Published in final edited form as:

IEEE Nucl Sci Symp Conf Rec (1997). 2009 January 1; 2009: 3458–3462. doi:10.1109/NSSMIC.2009.5401787.

An Investigation of Compressive-sensing Image Reconstruction from Flying-focal-spot CT Data

D. Xia, J. Bian, X. Han, E. Y. Sidky, and X. Pan

Department of Radiology, The University of Chicago 5841 S Maryland Avenue, Chicago, IL 60637

Abstract

Flying-focal-spot (FFS) technique has been used for improving the sampling condition in advanced clinical CT by collecting multiple cone-beam data sets with the focal-spot at different locations at each “projection view”. It has been demonstrated that the increased sampling rate in FFS scans can substantially reduce aliasing artifacts in reconstructed images. However, the increase of the sampling density through multiple illuminations at each view can result in the increase of radiation dose to the imaged subject. In this work, we have applied a compressive-sensing (CS)-based algorithm to image reconstruction from data acquired in FFS scans. The results of the study demonstrate that aliasing artifacts observed images reconstructed by use of analytic algorithms can be suppressed effectively in images reconstructed with this CS-based algorithm from only data acquired at one FFS scan.

I Introduction

A flying-focal-spot (FFS) technique, which allows for view-by-view deflections of the focal spot, has been used in some of advanced clinical CT scanners, for improving the sampling density in the rotation direction (i.e., xFFS), the longitudinal direction (i.e., the zFFS), or the combinations of the two. Studies have been conducted for image reconstruction from data obtained with the FFS technique by use of analytic algorithms [1,2]. Results of the studies demonstrate that the FFS technique can lead to a reduction of aliasing artifacts in reconstructed images. However, the increase of the sampling density through multiple illuminations at each view can result in an increase of radiation dose to the imaged subject. In this work, we have developed and applied a compressive-sensing (CS)-based algorithm to reconstruct an image through minimizing the image’s total variation (TV) subject to the data condition [3,4], from FFS-scan data. We have performed numerical studies using simulated FFS-scan data to demonstrate the algorithm performance.

II Flying-focal-spot Scanning Geometry

In an FFS scan, the x-ray tube rotates following the predescribed trajectory, such as a circular trajectory, while multiple data sets are acquired from different FFSs for each projection view. For simplicity of illustration, we consider an FFS scan with a circular trajectory in this work. However, the study method can readily be applied to FFS scans with other trajectories. The electron beam in the x-ray tube can be dynamically deflected with an electromagnetic control, and the focal spot is wobbled between different positions on the anode plate. For example, by adjusting the amplitude of the movement of the focal spot along rotation direction, one can collect two sets of cone-beam data at each projection view with a half detector-element shift along the rotation direction, as shown in Fig. 1a. Other FFS configurations can be realized through wobbling the focal spot among different positions on the anode plate, such as zFFS shown in Fig.1b. In this work, we have

performed image reconstruction from data acquired with xFFS scan, zFFS scan, and the combination of the two, which we refer to as xzFFS scan. In the paper, we present our preliminary study results for a circular scan, which can be written as

$$\vec{r}_0(\lambda) = [R_0 \cos \lambda, R_0 \sin \lambda, 0], \quad (1)$$

where λ is the rotation angle and R denotes the distance between the x-ray tube and rotation center. Each of the focal spot positions can be expressed as

$$\vec{r}_0(\lambda, \rho) = [R^\rho \cos \lambda + \Delta x^\rho \sin \lambda, R^\rho \sin \lambda - \Delta x^\rho \cos \lambda, \Delta z^\rho], \quad (2)$$

where $\rho = 1, 2, \dots, N_{\text{ffs}}$, N_{ffs} is the number of source positions for each projection view. Parameters Δx^ρ and Δz^ρ specify the source positions for different FFS modes. For an xFFS scan the parameters are given by $N_{\text{ffs}} = 2$, $R^\rho = R_0$, $\Delta x^\rho = (-1)^\rho \frac{1}{4} R \Delta \gamma \times \frac{D}{D-R}$, and $\Delta z = 0$; for a zFFS scan the parameters are given by $N_{\text{ffs}} = 2$, $R^\rho = R_0 + (-1)^\rho \frac{\Delta z}{\tan \phi}$, Δx^ρ and $\Delta z = (-1)^\rho \frac{1}{4} R \Delta \theta \times \frac{D}{D-R}$.

