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Multi-Excitation Magnetoacoustic Tomography with Magnetic Induction for Bioimpedance Imaging

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Abstract

Magnetoacoustic tomography with magnetic induction (MAT-MI) is an imaging approach proposed to conduct non-invasive electrical conductivity imaging of biological tissue with high spatial resolution. In the present study, based on the analysis of the relationship between the conductivity distribution and the generated MAT-MI acoustic source, we propose a new multi-excitation MAT-MI approach and the corresponding reconstruction algorithms. In the proposed method, multiple magnetic excitations using different coil configurations are employed and ultrasound measurements corresponding to each excitation are collected to derive the conductivity distribution inside the sample. A modified reconstruction algorithm is also proposed for the multi-excitation MAT-MI imaging approach when only limited bandwidth acoustic measurements are available. Computer simulation and phantom experiment studies have been done to demonstrate the merits of the proposed method. It is shown that if unlimited bandwidth acoustic data is available, we can accurately reconstruct the internal conductivity contrast of an object using the proposed method. With limited bandwidth data and the use of the modified algorithm we can reconstruct the relative conductivity contrast of an object instead of only boundaries at the conductivity heterogeneity. Benefits that come with this new method include better differentiation of tissue types with conductivity contrast using the MAT-MI approach, specifically for potential breast cancer screening application in the future.

Index Terms

Magnetoacoustic tomography; MAT-MI; electrical impedance imaging; finite element method; reconstruction

I. INTRODUCTION

Noninvasive electrical impedance imaging of biological tissues has been an active research area for several decades. One of the major motivations comes from the fact that electrical properties including conductivity and permittivity of biological tissue are sensitive to its physiological and pathological conditions [1]. For example, it has been shown that breast cancer tissue has significantly higher conductivity than the surrounding tissues [2]. In addition, the knowledge of biological tissue impedance is of great interest to researchers conducting electromagnetic source imaging [3]. Among all the techniques for imaging bioimpedance, electrical impedance tomography (EIT) [4], [5] was first developed using current injection and noninvasive surface voltage measurements. EIT has the benefits of low cost, real-time speed and safety. Its major limitations include low spatial resolution and degraded sensitivity in the center of an object. In addition, as with all other imaging

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techniques using current injection through surface electrodes, EIT has the problem of the “shielding effect” [6] caused by an insulating or low conductive region in the object, such as fat tissue. To avoid the problems associated with contact electrode measurements and the “shielding effect” of current injection in EIT, magnetic induction tomography (MIT) was introduced [7]. In MIT, an oscillating magnetic field is applied to the conductive sample and measurements of the secondary magnetic field produced by the induced eddy current are taken by small coils arranged around the object. However, the spatial resolution of current MIT technique is still quite limited. In order to achieve high spatial resolution conductivity imaging, magnetic resonance electrical impedance tomography (MREIT) was developed by combining EIT and magnetic resonance current density imaging (MRCDI) [8], [9]. In MREIT, the magnetic field disturbance caused by the injected current in the conductive sample is measured by a magnetic resonance imaging system. High spatial resolution conductivity images were obtained in both *in vitro* and *in vivo* experiments [9]. However MREIT is currently limited by its requirement of high level current injection to obtain an acceptable signal-to-noise (SNR) level.

Besides the electromagnetic imaging methods as EIT, MIT and MREIT, alternative approaches for noninvasive imaging of electrical current or conductivity are those utilizing the coupling between electromagnetic field and acoustic field as reported in magnetoacoustic tomography (MAT) [10], [11] and Hall Effect imaging (HEI) [6], [12]. In MAT and HEI, spontaneous or injected current flow is coupled to acoustic vibrations through Lorentz force with the existence of a static magnetic field. Ultrasound measurements are then collected for image reconstruction. Using a similar coupling mechanism, one can also apply ultrasonic energy to the sample and record voltage/current signals to obtain the sample’s conductivity information [12]–[14]. However, the use of current injection or voltage/current measurement through surface electrodes still make these methods limited by the “shielding effect”.

In order to avoid the problems associated with using surface electrodes and the corresponding “shielding effect”, magnetoacoustic tomography with magnetic induction (MAT-MI) [15]–[22] was proposed to achieve noninvasive electrical conductivity imaging with high spatial resolution. MAT-MI utilizes magnetic induction to induce eddy current in the conductive sample and generates acoustic vibrations through the same Lorentz force coupling mechanism as in MAT/HEI. Ultrasound waves are then sensed to reconstruct the conductivity related image. Theoretical and experiment studies have been conducted to demonstrate its feasibility.

Two major reconstruction algorithms have been previously developed for the MAT-MI imaging problem [16], [22]. As in the acoustic source reconstruction based algorithm reported in [16], according to the piecewise homogeneous assumption, we take an approximation in deriving the conductivity by ignoring the conductivity gradient term. This is equivalent to ignoring all the acoustic sources generated at the conductivity boundaries. However, these boundary sources have been demonstrated to be dominant sources in both computer simulation and experiment studies [17]–[19]. Without an ultrasound point receiver that has unlimited bandwidth, only the conductivity boundaries of the sample can be reconstructed in the MAT-MI image [19]. Some alternative strategies are also suggested in [16], but they either are numerically unstable or have obvious singularities. The other algorithm developed for MAT-MI is the vector field reconstruction algorithm reported in [22]. Using this algorithm, we can reconstruct the vector Lorentz force field from the acoustic measurements collected on spherical or cylindrical apertures. However, this algorithm requires rigid measurement geometry and requires the knowledge of the induced electric field in the conductive sample to derive the conductivity distribution. The induced

electric field, however, depends on the unknown conductivity distribution and is generally not known *a priori* especially for complex biological tissue samples.

