Fourier-based magnetic induction tomography for mapping resistivity

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Magnetic induction tomography is used as an experimental tool for mapping the passive electromagnetic properties of conductors, with the potential for imaging biological tissues. Our numerical approach to solving the inverse problem is to obtain a Fourier expansion of the resistivity and the stream functions of the magnetic fields and eddy current density. Thus, we are able to solve the inverse problem of determining the resistivity from the applied and measured magnetic fields for a two-dimensional conducting plane. When we add noise to the measured magnetic field, we find the fidelity of the measured to the true resistivity is quite robust for increasing levels of noise and increasing distances of the applied and measured field coils from the conducting plane, when properly filtered. We conclude that Fourier methods provide a reliable alternative for solving the inverse problem. © 2011 American Institute of Physics. [doi:10.1063/1.3524276]

I. INTRODUCTION

The experimentalist has several tomographic techniques available for measuring the passive electromagnetic properties of biological tissues, two of which are electrical impedance tomography (EIT) and magnetic induction tomography (MIT). Both techniques involve applying a known field to induce a current, measuring a resulting field, and solving the inverse problem to determine (for example) the resistivity of the material.^{1–3} In EIT, electrodes are employed that create a potential distribution that causes current to flow throughout the interior; the measured potential is a function of the resistivity of the material and the current distribution.^{1,3} This technique is well developed and its analysis may be found in articles exploring topics as diverse as clinical imaging^{4,5} and stealth cloaking.⁶

MIT is a relatively new tomographic technique. Holder and Griffiths note that the earliest references to MIT appear in the early 1990s.^{1,2} As such, this is a lesser developed technique. While a recent innovation, the essence of MIT is similar to EIT. In MIT, the fields are magnetic. Excitation coils in an apparatus produce a time-varying applied magnetic field^{1,7} which induces an eddy current density^{8,9} in the conductor. The measured magnetic field due to the induced eddy currents can be separated from the total measured field by examining the in-phase and out-of-phase components of the total measured signal.⁵ A schematic diagram of an MIT experiment is given in Fig. 1. The contribution to the measured field from the eddy currents will be a function of the resistivity of the material.^{1,2} In general, the magnetic field due to the eddy current density is quite small when compared to the applied field.^{7,10}

Our proposed technique for solving the inverse problem—determining resistivity from the applied and measured magnetic fields—is Fourier based. We consider a two-dimensional thin sheet in the xy -plane. The thickness of the sheet should be less than the skin depth of the material so that the induced eddy current density is independent of

depth. When properly filtered, our method of image processing is relatively robust at handling noise and measurement error and works well for moderate distances of the coils from the conducting plane; our thin sheet has a resistivity distribution of a magnitude representative of biological tissue.

II. METHODS

Consider a thin planar sheet at $z=0$ of thickness δ with a resistivity distribution $\rho(x,y)$. Assuming the sheet is periodic with period L in both directions, the resistivity can be expanded as the two-dimensional Fourier series¹¹

$$\rho(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \rho_{nm} e^{-i(2\pi/L)(nx+my)}. \quad (1)$$

FIG. 1. (Color) Diagram of the MIT experiment. In MIT, a set of coils produces an applied magnetic field that varies with time; the coils, here, are located at a distance of z_{applied} above the conducting plane. The conductor, of thickness δ , has the resistivity distribution shown in Fig. 2. The applied field induces an eddy current in the plane, the distribution of which is a function of the resistivity of the material. At a distance below the conducting plane, here located at z_{measured} , a set of coils detects the measured field which includes the applied field and the field associated with the eddy currents. For the theoretical calculations considered here, we assume the applied field is known in the plane of the conductor and is a simple product of sines and cosines. In practice, the coil geometry is that of current loops.