III TV-minimization reconstruction algorithm

We have recently developed CS-based approach to image reconstruction in CT through seeking the solution for the constrained optimization problem below:

$$\vec{f}^* = \operatorname{argmin} \|\vec{f}\|_{TV}, \quad (3)$$

subject to two constraints,

$$|\vec{M}\vec{f} - \vec{g}| \leq \epsilon \quad \text{and} \quad f_i \geq 0,$$

where TV indicates the total variation (TV) of the image, f_i denotes the i th element in vector $\vec{f}^*$ which represents the reconstructed image, and matrix M is the linear operator representing the cone-beam forward projection. The inequality used in the first constraint accounts for data inconsistency, such as noise and continuous-to-discrete inconsistency. The parameter ϵ can be selected for controlling the impact level of potential data inconsistency on image reconstruction. In our investigation of 3D image reconstruction, the TV of a discrete image $\vec{f}$ is defined as

$$\|\vec{f}\|_{TV} = \sum_{s,t,r} |\vec{\nabla} f_{s,t,r}| = \sum_{s,t,r} \left[(f_{s,t,r} - f_{s-1,t,r})^2 + (f_{s,t,r} - f_{s,t-1,r})^2 \right] \left[(f_{s,t,r} - f_{s,t,r-1})^2 \right]^{1/2} \quad (4)$$

where s , t , and r indicate the pixel indices of the 3D image. We refer to this algorithm as the TV-minimization algorithm. The implementation of the TV-minimization algorithm includes two major steps: gradient descent method and projection on convex sets (POCS) [5,6,7]. The gradient descent method is used for minimizing the image total variation, whereas the POCS is used for enforcing the constraints imposed by the known projection data. The reason the POCS is used here is that, even in the case of sparse sampling, the size of the projection data sets can be large, and POCS can efficiently handle large data sets.

It should be noted that in CT imaging with FFS scans, the acquired cone-beam data are generally rebinned and resampled so that the analytic algorithm can be applied [1,2]. The TV-minimization algorithm, however, can be applied directly to data acquired with any FFS scans.

IV Results

We have performed image reconstruction from data simulating xFFS and xzFFS scans based upon the configuration geometry of a typical clinical scanner. A circular scan with a source to rotation axis distance $R = 570$ mm was considered in this work. A spherical detector is placed at a distance $D = 1040$ mm from the source position. The spherical detector centers on the x-ray tube position for each projection view. The spherical detector consists of 128 rows along z direction and 672 channels along horizontal direction. The size of each detector element is $1.41 \text{ mm} \times 1.14 \text{ mm}$. The cone-beam data were acquired at 1200 projection views uniformly distributed over the angular range of 2π . For each projection view, two sets of projections ($\rho = 1$ and 2) were generated for the two FFSs. Cone-beam data were generated from an analytic phantom that contains high contrast objects, which may cause aliasing artifacts in reconstructed images by using analytic algorithms when the sampling is insufficiently dense, as shown in Fig 2. For comparison, we have also applied two analytic algorithms, i.e., the Feldkemp-Davis-Kress (FDK) algorithm [8,9] and the backprojection-filtration (BPF) algorithm [10,11], to reconstructing images.