In the present study, based on the analysis of the relationship between the conductivity distribution and the generated MAT-MI acoustic source, we propose a new multi-excitation MAT-MI approach and the corresponding reconstruction algorithms. In this method, multiple magnetic excitations using different coil configurations are employed and ultrasound measurements corresponding to each excitation are collected to derive the conductivity distribution inside the sample. A modified reconstruction algorithm is also proposed for the multi-excitation MAT-MI imaging approach when only limited bandwidth acoustic measurements are available. We have conducted computer simulation and phantom experiment studies to test the performance of the proposed method. It is shown that if unlimited bandwidth acoustic data is available, we can accurately reconstruct the internal conductivity contrast of an object. With limited bandwidth data and the use of the modified algorithm we can reconstruct the relative conductivity contrast of an object instead of only boundaries at the conductivity heterogeneity. Benefits that come with this new method include better differentiation of tissue types with conductivity contrast using the MAT-MI approach, specifically for potential breast cancer screening application in the future.

II. THEORY

A. Problem Description

The forward problem of the MAT-MI approach describes two major physical processes in its signal generation mechanism, i.e. magnetic induction in the conductive sample and acoustic wave propagation with the Lorentz force induced acoustic sources.

We consider a sample with isotropic conductivity $\sigma(\mathbf{r})$. The sample is placed in a static magnetic field with flux density $\mathbf{B}_0(\mathbf{r})$. In the proposed multi-excitation MAT-MI approach, we have N different excitation coil setups with $N \geq 2$. Denote the stimulating time-varying magnetic field generated from the j th coil setup as $\mathbf{B}_1^j(\mathbf{r}, t)$ for $j = 1, \dots, N$. The j th stimulating field applied to the conductive sample induces the corresponding electrical field $\mathbf{E}^j(\mathbf{r}, t)$ and eddy current density distribution $\mathbf{J}^j(\mathbf{r}, t)$. As in MAT-MI we are considering around μs level current pulses for driving the stimulating coil, the corresponding MHz skin depth in general biological tissue is at the level of meters, so the magnetic induction problem in MAT-MI can be considered quasi-static and magnetic diffusion can be ignored. This condition allows us to separate the spatial and temporal function of the time-varying magnetic field, i.e. $\mathbf{B}_1^j(\mathbf{r}, t) = \mathbf{B}_1^j(\mathbf{r})f_j(t)$. According to Faraday's Law and Ohm's Law, the similar spatial and temporal separation holds for the induced electrical field and eddy current density, i.e. $\mathbf{E}^j(\mathbf{r}, t) = \mathbf{E}^j(\mathbf{r})f_j'(t)$ and $\mathbf{J}^j(\mathbf{r}, t) = \mathbf{J}^j(\mathbf{r})f_j'(t)$ where the prime denotes first order time derivative. The quasi-static condition also indicates that the stimulating magnetic field in the sample can be well approximated by the field generated by the same coil configuration in free space [23]. In addition, the displacement current can be ignored as it is much smaller than the conductive current in biological tissue at MHz frequency [16]. Using the notations of magnetic vector potential $\mathbf{A}^j(\mathbf{r}, t)$ where $\mathbf{B}_1^j = \nabla \times \mathbf{A}^j$ and electrical scalar potential $\phi^j(\mathbf{r})$, the governing equation for magnetic induction in MAT-MI can be written as in (1) [18]

$$\nabla \cdot \left(\sigma \left(-\frac{\partial \mathbf{A}^j}{\partial t} - \nabla \phi^j \right) \right) = 0 \quad j=1, \dots, N \quad (1)$$

Because of the quasi-static condition the magnetic vector potential $\mathbf{A}^j$ and the corresponding flux density $\mathbf{B}_1^j$ depend only on the j th coil configurations and can be estimated with known coil geometry. The magnetic vector potential $\mathbf{A}^j$ in (1) is then considered to be known. Equation (1) subject to a Neumann boundary condition on the current density $\mathbf{J}^j$ as in (2) has a unique solution when we choose a reference position with zero potential [23].

$$\mathbf{J}^j \cdot \mathbf{n} = 0 \quad (2)$$

Here $\mathbf{n}$ is the unit vector norm of the outer boundary surface of the conductive object. This boundary condition requires the current density component that is normal to the bounding surface to vanish. With known σ and $\mathbf{A}^j$, we can solve for ϕ^j throughout the whole three dimensional (3D) conductive volume using the finite element method (FEM). The corresponding electrical field and current density can then be computed, as

$$\mathbf{E}^j = -\frac{\partial \mathbf{A}^j}{\partial t} - \nabla \phi^j, \text{ and } \mathbf{J}^j = \sigma \mathbf{E}^j.$$

