Published in final edited form as:

J Refract Surg. 2010 October ; 26(10): 796–802. doi:10.3928/1081597X-20100921-05.

Noise in WFE measurement from pupil center location uncertainty

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Abstract

Purpose—To examine the impact of pupil center location uncertainty in wavefront sensing on the variance in repeated measures of the high order RMS wavefront error (WFE).

Methods—Dilated WFE for one normal eye and one eye with keratoconus were measured using a custom Shack/Hartmann wavefront sensor (lenslet spacing 400 μm). Twelve measurements for each subject were averaged to form the best estimate of each eye's WFE and standard deviation. The percentage of the standard deviation of the actual measurements attributable to the uncertainty in pupil center location was modeled by inducing random offsets in the pupil center up to 50, 100, 150 and 200 μm .

Results—The percentage of standard deviation of the actual measurements accounted for by pupil location uncertainty form a complex interaction between magnitude and distribution of the HO-WFE with the magnitude of pupil centering error. The larger the WFE and the larger the pupil center uncertainty the greater is the effect.

Conclusions—As pupil center uncertainty increases, so does the WFE variation in repeated measurements. The larger the underlying WFE, the greater is the impact on measurement variation. Increasing measurement variation decreases the ability to detect changes in WFE (e.g., as a function of aging or clinical intervention) and decreases the accuracy with which WF guided corrections can be made.

Background

Ophthalmic aberrometers typically follow the recommendations of the Optical Society of America working group¹ and the ANSI standard² and quantify the wavefront error (WFE) with respect to the eye's entrance pupil center. There are several strategies for determining the pupil center that different manufacturer's use (e.g., In Shack/Hartmann wavefront sensors it is common to use the S/H spot pattern or a separate pupil camera to find the pupil center). Regardless of the method used to determine the pupil center, there are small variations in locating the pupil center from measurement to measurement even if pupil diameter is fixed (e.g., drug dilated). For example, if the aberrometer is a Shack/Hartman aberrometer which uses the captured spot pattern to estimate the pupil center, the uncertainty in finding the pupil center is principally due to the lenslet diameter (resulting sampling density), the magnification of the imaging system, and the centroiding algorithms used to

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Conflicts of Interest: Raymond A. Applegate, None

Jason D. Marsack, None

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decide whether a particular lenslet can participate or not and the accuracy with which the “true” centroid is found. On the other hand, if the aberrometer has a separate pupil monitoring camera that is used to find the pupil center, the uncertainty in finding the pupil center is principally due to the pixel density combined with the magnification of the imaging system, the precision and accuracy of the edge finding algorithm, and the accuracy with which the pupil monitoring system is registered to the aberration measurement system. Here we model our best guess of the likely range of pupil uncertainties likely to exist in commonly built systems. Uncertainty in the pupil center location leads to variation in the quantification of wavefront error as illustrated in Figure 1. In Figure 1, notice that neither the underlying wavefront error nor the diameter of the pupil size of interest is changing. The only changed factor is the location of the pupil center and therefore the location of the area of the optics over which the WFE is measured. Since the WFE is quantified with respect to the pupil center, its mathematical representation is changed. When using the Zernike expansion to quantify the WFE these changes are reflected as changes in coefficients of each term. It is worth noting that the impact of a pupil shift on the Zernike specification of the WFE can be calculated empirically^{3,4} or by re-sampling the WFE with respect to the new pupil center assuming the shift of the pupil is known and the WFE is known over a larger pupil diameter than the pupil diameter of interest. Recalculating the Zernike coefficients for a known pupil shift is not the fundamental topic of this paper. Instead the focus of this paper is on the impact of uncertainty with which the pupil center is located (i.e., unknown pupil shifts that occur because of uncertainty in the true pupil center location).

The induced variation in the quantification of the WFE from measurement to measurement caused by the pupil center uncertainty decreases the ability to detect significant change in ocular aberration (e.g., over time, as a result of a therapeutic intervention) and is a source of error in designing and implementing corrections (e.g., laser refractive platforms, custom contact lenses) of the eye’s aberrations. It is one factor in larger error budget necessary for successful implementation of wavefront guided corrections.

Uncertainty in the actual location of the pupil center impacts the design of wavefront guided corrections differently than an actual shift in the pupil that can occur as the pupil dilates and constricts.⁵ If we know the pupil center shift, we can recalculate the WFE. If we don’t know the shift, then we cannot recalculate the WFE and noise is introduced. The implications of this type of noise can be illustrated with a mental exercise. Imagine that you can measure and correct the WFE of the eye over the largest possible pupil with 100% accuracy and precision. After correction, this eye will have no WFE regardless of pupil size or small shifts in the pupil center. Now imagine that you make the same measurement over the largest pupil possible but this time there is uncertainty in the pupil center location. The uncertainty in the pupil location will lead to errors in the quantification of the WFE and thus the design of the correction.