By using data generated from the first xFFS (i.e., $\rho = 1$), which are referred to as single xFFS-data set, we reconstructed images by use of the FDK and TV-minimization algorithms. The images within the 2D slices at $z = 0$ mm, $x = 0$ mm, and $y = 0$ mm are displayed in Fig. 3. Artifacts can be observed in images reconstructed from the single xFFS-data set by use of the FDK algorithm, as shown in the left panel of Fig. 3, suggesting that the sampling density along the detector array is insufficiently dense for analytic algorithms. However, images in the right panel of Fig. 3 obtained by use of the TV-minimization algorithm are virtually free of aliasing artifacts observed in FDK reconstruction from the same data. Moreover, based upon the two data sets acquired from the two xFFSs (i.e., $\rho = 1$, and 2), we formed a combined data set that has an increased sampling density. From the combined data set, we also reconstructed the images by use of the FDK algorithm, as shown in Fig. 4. When the two xFFS data sets are appropriately combined, the resulting data set has improved sampling rate. This can be observed in images shown in the left panel of Fig. 4, in which the images reconstructed by use of analytic algorithms from the combined data have much reduced aliasing artifacts as compared to their counterparts obtained only from a single xFFS-data set (i.e., images in the left panel of Fig. 3). For the comparison, we displayed the images obtained by use of the TV-minimization algorithm from a single xFFS-data set in the right panel of Fig. 4. The results demonstrate that from a single xFFS-data set, the TV-minimization algorithm can yield images comparable to those obtained by used of analytic algorithms from two xFFS-data sets.

We have also reconstructed images by use of the FDK and TV-minimization algorithms from the single xzFFS-data set, the results are shown in Fig. 5. Again, the aliasing artifacts in TV-minimization reconstruction are substantially reduced. For comparison, we displayed the images obtained by use of the FDK algorithm from two xzFFS-data sets in Fig. 4. It can be observed again that the TV-minimization results obtained from the single xzFFS-data set are comparable to those obtained by used of analytic algorithm from two xzFFS-data sets.

V Conclusion

The FFS technique has been used in advanced clinical CT for increasing data sampling density, and such an increased sampling rate can lower significantly aliasing artifacts in analytic reconstructions. In this work, we have investigated the CS-based image reconstruction from data acquired at only one of the FFS in 2 FFSs for xFFS and xzFFS scans. We have shown and compared the results obtained by use of the FDK and CS-based TV-minimization algorithms from computer-simulated single FFS-data set and two FFS-data sets. The results of the study suggest that aliasing artifacts observed in images reconstructed by use of analytic algorithms can be reduced effectively in images reconstructed with the CS-based TV-minimization algorithm. In particular, images reconstructed from single-FFS data by use of the CS-based TV-minimization algorithm appear to have fewer aliasing artifacts than images reconstructed from two-FFS data by use of analytic algorithms. This observation seems to suggest that the CS-based TV-minimization algorithm may be used for image reconstruction from single-FFS data, thus reducing the scanning time and radiation dose.

Acknowledgments

This work was supported by NIH ROI grants Nos. EB000225 and CA120540. J. Bian was supported in part by a DOD Predoctoral training Grant No. BC083239. E. Y. Sidky was supported in part by a Career Development Award from NIH SPOR grant CA123183-03. Computations for this work were performed in part on a cluster, partially funded by NIH Grant Nos. S10 RR021029 and P30 CA14599. The authors thank Dr. Thomas Koehler and Dr. Stanislav Zabic for helpful discussions.