With the magnetically induced eddy current $\mathbf{J}^j$ and the static magnetic field $\mathbf{B}_0$, the Lorentz force acting on the eddy current can be described as $\mathbf{J}^j \times \mathbf{B}_0$. In MAT-MI the divergence of the Lorentz force acts as acoustic source of propagating ultrasound waves that can be sensed by ultrasonic transducers placed around the sample. The wave equation governing the pressure distribution is given in (3) [11]

$$\nabla^2 p_j - \frac{1}{c_s^2} \frac{\partial^2 p_j}{\partial t^2} = \nabla \cdot (\mathbf{J}^j \times \mathbf{B}_0) \quad j=1, \dots, N \quad (3)$$

where p_j is the pressure corresponding to the j th magnetic stimulation and c_s is the acoustic speed in the media. Here we assume the sample is acoustically homogeneous. Using the 3D Green's function, the solution to (3) can be written as in (4) [24]

$$p_j(\mathbf{r}_0, t) = -\frac{1}{4\pi} \int_V d\mathbf{r} \nabla_{\mathbf{r}} \cdot [\mathbf{J}^j \times \mathbf{B}_0] \frac{\delta(t - |\mathbf{r}_0 - \mathbf{r}|/c_s)}{|\mathbf{r}_0 - \mathbf{r}|} \quad (4)$$

where $\mathbf{r}_0$ is a position located on certain ultrasound detection aperture.

The inverse problem of MAT-MI concerns how to reconstruct the conductivity of the sample with the obtained acoustic measurements. First, with the acoustic measurements p_j obtained on certain acoustic aperture around the sample, we can reconstruct the acoustic source map, i.e. distributions of $\nabla \cdot (\mathbf{J}^j \times \mathbf{B}_0)$, in the 3D conductive volume using the time reversal back projection method [16], [19]. In addition, multiplying both sides of (4) with $-4\pi t$, (4) takes the form of a spherical Radon transform and the corresponding expectation maximization (EM) algorithm developed for reflective tomography [25] can also be used to reconstruct the acoustic source map. The EM algorithm generates fewer artifacts than the time reversal method when dealing with limited view angle data [25], but it is computationally more demanding.

After reconstructing the acoustic source map, we can then derive the conductivity distribution of the object, which is of more interest from the clinic application perspective. Taking the fact that the static magnetic field $\mathbf{B}_0$ in MAT-MI is generated from sources outside the conductive object, for example from some permanent magnets, we have $\nabla \times \mathbf{B}_0 = 0$ inside the object volume [16] and the acoustic source term on the right hand side of (3)

can be further simplified as $(\nabla \times \mathbf{J}^j) \cdot \mathbf{B}_0$. Denoting $AS^j = (\nabla \times \mathbf{J}^j) \cdot \mathbf{B}_0$ and by expanding it with Ohm's Law $\mathbf{J}^j = \sigma \mathbf{E}^j$, we can obtain (5)

$$AS^j = \sigma \left(-\frac{\partial B_1^j}{\partial t} \right) \cdot \mathbf{B}_0 + (\nabla \sigma \times \mathbf{E}^j) \cdot \mathbf{B}_0 \quad (5)$$

Let the static magnetic field sit in the Z direction i.e. $\mathbf{B}_0 = B_{0z} \hat{z}$ and note that $\mathbf{B}_1^j(\mathbf{r}, t) = \mathbf{B}_1^j(\mathbf{r}) f_j(t)$ and $\mathbf{E}^j(\mathbf{r}, t) = \mathbf{E}^j(\mathbf{r}) f_j'(t)$, (5) can be rewritten as (6)

$$AS^j = \left(\sigma (-B_{1z}^j) + \left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right) \cdot (E_y^j, -E_x^j) \right) B_{0z} f_j'(t) \quad j=1, \dots, N \quad (6)$$

where E_x^j and E_y^j are the x and y components of the induced electrical field vector $\mathbf{E}^j$, respectively. B_{1z}^j is the z component of $\mathbf{B}_1^j(\mathbf{r})$. Note here that this equation holds for every position inside the 3D sample volume and all acoustic sources have similar function in time i.e. $AS^j(\mathbf{r}, t) = AS^j(\mathbf{r}) f_j'(t)$. In addition, as shown in (6) the generated acoustic source in MAT-MI is related to both the conductivity distribution of the sample and its spatial gradient in XY planes as well. For numerical stability consideration, in the following we consider solving $\nabla \sigma = \left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right)$ first and then derive the conductivity $\sigma(\mathbf{r})$ itself.

Using matrix form (6) can be written as in (7)

$$\mathbf{U} \mathbf{x} = \mathbf{b} \quad (7)$$

where

$$\mathbf{U} = \begin{bmatrix} E_y^1 & -E_x^1 \\ \vdots & \vdots \\ E_y^N & -E_x^N \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \frac{\partial \sigma}{\partial x} \\ \frac{\partial \sigma}{\partial y} \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} \frac{AS^1(\mathbf{r})}{B_{0z}} + \sigma B_{1z}^1 \\ \vdots \\ \frac{AS^N(\mathbf{r})}{B_{0z}} + \sigma B_{1z}^N \end{bmatrix}$$

With N appropriately chosen coil setups, we can get the determinant of matrix $\mathbf{U}$ to be nonzero and obtain $\mathbf{x}$ using the regularized least square method

$$\mathbf{x} = (\mathbf{U}^T \mathbf{U} + \lambda \mathbf{I})^{-1} \mathbf{U}^T \mathbf{b} \quad (8)$$

where $\mathbf{U}^T$ is the transpose of $\mathbf{U}$, λ is a regularization parameter and $\mathbf{I}$ is a 2×2 identity matrix. Many methods can be used to determine the regularization parameter λ such as the L-curve method [26]. However, if the condition number of the matrix $\mathbf{U}$ is not large in the whole region of interest, we can select $\lambda = 0$ and (8) becomes the normal least square solution. Additionally, note here that the entries of matrix $\mathbf{U}$ are components of the induced electrical field, which depends on the unknown conductivity distribution $\sigma(\mathbf{r})$ and the vector $\mathbf{b}$ contains a term related to the conductivity distribution too. Thus, in order to calculate $\mathbf{x}$ and derive the conductivity $\sigma(\mathbf{r})$, an iterative algorithm is required, as we will discuss later in this paper.