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At position $\mathbf{x}=(x,y,z)$ in a plane above the sheet are a set of coils that produce an applied magnetic field^{1,7} $\mathbf{B}(\mathbf{x},t)$ with an associated vector potential^{8,9} $\mathbf{A}(\mathbf{x},t)$. Because the coils that produce the current are planar, the z -component of $\mathbf{A}$ is zero everywhere. If we use the Coulomb gauge^{8,9} ($\text{div } \mathbf{A}=0$), then $\mathbf{A}$ can be expressed in terms of a stream function¹² $\eta(\mathbf{x},t)$ such that $\mathbf{A}(\mathbf{x},t)=(\partial\eta/\partial y)\hat{i}-(\partial\eta/\partial x)\hat{j}$. Because there are no currents in this region, the stream function obeys Laplace's equation;¹¹ the Fourier expansion of the time derivative of the stream function in the region below the plane containing the applied coils is¹¹

$$\dot{\eta}(x,y,z,t) = \sum_{N'=-\infty}^{\infty} \sum_{M'=-\infty}^{\infty} \dot{\eta}_{N'M'} e^{2\pi i L \sqrt{N'^2+M'^2} z} e^{-i(2\pi/L)(N'x+M'y)}, \quad (2)$$

where $\dot{\eta}_{N'M'}$ is the Fourier coefficient of the stream function evaluated in the conducting plane at $z=0$. The eddy current distribution $\mathbf{j}(\mathbf{x},t)$ induced in the sheet^{8,9} (and obeying the continuity equation^{8,9} $\text{div } \mathbf{j}=0$) can also be represented with a stream function ξ such that $\mathbf{j}=(\partial\xi/\partial y)\hat{i}-(\partial\xi/\partial x)\hat{j}$; ξ can be expanded as the Fourier series

$$\xi(x,y,t) = \sum_{n'=-\infty}^{\infty} \sum_{m'=-\infty}^{\infty} \xi_{n'm'} e^{-i(2\pi/L)(n'x+m'y)}. \quad (3)$$

Below the sheet, at $z=z_{\text{measured}}$, a set of measurement coils detect the resulting magnetic field.^{1,7} The measured field $\mathbf{b}(\mathbf{x},t)$ (due solely to the eddy current distribution) has an associated vector potential $\mathbf{a}(\mathbf{x},t)$. Again, working with the Coulomb gauge ($\text{div } \mathbf{a}=0$) in this region, where there is no current, the vector potential can be expressed in terms of a stream function $\lambda(\mathbf{x},t)$ that obeys Laplace's equation such that $\mathbf{a}=(\partial\lambda/\partial y)\hat{i}-(\partial\lambda/\partial x)\hat{j}$. The Fourier expansion of λ is

$$\lambda(x,y,z,t) = \sum_{N=-\infty}^{\infty} \sum_{M=-\infty}^{\infty} \lambda_{NM} e^{2\pi i L \sqrt{N^2+M^2} z} e^{-i(2\pi/L)(Nx+My)}, \quad (4)$$

where λ_{NM} are the Fourier coefficients of the stream function evaluated in the conducting plane at $z=0$; the expansion is valid in the region below the conducting plane. For the tangential component of the measured magnetic field to have the proper discontinuity across the conducting plane,^{8,9} the Fourier components of the stream functions for the measured field and eddy current must be related by¹³

$$\lambda_{NM} = -\frac{L\mu_0\delta}{4\pi} \frac{1}{\sqrt{N^2+M^2}} \xi_{NM}. \quad (5)$$

We consider the tissue to be Ohmic; the eddy current induced in the sheet may be related to the scalar electrical potential ϕ and magnetic vector potential $\mathbf{A}$ by $\rho\mathbf{j}=\mathbf{E}=-(\nabla\phi+\dot{\mathbf{A}})$, where $\dot{\mathbf{A}}$ is the time derivative of the vector potential.^{8,9} If we take the curl of this equation and write $\dot{\mathbf{A}}$ and $\mathbf{j}$ in terms of their stream functions, ϕ is eliminated¹¹ and we obtain the relation

$$\frac{\partial}{\partial x} \left(\rho \frac{\partial \xi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\rho \frac{\partial \xi}{\partial y} \right) = - \left(\frac{\partial^2 \dot{\eta}}{\partial x^2} + \frac{\partial^2 \dot{\eta}}{\partial y^2} \right). \quad (6)$$

