented and then details of implementing and validating the method are given. Results are shown in Sec. III, for five spectra estimated using the EM method and the validation tests of the estimated spectra.

II. MATERIALS AND METHODS

II.A. Mathematical model

The transmission measurements of polychromatic x rays can be formulated as

$$\begin{aligned} p &= \frac{I}{I_0} = \int_{E_{\min}}^{E_{\max}} D(E) s(E) e^{-\int_L \mu(E, r) dl} dE \\ &= \int_{E_{\min}}^{E_{\max}} W(E) e^{-\int_L \mu(E, r) dl} dE, \end{aligned} \quad (1)$$

where p represents transmission data through linear attenua-

(a)

(b)

FIG. 1. Pictures of the step-wedge phantoms. (a) Aluminum step-wedge phantom. (b) The step units of polycarbonate phantom. Each unit consists of three plates. The units were manually stacked to form a thicker step. Four step units are shown here.

tion $\mu(E, \mathbf{r})$ and total path length L ; I_0 and I are incident and transmitted photon intensities, respectively. E is the photon energy and $E_{\min}$ and $E_{\max}$ are the minimum and maximum photon energy. $W(E)$ denotes the overall spectrum, which combines the x-ray source spectrum $s(E)$ and the detector response $D(E)$. $W(E)$ also includes any beam filters in the system.

After discretization, Eq. (1) is converted to a linear system

$$p_j = \sum_{i=1}^N A_{i,j} w_i, \quad j = 1, \dots, M,$$

$$A_{i,j} = \exp\left(-\int_L \mu_j(E_i, \mathbf{r}) dl\right), \quad (2)$$

where M is the total number of measurements, N is the number of samplings for the spectrum, A is the measurement matrix calculated from the linear attenuation coefficients and thickness of the phantom, and w_i is the spectrum sampling.

(a)

(b)

FIG. 2. Pictures of experimental setup. The phantoms were laid on the surface of the gantry and aligned with lasers. (a) Aluminum phantom. A Gafchromic XR-CT dosimetry film was used to measure the beam width (arrow). (b) Polycarbonate phantom. In the measurement of this picture, four step units were stacked into one thick step. The x-ray beam width was narrower than the smallest unit (top step).

The EM method was applied to solve the linear system.¹⁶ Based on statistical models, the EM method is robust to noise and it has inherent positivity, which results from a multiplicative update form

$$w_j^{(k+1)} = \frac{w_j^{(k)}}{\sum_i A_{i,j}} \sum_i \frac{A_{i,j} p_i}{\sum_s A_{i,s} w_s^{(k)}}, \quad (3)$$

where the index k represents the iteration number.

II.B. Phantoms and scanner

Two step-wedge phantoms, one made of aluminum (Al) and one made of polycarbonate ($C_{16}H_{14}O_3$), were used to represent different attenuation levels (Fig. 1 and Table I, where Table I uses commonly accepted mass densities for the materials and the thicknesses were measured using a caliper). The aluminum phantom was a standard step-wedge phantom,

TABLE I. Technical description of step phantoms.

Phantom material	Mass density (g/cm ³)	Number of steps	Minimum thickness (cm)	Maximum thickness (cm)
Aluminum	2.7	16	0.136	2.54
Polycarbonate	1.21	12	0.876	10.0

FIG. 3. A water tank laid on the gantry was used to validate the spectrum estimation.

which was originally used for radiographic quality control. The polycarbonate phantom consisted of multiple plates, each of about 2.7 mm thickness.

The spectrum estimation was performed on a dual-source CT scanner (SOMATOM Definition Flash, Siemens Healthcare, Germany). The phantoms were laid directly on the surface of the gantry and were aligned with the axial and sagittal lasers (Fig. 2). The tube current was 100 mA with 1 s exposure time and 12×1.2 mm collimation. A narrow beam collimation was used to allow more precise alignment and reduce scattering. Five spectra from two tubes (A and B) were estimated, including 80, 100, 120, and 140 kVp on tube A and 140 kVp with a tin filter on tube B. In the experiment,

FIG. 4. [(a)–(e)] X-ray spectra estimated from the transmission data of the step-wedge phantoms using the EM method. “Initial guess” refers to the initial guess for the EM algorithm. “Estimated” refers to the spectrum estimated from the transmission measurements. “Simulated” refers to the simulated spectrum using DRASIM software.

TABLE II. Comparison of estimated and simulated spectra.

Spectrum	Mean energy difference (keV)	NRMSD (%)
80 kVp	-0.23	8.6
100 kVp	-0.01	4.8
120 kVp	0.55	4.8
140 kVp	0.27	4.7
140 kVp /w tin filtration	0.81	9.0

the scanner was operated in service mode and the measured x-ray tube remained stationary on the bottom of the gantry (6 o'clock position). In this geometry, the phantoms were much closer to the x-ray tube than to the detector, so the width of x-ray beam was narrower than the width of each step of the phantoms, which was also verified by a Gafchromic[®] XR-CT dosimetry film [Fig. 2(a)]. Therefore, for each measurement, only one step of the phantom was exposed to the x-ray beam. This geometry also helped to suppress the measurement of scattered radiation by the detector.

After measurement, all the raw projection data were exported from the scanner to an external computer to calculate the spectra. The value of raw data were scaled as

$$p_{\text{raw}} = -k \ln \frac{I}{I_0},$$

where k is a constant. Thus, the projection data directly used for solving the equations [Eq. (2)] were calculated as

$$p = \frac{I}{I_0} = e^{-p_{\text{raw}}/k}.$$

TABLE III. The results of validation experiment using water as an attenuator. (E indicates calculated transmission using estimated spectra and S indicates using simulated spectra.)