In order to compute σ from $\nabla\sigma = (\frac{\partial\sigma}{\partial x}, \frac{\partial\sigma}{\partial y})$ in all the imaging slices, a two dimensional (2D) layer potential integration technique can be used as in (9) [27]

$$\sigma(\mathbf{r}) = - \int_S \nabla_{\mathbf{r}'} \Phi(\mathbf{r} - \mathbf{r}') \cdot \nabla\sigma(\mathbf{r}') d\mathbf{r}' + \int_{\partial S} \mathbf{n}_{\mathbf{r}'} \cdot \nabla_{\mathbf{r}'} \Phi(\mathbf{r} - \mathbf{r}') \sigma_{\partial S}(\mathbf{r}') dl_{\mathbf{r}'} \quad (9)$$

where $\Phi(\mathbf{r} - \mathbf{r}') = \frac{1}{2\pi} \log|\mathbf{r} - \mathbf{r}'|$ is the two dimensional Green's function of the Laplacian

operator and $\nabla_{\mathbf{r}'} \Phi(\mathbf{r} - \mathbf{r}') = -\frac{1}{2\pi} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^2}$. S denotes the imaging region of interest (ROI) in the imaging slice where $\nabla\sigma$ is obtained and ∂S denotes its boundary. $\sigma_{\partial S}$ is the conductivity value restricted at the boundary ∂S . This integration can be applied in a whole 3D volume slice by slice.

B. Image Reconstruction Algorithm

Here we describe the image reconstruction algorithm for the proposed multi-excitation MAT-MI approach. For $j = 1, \dots, N$, we apply different magnetic excitations on the sample through different coil setups and collect the corresponding pressure measurements on certain acoustic aperture. The conductivity value $\sigma_{\partial S}$ at the boundary of the imaging ROI is measured experimentally. In practice, this can be done by applying certain coupling material with known conductivity value on the sample surface, and letting ∂S reside in the area filled with this coupling material. Then the multi-excitation algorithm is as follows:

- Step 1** Calculate the acoustic source map $A\tilde{S}^j$ in the whole object volume using the time reversal back projection algorithm or EM algorithm.
- Step 2** Let $i = 0$ and assume an initial conductivity distribution σ_0 .
- Step 3** Solve the following differential equation with Neumann boundary condition in the whole conductive volume for $j = 1, \dots, N$.

$$\begin{cases} \nabla \cdot (\sigma_i (-\frac{\partial \Lambda^j}{\partial t} - \nabla \phi^j)) = 0 \\ \mathbf{J}^j \cdot \mathbf{n} = 0 \end{cases} \quad (10)$$

- Step 4** Compute $\mathbf{E}^j$ based on the solution from Step 3.
- Step 5** Compute σ_{i+1} using (8) and (9) on every imaging slice.
- Step 6** If the relative error between σ_{i+1} and σ_i is larger than the given tolerance ϵ ,

$$\text{i.e. } \frac{\|\sigma_{i+1} - \sigma_i\|_2}{\|\sigma_{i+1}\|_2} > \epsilon, \text{ replace } i \text{ by } i+1 \text{ and go to Step 3. Otherwise finish the procedure and use } \sigma_{i+1} \text{ as the solution.}$$

C. Modification with limited bandwidth measurements

One of the major technical limitations of the MAT-MI approach comes from limited bandwidth acoustic measurements. Generally, in order to achieve millimeter level spatial resolution, ultrasound transducers with around MHz central frequency are needed. Using these transducers, DC to very low frequency signal components are usually not available. This effect can be considered as a band pass filtering procedure in the MAT-MI forward problem. With this measurement data, denoting it $\tilde{p}_j$, we can only reconstruct part of the acoustic source distribution, denoting it $A\tilde{S}^j$. As shown in (6), the generated MAT-MI

acoustic source is related to both the conductivity distribution and its spatial gradient. However, if only limited bandwidth acoustic measurements are available, these two subtypes of sources can not be detected equally. Assuming the sample is piecewise homogenous, the gradient source (i.e. the acoustic source related to the conductivity gradient term $\nabla\sigma$, shown in the second term on the RHS of (6)) can be considered as a wide-band source. On the contrary, the conductivity source (i.e. the acoustic source related to the conductivity σ itself, shown in the first term on the RHS of (6)) is a narrow-band source whose central frequency depends on the object geometry. In addition, the gradient source is generally much larger than the conductivity source as shown in our previous computer simulation study [18] and experiment studies [17], [19]. In consequence, with limited bandwidth measurements, the acoustic source we can reconstruct will mainly be determined by the conductivity gradient term as in (11)

$$A\tilde{S}^j \approx ((\frac{\partial\sigma}{\partial x}, \frac{\partial\sigma}{\partial y}) \cdot (E_y^j, -E_x^j))B_{0z}f_j'(t) \quad j=1, \dots, N \quad (11)$$