Purpose

The purpose of this study is to examine the impact of pupil center location uncertainty on the variability (standard deviation) in RMS WFE for a person with normal high order (above 2nd radial order) wavefront error and a person with large high order aberrations.

Methods

Overview

It is expected that the impact of pupil center uncertainty on the standard deviation of repeated measures will increase with increasing WFE. To examine this expectancy, the

wavefront error of one eye in each of two subjects was measured 12 times using a custom Shack/Hartmann ophthalmic wavefront sensor described below.

Subjects

Subject one was a normal 59 year old myopic astigmat ($-2.50\text{ D} - 2.00\text{ D} \times 121$). Subject two was a 33 year old moderate keratoconic. The test eyes were drug dilated using one drop of 1% tropicamide. The WFE for each measurement was fit with a Zernike expansion following the ANSI Z80.28 standard through the 10th radial order for both 7mm and 6mm pupil diameters. The average normalized coefficient value for each of the 66 Zernike modes served as the best estimate of the WFE of the eye. Subjects signed an informed consent approved by the Institutional Review Boards of the University of Houston for the Protection of Human Subjects.

Custom Shack/Hartmann wavefront sensor and pupilometry

The S/H wavefront sensor used in this study images the entrance pupil of the subject through a 1:1 relay telescope onto a lenslet array. The lenslet array is a single optical element composed of a 65×65 matrix of small lenses each having a pitch of $400\text{ }\mu\text{m}$ and a focal length of 24 mm . Each lenslet within the array samples a portion of the wavefront originating from a small retinal point source created by a super luminescent diode (SLD) ($\lambda = 830\text{ nm}$) and images the retinal point source onto a CCD camera. Subject alignment is maintained using a pupil camera, which allows the operator to align the subject's entrance pupil conjugate with the entrance aperture of the lenslet array. The subject's head is held in place by the typical forehead rest and chin cup. Pupil center location was defined by the best fitting circle to the outer most centroids of the spot diagram.

Modeling pupil center uncertainty

To simulate pupil center uncertainty relative to the eye's optics (assumed to be in a fixed state for the purposes of modeling), a custom program was written to densely resample the 7mm diameter WFE over a 6mm sub-aperture in any location desired within the following constraints. To model different levels of pupil center uncertainty, the pupil center was allowed to vary randomly 12 times with up to 200, 150, 100 or 50 micrometers of translation in any direction from the center of the underlying 7mm WFE of the eye (angle and radii selected randomly). Standard deviations were calculated for the 12 sample values for each of the 4 movement conditions for: 1) total RMS; 2) defocus; 3) astigmatism 4) higher order RMS; 5) trefoil; 6) coma; 7) tetrafoil; 8) secondary astigmatism; and 9) spherical aberration. A ratio between the SD in WFE for 12 simulations of pupil uncertainty for each uncertainty magnitude and for each subset of aberrations and the equivalent SD of the actual 12 ocular WFE measurements for each movement and type of aberration was calculated to describe the percentage of the actual on-eye measurement variability that can be accounted for by varying levels of pupil center uncertainty as follows:

$$\% \text{ uncertainty} = \frac{SD_x}{SD_a} \times 100$$

Where:

SD_x = SD of the WFE for the aberration of interest resulting from 12 random pupil shifts for level of maximum pupil uncertainty of interest (i.e., 200, 150, 100 and 50 micrometers)

SD_a = SD of the WFE for the aberration of interest resulting from 12 actual WFE measurements for the subject of interest.

Finally, change in total RMS WFE as a function of pupil location uncertainty up to 200 micrometers (the estimated maximum uncertainty in the pupil center location of the custom wavefront sensor used to measure the two subjects' WFE) will be plotted for both the normal and abnormal eye of the present study. To generate the display, pupil center displacement was allowed to vary randomly in x and y up to 200 micrometers 1000 times and the resulting change in total RMS WFE was plotted as a function of pupil center displacement

Results

Tables 1 and 2 (for the normal myopic-astigmatic eye and the eye with keratoconus, respectively) present a summary of basic statistics of the 12 measurements on each eye over 6mm pupil. Figure 2 displays the average high order (HO) RMS WFE and average spherocylindrical (calculated from the 2nd radial order) for the two subjects. Note that color scale is 5 times larger for the eye with keratoconus.

Table 3 lists the basic statistics for the vector lengths of pupil center uncertainty in micrometers from the pupil center for maximum pupil center uncertainties of 200, 150, 100 and 50 micrometers. As can be seen in Table 3 the average vector length of the pupil center uncertainty is less than ½ the maximum allowed displacement and ranges from near zero displacement to nearly the maximum displacement.