Water thickness (cm)	Relative difference of measured and calculated transmission (%)									
	80 kVp		100 kVp		120 kVp		140 kVp		140 kVp tin	
	E	S	E	S	E	S	E	S	E	S
2.25	3.0	3.6	-0.04	0.3	-0.2	0.5	1.8	2.6	1.7	2.0
4.25	4.1	4.9	0.9	1.4	0.2	1.4	2.9	4.2	3.2	3.7
6.25	3.3	4.2	2.9	3.7	1.5	3.0	2.0	3.6	2.5	3.1
8.25	4.4	5.3	3.0	3.9	3.2	5.1	3.9	5.8	3.7	4.5
10.25	4.6	5.4	4.3	5.2	3.6	5.6	4.6	6.6	3.7	4.7
12.25	7.6	8.2	4.4	5.4	3.9	6.1	5.5	7.6	5.1	6.2
14.25	5.9	6.2	5.6	6.5	5.0	7.3	6.3	8.3	6.3	7.6
16.25	8.5	8.5	7.3	8.2	6.8	9.1	8.3	10.3	6.4	7.7
18.25	10.7	10.2	5.6	6.4	6.0	8.4	8.4	10.3	6.5	7.9
20.25	13.6	12.7	8.6	9.3	9.1	11.4	10.8	12.5	7.8	9.3
P value ^a	0.11		6.8×10^{-7}		2.1×10^{-6}		2.0×10^{-7}		2.4×10^{-5}	

^aTwo-tailed paired t -tests were used to evaluate the statistical significance of the difference between the spectra estimated using the method described here and the spectra provided by the manufacturer. $P < 0.05$ were considered statistically significant.

II.C. Experimental procedure

The experiment was performed in two steps: (1) After transmission measurements, estimated spectra were reconstructed using the EM method; (2) the accuracy of the estimated spectra was evaluated by comparing measured and calculated transmission data through different depths of water.

As mentioned in Sidky's work, the EM method cannot recover the details of the spectrum such as a characteristic peak if the initial guess does not contain a peak at the same energy.¹⁶ Therefore, the initial guess for the EM method was constructed by multiplying a generic tungsten anode x-ray tube spectrum³ by the detector response function ($\text{Gd}_2\text{O}_2\text{S}$; area mass density 1.05 g/cm^2), so that the spectrum contained both the characteristics from the x-ray tube and the K-edges from the detector.

In the validation step, a water tank filled with different depths of water was placed on the surface of the gantry and a ruler was used to measure the depth of water (Fig. 3). Transmission data were measured for different thickness of water (ten thicknesses, 2.25–20.25 cm). The measured transmission data were compared to the transmission data calculated using the estimated spectra and the spectra simulated using software provided by the manufacturer (DRASIM, Siemens Healthcare, Germany).

III. RESULTS

The estimated spectra are shown in Fig. 4. The total photon counts were normalized to unity and the energy interval was 1 keV. In each figure, three spectra are compared: The initial guess for the EM algorithm, the estimated spectrum from transmission measurements, and the spectrum simulated using software provided by the manufacturer. As shown in Fig. 4, the estimated and simulated spectra were well

matched. The quantitative comparison is shown in Table II, where the mean energy difference is defined as the mean energy of the estimated spectra minus the mean energy of the simulated spectra and the normalized root mean squared deviation (NRMSD) is defined as

$$\text{NRMSD} = \frac{\sqrt{\sum_{i=1}^N (W_i^e - W_i^s)^2 / N}}{\max(W_i^e) - \min(W_i^e)},$$

where W_i^e is estimated spectrum, W_i^s is simulated spectrum, and N is the number of data points. The initial guesses were significantly softer than the other two spectra because additional filters installed in the scanner were not included in the initial guess.

The results of the validation experiment are shown in Table III. The relative difference is defined as $(p_m - p_c)/p_c$, where p_m is the measured transmission and p_c is the calculated transmission. The transmission data from measurements and calculations were well matched. The residual differences mainly resulted from scattering in the measurements. Smaller differences were achieved by using the estimated spectra than the simulated spectra.

IV. DISCUSSION AND CONCLUSION

Spectrum estimation using transmission data and the EM method is an accurate and robust technique to estimate the spectrum of a CT system. It is also easy to implement in practice. In our experiment, only two step-wedges were used and no additional instrument was involved.

General principles for choosing step-wedge phantoms included known composition, regular geometry, and convenience for measuring dimensions and ensuring alignment. Because there are two major mechanisms of x-ray interactions with matter in the diagnostic x-ray energy range, photoelectric effect and Compton scattering, using two materials with sufficiently different atomic numbers (e.g., plastic and metal) gave improved equation stability compared to using only one material.

For a clinical CT scanner, transmission measurements are most easily performed in service mode. In our experiment, the measurement was performed in static mode, i.e., neither the gantry nor the patient table moved. If the service mode is not accessible, an alternative method would be to use the localization features of a scanner, e.g., "Scout," "Pilot," "Topogram," or "Preview." In this scheme, one should make sure the patient table does not move over the step-wedge when taking transmission measurements. Alternatively, we have used dosimeters, such as ion chambers, for transmission measurements. However, additional collimation was needed to collimate the x rays into a narrow beam before irradiating step-wedges. The accuracy of the results was similar to the service mode method.

The spectra estimated in our study were weighted by the energy response function of the detectors. If the spectrum

from the x-ray tube is needed, the estimated spectrum should be divided by the detector response function.

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