The corresponding matrix form can be written as

$$\mathbf{U}\mathbf{x} \approx \tilde{\mathbf{b}} \quad (12)$$

where $\mathbf{U}$ and $\mathbf{x}$ take the same definitions as in (7) and

$$\tilde{\mathbf{b}} = \begin{bmatrix} \frac{A\tilde{S}^1(\mathbf{r})}{B_{0z}} \\ \vdots \\ \frac{A\tilde{S}^N(\mathbf{r})}{B_{0z}} \end{bmatrix}$$

Replacing vector $\mathbf{b}$ with vector $\tilde{\mathbf{b}}$, a similar reconstruction procedure can be applied to estimate the conductivity distribution of the object. However, as we will show in the simulation and experiment studies, with limited bandwidth acoustic measurements, we are not able to quantitatively reconstruct the absolute conductivity values. What will be visible in the reconstructed MAT-MI image is the relative conductivity contrast.

III. METHODS

In order to verify the proposed multi-excitation MAT-MI method, we have conducted computer simulation and phantom experiment studies. In these studies, the conductive samples were homogeneous in the Z direction. The static and dynamic magnetic fields were also approximately uniform in the Z direction. With these setups and an appropriate setup for acoustic measurements, we can simplify the corresponding 3D MAT-MI problems to 2D problems. The corresponding 2D system model and reconstruction algorithms can be easily derived from their 3D counterparts.

A. Computer Simulation Method

Fig. 1 shows the diagram of the multi-excitation MAT-MI system setup used in our computer simulation study. The static magnetic field is assumed to be uniform in the imaging area and pointing in Z direction. The flux density B_{0z} is set to 1 Tesla. In this simulation study we considered a conductive sample that is homogeneous in Z direction, i.e. the conductivity $\sigma(\mathbf{r})$ is independent of z . The conductive sample is placed around the center

of the coordinate system. Three groups of coils are selected to sequentially send three different magnetic excitations, i.e. $N = 3$. As shown in Fig. 1(a), coil group A contains two figure eight coils located in planes of $Z = 5$ cm and $Z = -5$ cm, respectively. Each figure eight coil pair is arranged along the X axis and every coil in the group has a radius of 10 cm. The distance between the two coils in the figure eight coil pair is 4 cm. In addition, coils A-1 and A-3 are placed in the manner of a Helmholtz coil pair and similar arrangement is applied to coils A-2 and A-4. Coil group B is similar to group A, but is arranged along the Y axis. Coil group C contains a pair of Helmholtz coils with 10 cm coil radius and its axis is the Z axis. Fig. 1(b) shows the top view of the system and the directions of the stimulating current flow in each coil. We assume each coil has one turn and has the same current flow amplitude. For simplicity we set $f_j'(t) = \delta(t)$ and the maximum current changing rate in every coil is set to be $1e8$ A/s. With this current changing rate, the stimulating magnetic field B_{1z} generated by excitation Group C has a changing rate of 900 T/s at the coordinate center. Ultrasound transducers are assumed to be located on a circular orbit with radius of 20 cm around the sample in the $Z = 0$ plane.

With this setup, the MAT-MI system can then be approximated as a two dimensional system and the imaging slice at $Z = 0$ plane were selected for us to do the forward and inverse calculation. As shown in Fig 1. (c), the 2D object model is divided into three regions. The insulating region models the de-ionized water area surrounding the object in experiment. The coupling region is a conductive region with known conductivity value. The imaging region contains the conductive object and is our imaging region of interest, where the domain S is defined.

In order to obtain the forward solution, the whole object model was discretized into a finite element mesh. A 2D forward solver was developed using the Matlab PDE Toolbox. Triangular three-node linear element was used in this forward solver. The magnetic vector potential produced by each current carrying coil in one excitation group was calculated in each element in terms of elliptic integrals [28], [23] and was added together to obtain the total magnetic vector potential produced by this excitation group. The magnetic induction problem as shown in (10) was then solved in the non-insulating regions. The solution of electrical potential ϕ^j was obtained on each element node and the corresponding electrical field $\mathbf{E}^j$ and current density $\mathbf{J}^j$ were calculated at the center of each element. To calculate the acoustic source $AS^j = (\nabla \times \mathbf{J}^j) \cdot \mathbf{B}_0$, the current density value was interpolated to each element node and the acoustic source was then calculated at the center of each element. With the simulated acoustic source distribution, the acoustic pressure measurements p_j can then be calculated using the 2D version of (4). In addition, in order to simulate the limited bandwidth measurements $\tilde{p}_j$, we used an impulse response function $IR(t)$ that has a central frequency at 500KHz and around 100% bandwidth to convolve with the pressure signal p_j , i.e. $\tilde{p}_j = p_j \otimes IR(t)$.

In applying the proposed iterative algorithm, the initial conductivity distribution σ_0 was set to be uniform with the conductivity value to be σ_s , i.e. the conductivity value of the media in the coupling region. (8) and (9) were calculated only in the imaging region. The tolerance value ϵ was set to be 0.001. In the process of choosing the regularization parameter as in (8), we set a condition number threshold for matrix $\mathbf{U}$. Basically, if the condition numbers of $\mathbf{U}$ at all the pixels in the imaging region are smaller than 50, we set $\lambda = 0$ for all the pixels, otherwise we use the L-curve method to determine λ . Actually, it is observed that the induced electric fields under different magnetic excitations would become parallel to each other mainly near the boundary between the insulating region and non-insulating region. In our simulation and experiment studies the condition numbers of $\mathbf{U}$ at all the pixels in the imaging ROI were under the threshold. In the simulation using limited bandwidth data, a least-square deconvolution [29] filter was applied before doing the image reconstruction.