References

- [1]. Flohr T, Stierstorfer K, Ulzheimer S, Bruder H, Primak A, McCollough C. Image reconstruction and image quality evaluation for a 64-slice CT scanner with z-flying focal spot. *Med. Phys* 2005;vol. 32:2536–2547. [PubMed: 16193784]
- [2]. Kachelrieß M, Knaup M, Penbel C, Kalender WA. Flying focal spot (FFS) in cone-beam CT. *IEEE Trans. Nucl. Sci* 2006;vol. 53:1238–1247.
- [3]. Sidky EY, Kao C-M, Pan X. Accurate image reconstruction from few-views and limited-angle data in divergentbeam CT. *J. X-ray Sci. Tech* 2006;vol. 14:119–139.
- [4]. Sidky EY, Pan X. Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization. *Phys. Med. Biol* 2008;vol. 53:4777–4807. [PubMed: 18701771]
- [5]. Barrett, HH.; Myers, KJ. *Foundations of Image Science*. John Wiley & Sons; Hoboken, NJ: 2004.
- [6]. Gordon R, Bender R, Herman GT. Algebraic reconstruction techniques (ART) for three-dimensional electron microscopy and X-ray photography. *J. Theor. Biol* 1970;vol. 29:471–481. [PubMed: 5492997]
- [7]. Guan H, Gordon R. A projection access order for speedy convergence of ART (algebraic reconstruction technique): A multilevel scheme for computed tomography. *Phys. Med. Biol* 1994;vol. 39:2005–2022. [PubMed: 15560007]
- [8]. Feldkamp LA, Davis LC, Kress JW. Practical cone-beam algorithm. *J. Opt. Soc. Am* 1984;vol. A1:612–619.
- [9]. Turbell, H. Ph.D. thesis. Linköping University; 2001. Cone-beam Reconstruction Using Filtered Back-projection.
- [10]. Yu L, Zou Y, Sidky EY, Pelizzari CA, Munro P, Pan X. Region of interest reconstruction from truncated data in circular cone-beam CT. *IEEE Trans. Med. Imaging* 2006;vol. 25:869–881. [PubMed: 16827488]
- [11]. Zou Y, Pan X, Sidky EY. Theory and algorithms for image reconstruction on chords and within region of interests. *Journal of the Optical Society of America A* 2005;vol. 22:2372–2384.

Fig. 1.

Schematic illustration of the xFFS (left) and zFFS (right) techniques for increasing sampling density. Parameters R and D represent the source to rotation center distance and source to detector distance. The physical size of each detector element on the spherical detector is $D\Delta\gamma$ along rotation direction and $D\Delta\theta$ along z direction. Without the FFS, the sampling distance at the rotation center is determined by the physical size of each detector element, which is $R\Delta\gamma$ along rotation direction and $R\Delta\theta$ along z direction. In an xFFS scan one acquires data with two focal spots $\frac{1}{2}R\Delta\gamma$, and in a zFFS scan one acquires data with two focal spots $\frac{1}{2}R\Delta\theta$. Based upon the geometry shown, the length of the focal spot deflection along rotation direction and z direction can be expressed as $\Delta x = \frac{1}{4}R\Delta\gamma \times \frac{D}{D-R}$ and $\Delta z = \frac{1}{4}R\Delta\theta \times \frac{D}{D-R}$, respectively.

Fig. 2. Phantom images within 2D slices at $z = 0$ mm (a), $x = 0$ mm (b), and $y = 0$ mm (c). The display window is [1.025,1.07].

Fig. 3. Images within 2D slices at $z = 0$ mm (top row), $x = 0$ mm (middle row), and $y = 0$ mm (bottom row) reconstructed from the single xFFS-data set by use of the FDK algorithm (left) and the TV-minimization algorithm (right). The display window is [1.025,1.07].

Fig. 4. Comparison between FDK reconstruction from two xFFS-data sets and TV reconstruction from single xFFS-data set. The Images within 2D slices at $z = 0$ mm (top row), $x = 0$ mm (middle row), and $y = 0$ mm (bottom row) reconstructed by use of the FDK algorithm from the two xFFS-data sets (left) and by use of the TV-minimization algorithm from the single xFFS-data set (right). The display window is [1.025,1.07].

Fig. 5. Images within 2D slices at $z = 0$ mm (top row), $x = 0$ mm (middle row), and $y = 0$ mm (bottom row) reconstructed from the single-xzFFS data set by use of the FDK algorithm (left) and the TV-minimization algorithm (right). The display window is [1.025,1.07].

Fig. 6.

Comparison between FDK reconstruction from two xzFFS-data sets and TV reconstruction from single xzFFS-data set. The Images within 2D slices at $z = 0$ mm (top row), $x = 0$ mm (middle row), and $y = 0$ mm (bottom row) reconstructed by use of the FDK algorithm from the two xzFFS-data sets (left) and by use of the TV-minimization algorithm from the single xzFFS-data set (right). The display window is [1.025,1.07].