If we replace the stream functions and the resistivity by their Fourier expansions [Eqs. (1)–(3)] and evaluate the expres-

sions in the conducting plane $z=0$, then Eq. (6) becomes

$$\begin{aligned} & -\sum_n \sum_m \sum_{n'} \sum_{m'} \rho_{nm} \xi_{n'm'} [n'(n+n') + m'(m+m')] \\ & \times e^{-i(2\pi/L)[(n+n')x + (m+m')y]} \\ & = \sum_{N'} \sum_{M'} \dot{\eta}_{N'M'} (N'^2 + M'^2) e^{-i(2\pi/L)(N'x+M'y)}. \end{aligned} \quad (7)$$

We multiply both sides of Eq. (7) by $\exp(i2\pi(Nx+My)/L)$ and integrate over x and y , exploiting orthogonality¹¹ to find

$$-\sum_n \sum_m \rho_{nm} \xi_{N-n,M-m} (N(N-n) + M(M-m)) = \dot{\eta}_{NM} (N^2 + M^2), \quad (8)$$

which is a linear system relating the Fourier coefficients of the resistivity to the Fourier coefficients of the stream functions of the applied magnetic field and the current density [which through Eq. (5) relates the current density to the measured magnetic field].

Experimentalists may prefer to express this relationship solely in terms of the applied and measured magnetic fields

$$\begin{aligned} & \sum_n \sum_m \rho_{nm} b_{N-n,M-m} \frac{N(N-n) + M(M-m)}{\sqrt{(N-n)^2 + (M-m)^2}} \\ & \times e^{-2\pi i L \sqrt{(N-n)^2 + (M-m)^2} z_{\text{measured}}} \\ & = \frac{L\mu_0\delta}{4\pi} \dot{B}_{NM} e^{2\pi i L \sqrt{N^2+M^2} z_{\text{applied}}}, \end{aligned} \quad (9)$$

where $b_{N-n,M-m}$ are the Fourier coefficients of the measured magnetic field evaluated at the measuring coils in the plane at $z=z_{\text{measured}}$ and $\dot{B}_{NM}$ are the Fourier coefficients of the applied magnetic field evaluated in the plane at $z=z_{\text{applied}}$; we have assumed in our experiments that $z_{\text{applied}}=0$. In practice, the relevant summations are finite results of a discrete Fourier transform¹¹ and Eq. (8) or (9) is solved with least-squares Gaussian elimination.^{14,15}

Noise is always present in measured signals and coping with noise is central to image processing.¹⁶ In magnetic imaging modalities such as MIT or magnetic resonance imaging, noise may appear from the thermal motion of free electrons in the measuring apparatus¹⁶ or in the conducting plane itself.¹³ The MIT signal is quite weak,^{7,10} so even a small amount of noise can be a big problem. Regardless of the source, we shall model the noise as a stochastic contribution to the measured magnetic field. The random variable is Gaussian and has a mean of zero and a standard deviation of $Q \times [\max(\mathbf{b}) - \min(\mathbf{b})]$ where $\max(\mathbf{b})$ and $\min(\mathbf{b})$ are the maximum and minimum values, respectively, of the measured magnetic field without noise and Q is the noise-to-signal ratio.

Filtering of the signal is usually necessary to obtain a high fidelity image. With exponential terms $\exp(kz)$ in Eq. (9) (where k is the spatial frequency) any noise in the high frequency region of k -space (including numerical noise on the order of the precision error) will be greatly magnified, particularly for large values of kz . Our strategy for filtering will be to first calculate ξ and $\dot{\eta}$ in k -space, in the plane at $z=0$, and filter the stream functions with the filter function¹⁷

$$S' = S \times \begin{cases} \cos^2\left(\frac{\pi}{2} \frac{k}{F\Delta k}\right), & k < F\Delta k, \\ 0, & k > F\Delta k, \end{cases} \quad (10)$$

where F is a filter threshold; k is the spatial frequency with $k = \sqrt{k_x^2 + k_y^2}$ where $k_x = p\Delta k$ and $k_y = q\Delta k$ (p and q are integers); $\Delta k = 2\pi L/W$ (W is the number of points along the x -axis in the $W \times W$ discrete mesh); S is the signal in k -space before filtering and S' is the signal in k -space after filtering. High frequency information about the resistivity will have to be sacrificed due to amplification of high frequency noise in the stream functions. Consequently, filtering will produce best results with low spatial frequency magnetic fields and resistivities.