The performance of the proposed algorithm was evaluated using simulated pressure data under different noise level. The SNR used here was defined as the ratio of the maximum pressure signal amplitude over the standard deviation of the added Gaussian random noise. In addition, a numerical phantom with objects of different sizes was used to test the spatial resolution that can be achieved using the proposed algorithm.

B. Experimental Method

The experiment system setup is similar to the setup used in our simulation study only with different parameters. The static magnetic field is generated from two permanent magnets and the field strength was measured to be 0.26 Tesla (Gaussmeter, Alpha Lab) at the coordinate center where the object is located. All the coils have radius of 4.45 cm. The Helmholtz coil pair of excitation group C has 3 turns in each coil and the figure eight coil pair of excitation group A and B has 2 turns in each coil. The distance between the upper coils and lower coils in each group is around 5 cm. The coils were driven by a home made stimulator, with $1\mu s$ pulse width. The dynamic magnetic excitation was measured by a sensing coil with radius of 1.5cm connected to an oscilloscope. The estimated maximum current changing rate in the Helmholtz coil pair of excitation group C is $1.4e8$ A/s, which corresponds to a magnetic field changing rate of $7e3$ T/s at the coordinate center. Considering the $1\mu s$ pulse width, the maximum dynamic magnetic field strength B_{1z} is around 0.007 T at the coordinate center. The maximum current changing rates of the other two excitation groups are at similar levels. A 500 KHz flat ultrasound transducer (Panametrics V301) with around 60% bandwidth was used in our MAT-MI experiment system. The transducer was mounted to a scanning frame and can scan around the sample with 330 degree view angle. The scanning step used in our experiment study was 2.5 degree. The scanning radius, i.e. the distance between the transducer and the scanning center is 22.8 cm. A 3 cm thick sample is submerged in 3 cm thick deionized water media for acoustic coupling. During the ultrasound scanning the magnetic excitation coil and the sample are both fixed in their positions to ensure reliable acoustic measurements. Acoustic data collections are synchronized with the magnetic excitation. This setup makes the corresponding MAT-MI problem valid to be simplified to a 2D problem both electrically and acoustically. The acoustic signal collected using the transducer was fed into preamplifiers with 90dB gain and digitized by a 5MHz data acquisition card. Signal averaging was used to increase SNR.

With the collected ultrasound data under each of the three excitation conditions, the corresponding acoustic source maps were firstly reconstructed using the time reversal back projection algorithm or EM algorithm. In order to build the FEM mesh for the proposed multi-excitation reconstruction algorithm, one of the reconstructed acoustic source images was post-processed and conductive regions including the coupling region and the imaging region were extracted and discretized into finite element meshes.

For comparison, we also performed ultrasound pulse echo imaging using the same transducer and a pulser-receiver (Panametrics 5077PR). The RF pulse echo data was collected at the same respective scanning positions MAT-MI data was collected. A simple back projection algorithm was used to form the pulse echo image.

IV. RESULTS AND DISCUSSION

A. Computer Simulation Results

To validate the proposed multi-excitation MAT-MI method, we first did a computer simulation using unlimited bandwidth data and under noise free conditions. The result is shown in Fig. 2. Fig. 2(a) shows the target conductivity distribution and (b) shows the reconstructed conductivity image. Fig. 2(c) shows the profile comparison at $y = 0.02$ m. In

this simulation, the whole model area is a circular region with 0.1m radius. The non-insulating area is a circular region with 0.085 m radius containing a circular imaging region that has 0.075 m radius. The conductivity values of the object are set to be in the same range of biological tissue conductivity. The finite element mesh of this model has 17,109 nodes and 33,856 elements. The iterative reconstruction algorithm took 8 steps to converge to its final solution.

As shown in Fig. 2, the conductivity distribution is accurately reconstructed. The correlation coefficient (CC) between the target image and the reconstructed image is 99.5% and the relative error (RE) is 6.5%. Here relative error is defined as in (13)

$$RE = \sqrt{\sum_{n=1}^N (\sigma_n - \sigma_{r,n})^2 / \sum_{n=1}^N (\sigma_n)^2} \quad (13)$$

where σ_n is the target conductivity value of the n th element and $\sigma_{r,n}$ is the corresponding reconstructed conductivity value. N is the total number of pixels or elements in the ROI. This result shows that a much better performance can be obtained using the proposed multi-excitation algorithm as compared to the results obtained using other previous algorithms [18], [22].