Figure 4 displays change in total RMS WFE (2nd through 10th radial orders) as a function of pupil center location uncertainty up to 200 micrometers for a normal astigmatic eye and an eye with moderate keratoconus.

Discussion

As illustrated in Figure 3, pupil center uncertainty increases the standard deviation of repeated wavefront measurements. Pupil uncertainty that ranges up to 200 micrometers accounted for 45% of the SD of the actual measurements of coma in the normal eye and 87% of the SD of trefoil in the eye with keratoconus. Pupil center uncertainty in the highly aberrated eye accounted for a greater percentage of SD of actual measurements than the same pupil center uncertainty in the normal eye.

As illustrated in Figure 4, for any WFE one can determine the precision with which the pupil center needs to be maintained to keep the measurement error within a predefined error limit. For example, assume the goal is to keep the measurement error due to pupil location uncertainty below 0.1 micrometer of total RMS WFE. (Note: Any metric of retinal image quality can be used; here we use total RMS WFE because it is the most common, if not accurate, metric of retinal image quality). For the normal myopic astigmatic eye used in this experiment, the pupil center uncertainty can be as large as 120 micrometers before there is a risk of the error in RMS WFE exceeding 0.1 micrometers. For the eye with keratoconus applying the same limit (0.1 micrometers in total RMS WFE), the maximum allowable uncertainty in pupil center location is 46 micrometers. Each individual eye's WFE will have a different allowable amount of pupil center uncertainty for any given criteria of image quality. If manufacturers specified the typical level of pupil center uncertainty, an internal program could estimate resulting measurement variability for any given retinal image quality metric due to the pupil center uncertainty and the number of measurements necessary to get a good estimate of the true WFE. Such information is particularly useful for designing wavefront guided corrections for the highly aberrated eye as well as determining the number of measurements necessary to detect a fixed level of anticipated change over time (as in aging experiments⁶) or due to therapy designed to improve the WFE of the eye.

The clinical significance of pupil center uncertainty can be demonstrated visually by assuming that the measured WFE for any given pupil uncertainty is used to design a correction for the measured eye. The resulting residual error from such a correction can in turn be used to make a retinal image simulation of a log MAR acuity chart (or any other object of interest). Figure 5 panel left displays a simulation of diffraction limited log MAR acuity chart viewed through a 6mm pupil and panel right displays a simulation of the largest residual RMS WFE for the normal eye when pupil center uncertainty was allowed to range up to 200 micrometers. Similarly, Figure 6 panel left displays a simulation for the largest case residual RMS WFE and average residual RMS WFE for the eye with keratoconus when pupil center uncertainty was allowed to range up to 200 micrometers in any given direction.

Although not the specific topic of this paper, it is worth noting that even if perfectly accurate WFE measurements could be made, similar errors occur when implementing wavefront guided corrections if the correction is not perfectly registered with WFE of the eye. Further, one can argue that it does not matter if the WFE is measured around a location that is not exactly at the center of the pupil as long as the correction designed from this measurement is implemented around the same exact location. In reality, there are several sources of compounding error when implementing wavefront guided corrections and a detailed analysis of the error budget necessary to reliably reduce the high order aberration in wavefront guided corrections is overdue. It may be the case that all errors combined simply make it impossible to reliably reduce the high order aberrations particularly in eyes with typical levels of high order aberrations.

Future directions

It is our hope that this paper stimulates an open discussion of the error budget (WFE measurement uncertainties, correction registration uncertainties, etc.) necessary to successfully implement wavefront guided corrections. In the modeling of pupil center uncertainty presented here, only translation errors were allowed. In the real world there will also be small rotation errors (e.g., due to slight head tilts between measurements combined with small cyclo-rotations). We are in the process of adding small random rotations to determine the impact of such errors on the variability of repeated measures of WFE.

Additionally, in Figure 5 the correction was assumed to be perfectly registered to the WFE. However, when actually implementing a correction, there will be centering and rotation errors in the implementation of the correction as well. We are developing the necessary programming to model the combined effects of pupil center uncertainty in the measurement and registration uncertainty in the implementation of the correction and will report these findings in a separate publication. One can easily see how such a model can be expanded by adding more and more sources of error. The key to such a model will be accurately defining the magnitude and likely distribution of each type of error.

Conclusions

As pupil center uncertainty increases so does the WFE variation. The larger the underlying WFE, the greater is the impact on measurement variation of any given level of pupil center uncertainty. Increasing measurement variation decreases the ability to detect changes in WFE (e.g., as a function of aging or clinical intervention). It would be useful if instrument manufacturers provided data on the precision and accuracy with which the pupil center is located in real eyes during clinical measurement. Providing such data allows the clinician and or investigator to better determine if a particular instrument will meet their needs.