The experimentalist facing the problem of determining the resistivity by MIT would apply a time-varying magnetic field using a set of coils^{1,2,7} (current loops). As a theoretical calculation, we are free to choose any applied field distribution. Note in Eq. (8) or (9) that there is at least one equation in the linear system with zero on the left side (where $N=M=0$). When formulated as a matrix-vector system for solution of the resistivity in k -space, Eq. (8) or (9) results in a singular matrix.¹¹ Our approach is to repeat the experiment with four different applied magnetic fields and use Gaussian elimination with least-squares to solve the linear system.^{14,15} The first derivative with respect to time of the applied magnetic fields we use are

$$\begin{aligned} \dot{\mathbf{B}}_{\text{applied1}} &= \dot{B}_0 \sin\left(4\pi \frac{x}{L}\right) \sin\left(4\pi \frac{y}{L}\right), \\ \dot{\mathbf{B}}_{\text{applied2}} &= \dot{B}_0 \sin\left(4\pi \frac{x}{L}\right) \cos\left(4\pi \frac{y}{L}\right), \\ \dot{\mathbf{B}}_{\text{applied3}} &= \dot{B}_0 \cos\left(4\pi \frac{x}{L}\right) \sin\left(4\pi \frac{y}{L}\right), \\ \dot{\mathbf{B}}_{\text{applied4}} &= \dot{B}_0 \cos\left(4\pi \frac{x}{L}\right) \cos\left(4\pi \frac{y}{L}\right), \end{aligned} \quad (11)$$

where $\dot{B}_0 = 1 \text{ T}/\mu\text{s}$ and the fields are evaluated at $t=0$. Here and henceforth, all field and stream functions are evaluated at time $t=0$. Our experiments are repeated for different ratios of noise to signal Q , different levels of filtration F , and different values of z_{measured} . In each case the fields given in Eq. (11) are applied. In every experiment, we use the same resistivity distribution shown in Fig. 2 (Ref. 18)

$$\rho_{\text{true}} = 1.0 + \sum_{n=1}^5 r_n e^{-((x-X_n)^2 + (y-Y_n)^2)/R_n^2}, \quad (12)$$

where $r_n = \{0.04375, -0.4, 0.4, -0.3, 0.2\}$, $X_n = \{4.5, 5.5, 6, 5, 4\}$, $Y_n = \{5.5, 6, 3.5, 4.5, 4.5\}$, and $R_n = (2/3)\{4, 2, 1, 1, 2\}$. Our conducting plane is assumed to be periodic with $L = 10 \text{ cm}$ and a thickness $\delta = 0.1 \text{ cm}$ with units of resistivity $k\Omega \text{ cm}$, on the order of the resistivity of biological tissue.

Our steps for simulating the MIT experiment and solving the inverse problem are as follows: given the applied magnetic field and resistivity distribution, we numerically solve the forward problem [Eq. (6)] for the measured magnetic

FIG. 2. (Color) Resistivity distribution. The resistivity distribution used in all of the experiments consists of a sum of five Gaussian distributions, given by Eq. (12), and is shown here as the true resistivity. The units of resistivity of the given distributions are $k\Omega \text{ cm}$. With a noise-to-signal ratio of $Q = 0.01$ and a measured coil distance of $z_{\text{measured}} = -0.50 \text{ cm}$, we reconstruct the resistivity from the applied and measured magnetic fields with filtration levels of $F=4, 8$, and 16 .

field using a relaxation method,⁸ repeating this step for each of the applied fields of Eq. (11); we add noise to the measured fields that result; given these applied magnetic fields and measured magnetic fields with noise, we solve the inverse problem [Eq. (8)]. We characterize the fidelity of the calculated resistivity to the true resistivity using the mean deviation¹⁹

$$MD = \sqrt{\frac{\iint (\rho_{\text{true}} - \rho_{\text{calculated}})^2 dx dy}{\iint (\rho_{\text{true}})^2 dx dy}}. \quad (13)$$