We have also conducted a simulation study to test the performance of the modified reconstruction algorithm when only limited bandwidth measurement data is available. Fig. 3(a) shows the simulated impulse response function $IR(t)$ that centers at 500KHz. Using the simulated limited bandwidth pressure data $\tilde{p}_j$ and the modified multi-excitation reconstruction algorithm, we reconstructed the corresponding conductivity image under different noise levels as shown in Fig. 3(b), (c) and (d). The SNR of the simulated pressure data used to calculate these images are 1000, 100 and 10, respectively. The CCs between the target conductivity image and the reconstructed images in the imaging ROI under these noise conditions are 76.7%, 72.7% and 23.1%, respectively. The corresponding REs are 32.6%, 32.6% and 34.2%. Note the different color scales used in these images. As compared with the target conductivity distribution shown in Fig. 2(a), only the relative conductivity contrast can be seen in these images. Quantitative conductivity values of different regions are not accurately reconstructed. In addition, some artifacts are seen at those conductivity boundaries and at the centers of some conductive pieces. Furthermore, it is shown that the error that comes from the bandwidth limitation is much larger than that comes from the added random noise. Even when the measurement SNR is 1000 the RE between the reconstructed image and the target image is still 32.6% and lower SNR values do not increase the relative errors significantly. With these limitations, however, the relative conductivity contrast shown in these images still has values in certain potential clinical applications such as cancer detection.

To test the resolution that can be obtained by using the proposed method, a computer simulation was conducted using a numerical phantom containing objects of different sizes as shown in Fig. 4(a). The conductivity value in the background non-insulating region is set to be 0.2 S/m. Circular objects with radii to be 1mm, 3mm, 5mm, 8mm, 10mm and 15mm are presented in the imaging region. For each object size, the conductivity values are either 0.6 S/m or 0.04 S/m, creating different conductivity contrast. The simulation was done using bandwidth limited data with SNR=30 and the conductivity image was reconstructed using the modified multi-excitation algorithm. As shown in Fig. 4(b), the overall relative conductivity contrast pattern is well reconstructed, with some artifact presented at the conductivity boundaries and at the centers of some internal conductive objects with large

sizes. The small objects with 1mm radius can be clearly seen in the reconstructed conductivity image.

B. Experimental Results

Using the developed multi-excitation MAT-MI experiment system, we have conducted phantom experiments to demonstrate the benefits of the proposed method. Results from an example gel phantom experiment are shown in Fig. 5. A photo of the gel phantom is shown in Fig. 5(a). The phantom contains a background region made from 5% salinity gel. Two cylindrical columns with diameter of 12 mm are embedded in the gel. Marked by the red and blue circles in the photo are two high conductive regions filled with 20% and 10% salinity gels, respectively. These two regions have diameter of 8 mm. The two annular areas sitting between the two high conductive regions and the background are made from beef suet, which has low conductivity value as fat tissue.

Fig. 5(b) shows the ultrasound pulse-echo image we obtained from the gel phantom. This image indicates the acoustic impedance contrast of the phantom and boundaries between structures with different acoustic impedances can be seen in the image. As shown in this image, the echoes at the boundary between the 20% salinity gel and the fat layer are much stronger than echoes at other boundaries, indicating a larger acoustic impedance change here. This is consistent with the fact that the 20% salinity gel is much softer than the 10% and 5% salinity gels. However, the overall contrast of the pulse echo image is not strong and it would be challenging to differentiate tissue types from this type of image.

Using the multi-excitation MAT-MI system, we applied three different groups of magnetic excitations on the gel phantom and the corresponding acoustic source images were reconstructed as shown in Fig. 5(c), (d) and (e). Spatial resolution of 3 mm was achieved in these images. As expected, with limited bandwidth acoustic measurements the reconstructed acoustic sources are mainly distributed around conductivity boundaries i.e. where $\|\nabla\sigma\|$ is large. In addition, higher contrast can be seen in these images as compared to the pulse-echo image. This is mainly because of the stronger conductivity contrast existed in the gel phantom. However, as the reconstructed acoustic source maps emphasize conductivity boundaries, it is hard to tell which part of the object has high conductivity values. Using the modified multi-excitation algorithm, the conductivity image of the gel phantom was reconstructed as shown in Fig. 5(f). From this image, we can clearly see the relative conductivity contrast, while the fat layer shows lower conductivity than the surrounding background, the 10% salinity gel shows higher conductivity value and the 20% salinity gel shows the highest conductivity. A conductivity profile at $y = 0.01$ m is given in Fig. 5(g) showing the comparison between the target and reconstructed conductivity values. The geometry of the target distribution was estimated from the pulse-echo image and the conductivity values of each piece were estimated from corresponding conductivity measurements. As compared with the pulse echo image and the MAT-MI acoustic source images, the reconstructed conductivity image using the modified multi-excitation algorithm gives a more informative conductivity contrast map and enables us to better differentiate the material types in the phantom. In addition, we can also see similar artifacts at the conductivity boundaries and object centers in consistent with our computer simulation results.

For better comparison, we have also conducted a computer simulation using a conductivity distribution similar to that of the experiment phantom. The same excitation parameters as those used in the experiment were also employed. The simulated acoustic sources corresponding to the three magnetic excitations, i.e. $A\vec{S} = (\nabla \times \vec{J}) \cdot \vec{B}_0$ for $j = 1, 2, 3$, are shown in Fig. 6(a), (b) and (c), respectively. The target conductivity distribution is shown in Fig. 6(d). It is shown that the reconstructed acoustic source patterns in Fig. 5(c), (d) and (e)

are in general similar to those simulated source patterns shown in Fig. 6(a), (b) and (c), respectively. However, the reconstructed acoustic sources are lack of those low frequency components observed in the simulated acoustic sources. As we mentioned in the theory, this difference mainly comes from the bandwidth limitation in acoustic measurements. In spite of this, the reconstructed conductivity image shown in Fig. 5(f) represents well the relative conductivity contrast of the target distribution presented in Fig. 6(d).