Acknowledgments

Support: NIH/NEI R01 EY08520 (RAA); NIH/NEI R01 EY019105 (RAA); R44 EY015008 (EJS); NIH/NEI P30 EY 07551 (Core Grant to the College of Optometry); Borish Endowment to the College of Optometry

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Figure 1.

(Above, left) Assume we know the WFE of an eye. Now a wavefront sensor measures the WFE of this eye over a smaller pupil diameter two times. During the first measurement the wavefront sensor located the pupil center at the location of the solid cross and fit the WFE over the area defined by the solid circle. During the second measurement the smaller pupil did not move but the wavefront sensor located the pupil center at the location of the dashed cross and fit the WFE over the area defined by the dashed circle. More specifically, a fixed set of wavefront slopes were used to compute Zernike coefficients for two different estimates of the pupil center. (Above, right) Change in coefficient value induced by a shift in the pupil location of $x = 0.2\text{mm}$, $y = -0.2\text{mm}$ for the WFE shown (figure left) as a function of coefficient number through the 4th radial order.

Figure 2.

The left panel displays the average high order RMS WFE along with the average sphere and average cylindrical correction (calculated from the 2nd radial order) over a 6 mm pupil for the normal subject. The right panel displays the same data for the eye with keratoconus. Note that the scale is 5 times larger for the eye with keratoconus.

Figure 3.

Percent of the SD in actual repeated measures that can be attributed to pupil center uncertainty (PCU) as a function of the maximum pupil center uncertainty in mm. Left panel is for normal eye, right panel is for eye with keratoconus. Basic statistics for vector length of the pupil center displacement for each maximum pupil uncertainty are given in Table 3.

Figure 4.

Change in total RMS WFE as a function of pupil location uncertainty up to 200 micrometers. The left panel is for the normal eye. The right panel is for the eye with keratoconus. Red arrows indicate the maximum displacement allowable to keep the error in total RMS WFE below 0.1 micrometers.

Figure 5.

Retinal image simulations for a diffraction (no WFE) limited eye 6mm pupil (left panel) and the largest residual RMS WFE (right panel) for the normal eye where the maximum allowable pupil center error was up to 200 micrometers.

Figure 6. Retinal image simulations for the largest residual RMS WFE (left panel) and average residual RMS WFE (right panel) for where the maximum allowable pupil center error was up to 200 micrometers.

Table 1

Basic statistics from 12 measurements for total RMS WFE, high order RMS wavefront error (HO RMS WFE), and type of aberration for the normal myopic astigmatic eye over 6mm diameter pupil.

	Total RMS WFE (μm)	HOA RMS WFE (μm)	Sphere RMS WFE (μm)	Astig. RMS WFE (μm)	Trefoil RMS WFE (μm)	Coma RMS WFE (μm)	Tetrafoil RMS WFE (μm)	2nd Astig RMS WFE (μm)	Sph. Ab RMS WFE (μm)
Average	5.293	0.347	4.900	1.969	0.140	0.160	0.045	0.081	0.208
SD	0.101	0.062	0.074	0.097	0.036	0.081	0.027	0.024	0.033
MAX	5.449	0.430	4.996	2.166	0.205	0.269	0.091	0.131	0.287

Table 2

Basic statistics from 12 measurements for total RMS WFE, high order RMS wavefront error (HO RMS WFE), and type of aberration for the eye with keratoconus over a 6mm diameter pupil.

	Total RMS WFE (μm)	HOA RMS WFE (μm)	Sphere RMS WFE (μm)	Astig. RMS WFE (μm)	Trefoil RMS WFE (μm)	Coma RMS WFE (μm)	Tetrafoil RMS WFE (μm)	2nd Astig RMS WFE (μm)	Sph. Ab RMS WFE (μm)
Average	3.366	1.569	2.401	1.760	0.540	1.346	0.188	0.508	0.185
SD	0.274	0.091	0.227	0.159	0.057	0.110	0.028	0.019	0.026
MAX	3.847	1.726	2.716	2.108	0.587	1.501	0.235	0.536	0.218
MIN	2.845	1.386	1.962	1.523	0.391	1.126	0.148	0.475	0.128
Range	1.002	0.339	0.754	0.585	0.196	0.375	0.087	0.061	0.089

Table 3

Basic statistics of the vector lengths of pupil center movement in micrometers for maximum pupil center uncertainties of 200, 150, 100 and 50 micrometers.

	Maximum Pupil Center Uncertainty			
	200 (μm)	150 (μm)	100 (μm)	50 (μm)
Average	89.9	67.4	44.9	15.4
SD	64.3	48.3	32.2	13.3
Max	174.1	130.6	87.1	40.9
Min	1.4	0.9	0.6	0.1