III. RESULTS

Let us consider a single experiment. Using a set of applied magnetic fields [given in Eq. (11)] with the z -component of $\dot{\mathbf{B}}_{\text{applied1}}$ [shown in Fig. 3, panel A] applied to the resistivity distribution [given by Eq. (12) and shown in Fig. 2] we obtain a set of measured magnetic fields [with $\mathbf{b}_{\text{measured1}}$ shown in Fig. 3, panel B]. The noise Q that we add to $\mathbf{b}_{\text{measured1}}$ is shown in Fig. 3, panel C; similar Gaussian distributed noise is also added to the other three fields. Our applied field is known in the conducting plane ($z_{\text{applied}}=0$) and the measurement coils are quite close to the conducting plane (here $z_{\text{measured}}=-0.25 \text{ cm}$). When we process the applied and measured (with noise) magnetic fields using the FFT routines provided by MATLAB and solve the inverse problem of Eq. (8) with summations over -16 to $+15$, filtered with $F=8$ in Eq. (10), we obtain the resistivity map shown in Fig. 3, panel D. In general, the fidelity of the measured resistivity to the true resistivity seems quite reasonable. In this case $MD=0.0363$.

In Fig. 2, we observe how different levels of filtration can affect the reconstructed resistivity. Filtration of $F=4$ pro-

FIG. 3. (Color) Sample calculation of image processing of resistivity. The z -component of the applied magnetic field (panel A, units of $T/\mu s$) induces an eddy current distribution, which, in turn, contributes to the z -component to the measured magnetic field (panel B, units of μT). In our image processing we consider the measured magnetic field to be only due to the eddy current. We add Gaussian distributed random noise to the measured magnetic field with a noise-to-signal ratio $Q=0.01$ (panel C, units of μT). Relating the filtered ($F=8$) magnetic fields to their vector potentials and then to the associated stream functions and finally to the Fourier expansions of the stream functions, we solve Eq. (8) to obtain the measured resistivity distribution (panel D, units of $k\Omega\text{ cm}$) which is a reasonable reproduction of the true resistivity (Fig. 2) with $MD=0.0363$.

duces an over-filtered image that is smoothed out and contains little detail; the MD value is 0.0576. With $F=16$, the image is under-filtered and noise dominates the reconstructed image; the MD value is 0.1040. With $F=8$ we observe the optimum reconstruction of the true resistivity; the value of MD is 0.0368.

In Table I, we observe that the lowest MD is found with a filtration of approximately $F=8$. As the image is less and

TABLE I. MD Values for various experiment parameters. Here we vary the level of the noise-to-signal ratio (Q) that we add to the system, the level of filtration (F) used in our filter defined by Eq. (10), and the position of the measurement “coil” (z_{measured}) with the applied magnetic field known in the conducting plane. II indicates an Improper Image.

Q	F	$MD, z_{\text{measured}}=-0.5\text{ cm}$	$MD, z_{\text{measured}}=-1.0\text{ cm}$
0.00	32	0.0035	0.0035
0.01		II	II
0.10		II	II
0.00	16	0.0125	0.0125
0.01		0.1040	II
0.10		II	II
0.00	8	0.0361	0.0361
0.01		0.0368	0.0386
0.10		0.0696	0.1704
0.00	6	0.0486	0.0486
0.01		0.0488	0.0497
0.10		0.0580	0.0722
0.00	4	0.0575	0.0575
0.01		0.0576	0.0572
0.10		0.0581	0.0584

TABLE II. The formation of an improper image. In Table I, we observe the formation of an improper image for certain levels of noise and filtration and for certain values of position of the measurement coil. The formation of an improper image is examined with no noise ($Q=0.00$) and fixed filtration ($F=8$) for various positions of the measurement plane. MD values as well as a qualitative comment on the quality of the measured resistivity are listed, where the qualitative assessment is relative to the true resistivity.