C. Discussion

The MAT-MI imaging approach was previously proposed to do non-invasive conductivity imaging with high spatial resolution. However, using previously developed reconstruction algorithms, we are not able to reconstruct the conductivity contrast accurately and only conductivity boundaries at the conductivity heterogeneity can be reconstructed in experiment. In this study, we demonstrate that using the proposed multi-excitation MAT-MI method, we can reconstruct the internal conductivity contrast of the object with good accuracy if unlimited bandwidth acoustic measurement data is available. In addition, in the MAT-MI experiments, one of the major technical limitations comes from limited bandwidth acoustic measurements. As shown in our computer simulation and experiment studies, using the modified multi-excitation MAT-MI algorithm, we can reconstruct the relative conductivity contrast fairly well. In consequence, this would bring us the potential ability to better identify different tissue types based on their conductivity contrast and would significantly benefit potential MAT-MI applications such as cancer detection.

In MAT-MI, reliable acoustic measurements are essential for reliable conductivity reconstruction. Generally it requires a stable excitation and scanning frame and good synchronization between magnetic excitation and acoustic data collection. Ultrasound transducers with different sensitivity and focusing patterns may need different calibrations in practice. A good EM shielding of the transducer is also needed to reduce signal contamination that comes from the excitation coil. This can also be achieved by holding the transducer at certain distance from the imaging ROI and let the EM interference fade out before the real MAT-MI signal arrives at the transducer.

For the excitation number and coil configurations of the proposed multi-excitation MAT-MI method, we chose $N=3$ in this study with one Helmholtz coil setup and two figure eight coil setups arranged at different directions. We have tested in simulation that using more figure eight coil setups arranged at different directions does not significantly decrease the condition number of the system inverse matrix and does not speed up the convergence in the iteration. Of course, using more excitation setups would add an average effect to help handle the measurement noise. Different coil configurations that generate different excitation patterns may need to be further explored and optimized in the future research.

In this study, as a premier verification of the proposed multi-excitation MAT-MI method, the computer simulations and experiments were conducted on a simplified two dimensional space. This 2D simplification is valid under conditions that the conductivity distribution of the object is homogeneous in the Z direction, the magnetic fields are approximately uniform in the Z direction and the sample is placed between two plates, i.e. air and tank bottom, as in our experiment setup. However, note that the proposed reconstruction algorithm itself can also be applied to 3D MAT-MI imaging. For a conductive object with 3D structure, we need to acquire acoustic measurements on a 3D acoustic aperture first and reconstruct the MAT-MI acoustic source map in the whole 3D volume. The multi-excitation algorithm can then be applied to reconstruct the conductivity distribution in every imaging slice.

The acoustic homogeneous assumption used in the theoretical derivation would still limit the application of the proposed MAT-MI approach to soft tissue imaging. As the acoustic

heterogeneity in soft tissue is less than 10%, its effect can be considered negligible in MAT-MI [16], [22].

From the bioimpedance imaging perspective, as the electrical properties of biological tissue are frequency dependent, the conductivity property obtained from the MAT-MI imaging approach only indicates the tissue conductivity value at a certain frequency range determined by the central frequency of the magnetic excitation and acoustic measurements. Thus, the reconstructed MAT-MI image obtained in the present experiment study indicates conductivity properties of the phantom at around 500 KHz.

In summary, we have developed a multi-excitation MAT-MI imaging approach and the corresponding reconstruction algorithms. Computer simulation and phantom experiment studies have been conducted to demonstrate the promise of the proposed method in reconstructing the conductivity contrast using ultrasound measurements.

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FIG. 1.

System diagram of the multi-excitation MAT-MI system used in the computer simulation study. Each coil is labeled with its group number (A, B or C) and coil number in its group. Coils belonging to the same group are used together to generate a specific excitation pattern. (a) is the 3D view of the system and (b) is the top view showing the directions of stimulating current flow in each coil. (c) shows different regions of the object model used in the forward solver and reconstruction algorithm.

FIG. 2.

(a) Target conductivity image. (b) Reconstructed conductivity image using unlimited bandwidth data and under noise free conditions. (c) Conductivity profile along $y = 0.02$ m showing the comparison between the target and reconstructed conductivity distribution.

FIG. 3.

(a) Simulated impulse response function $IR(t)$. (b), (c) and (d) are reconstructed conductivity images using limited bandwidth data with SNR to be 1000, 100 and 10, respectively.

FIG. 4.

(a) Target conductivity image. (b) Reconstructed conductivity image using limited bandwidth data and 30 SNR. (c) Conductivity profile along $y = -0.025$ m showing the comparison between the target and reconstructed conductivity distribution.

FIG. 5.

(a) Photo of a gel phantom used in the experiment study. The red circle marks a region containing 20% salinity gel and the blue circle marks a region containing 10% salinity gel. (b) An ultrasound pulse-echo image of the phantom showing the acoustic impedance contrast. (c), (d) and (e) are reconstructed MAT-MI acoustic source images under different magnetic excitations. (f) Reconstructed conductivity image showing the relative conductivity contrast. (g) Conductivity profile along $y = 0.01$ m showing the comparison between the target and reconstructed conductivity values.

FIG. 6.

(a), (b) and (c) are computer simulated acoustic source distributions corresponding to three different magnetic excitations. These excitations have similar setups and parameters to those used in the phantom experiment. The target conductivity distribution is shown in (d).