z_{measured} (cm)	MD	Qualitative assessment
-4.00	II	Highly improper
-3.00	II	Highly improper
-2.75	II	Highly improper
-2.60	0.6703	Borderline improper
-2.55	0.5232	Borderline improper
-2.53	0.2757	Reasonably okay
-2.50	0.1316	Reasonably okay
-2.45	0.0735	Good
-2.40	0.0173	Good
-2.35	0.0098	Very good
-2.25	0.0025	Very good
-2.00	0.0002	Nearly perfect

less filtered there is the propensity of the inverse solution method to produce an improper image. We define an improper image as a resistivity that has a small amplitude variation with intermediate to high spatial frequencies about a mean value of resistivity considerably less than the mean value of the true resistivity. In Table II, we examine the effect that increased distance of the measurement coils from the plane has on the quality of the reconstructed image. We sometimes obtain improper images, even though no noise has been added. We observe that the exponential term involving z_{measured} in Eq. (9) is amplifying specific numerical noise from precision errors and small errors in the solution of the forward problem at high spatial frequencies, and that the least-squares solution method^{14,15} we use for the system of linear equations “distributes the error” throughout k -space in ρ leading to the formation of an improper image.

Apart from the formation of improper images, our solution of the inverse problem behaves much as we expect: for a given filtration and z_{measured} , the MD value generally increases with increasing noise levels (a minor deviation from this trend is attributable to the stochastic nature of the noise added). In Table I, we see that as z_{measured} is moved slightly from -0.50 to -1.00 cm there is marginal change in the MD for most of the experiments and an increase in the MD for certain noise and filtration levels.

Finally, we observed the effect of measurement error in z by first calculating the measured field (with and without noise added) for specific values of z_{measured} and z_{applied} and then solving the inverse problem using deliberately wrong values of z_{measured} and z_{applied} . In every case, we were able to reconstruct effectively the resistivity with a distribution that was qualitatively similar to that of Fig. 2; however, a nearly spatially uniform decrease (or increase) in the calculated value of resistivity was evident when the measurement of z was larger (or smaller) than the true values. This was consistently observed for errors in the measurement of z as large as the value of z itself. Thus, error in the measurement of posi-

tion of the coils is much less of a concern than noise or filter level and, when concerned only with a qualitative map of resistivity, is not important at all.

IV. CONCLUSION

Our analysis suggests that Fourier-based image processing in MIT is effective at reconstructing resistivity. The reconstruction is frustrated by increasing levels of noise added to the system, as one would expect. There is an optimum level of filtration for reconstruction of the resistivity. Over-filtration removes the higher spatial frequencies necessary for reconstruction of details in the resistivity and smoothes out the image. Under-filtration retains the higher spatial frequencies of the image while also retaining much of the noise in the system. Here, we have noted an optimum filtration level of $F=8$. This level of filtration is due to our simulated resistivity, measured coil distance and applied magnetic fields; experimentalists will encounter a different optimum level of filtration unique to their system. We have chosen four sinusoidal applied fields with no *a priori* justification for using four fields in the least-squares solution. The experimentalist is free to use fewer or more applied fields though, of course, the functional form of the applied fields will be a result of the geometry of the coils used to apply the magnetic field. Noise, filtration, and measurement coil distance are the primary determinants of the fidelity of the reconstructed resistivity to the true resistivity (a result analogous to a similar Fourier method to obtain current images from measured magnetic fields¹⁹). Parameters such as error in the position of the measurement coils are of lesser importance.

We note several limitations of this method of image processing. First, this system was assumed to be two dimensional; a three-dimensional system may be more difficult to handle because of the nonuniqueness of the solution. Three-dimensional calculations may be further complicated by excessively high computational needs. Here, with two dimensions, we solve the linear system in frequency space (with four applied fields) using least-squares. This method is computationally intensive (a calculation on the order of W^4 where W is the number of points along the x - or y -axis in the discrete mesh) and is a memory intensive algorithm. Finally, while our method seems robust at handling noise with a noise-to-signal ratio of 0.10 or less, higher levels of noise are not well tolerated. A low noise-to-signal level is necessary. Measurement coils presented here are within a distance of 2 cm from the conducting plane (of length and width 10 cm)

and the measured fields are extremely small when compared to the applied field. Measurement of magnetic fields this small at very close distances to the conducting plane presents challenges to the experimentalist, though such recordings are certainly not unprecedented (see for example a discussion of superconducting quantum interference devices for nondestructive evaluation²⁰). When these limitations are considered, resistivity seems well reconstructed using Fourier methods in MIT.

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