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Statistical Three-Dimensional Positioning Algorithm for High-Resolution dMiCE PET Detector

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Abstract

Our laboratory is developing a high-resolution PET detector capable of providing depth-of-interaction information (dMiCE) by tailoring the light sharing between two adjacent detector elements. Each detector element in the prototype system has a 2×2 mm² cross section and is directly coupled to a micro-pixel avalanche photodiode (MAPD). In this setup the distribution of the ratio of light shared between two adjacent detector elements can be expressed as a function of the depth of interaction. The three-dimensional points of interaction of a coincidence pair of photons within the detector module is estimated by numerical calculation of an expectation of the points of interaction conditioned on the signals measured by the MAPDs (Bayesian estimate). This conditional expectation is computed from estimates of the probability density function of the light collection process and a model of the kinetics of photon interactions in the detector module. Our algorithm is capable of handling coincidences where each photon interacts any number of times within the detector module before being completely absorbed or escaping. In the case of multiple interactions our algorithm estimates the position of the first interaction for each of the coincidence photons.

I. Introduction

Our laboratory is developing a high-resolution PET detector capable of providing depth-of-interaction information (dMiCE [1], [2]) by tailoring the light sharing between two adjacent detector elements. Each detector element in the prototype system has a 2×2 mm² cross section and is directly coupled to a micro-pixel avalanche photodiode (MAPD- Zecotek Photonics). See figure 1. In this setup the distribution of the ratio of light shared between two adjacent detector elements can be expressed as a function of the depth of interaction. The three-dimensional points of interaction of a coincidence pair of photons within the detector system is estimated by numerical calculation of an expectation of the points of

interaction conditioned on the signals measured by the MAPDs (Bayesian estimation). This conditional expectation is computed from estimates of the probability density function of the light collection process and a model of the kinetics of photon interactions in the detector module. Our algorithm is capable of handling coincidences where each photon interacts any number of times within the detector module before being completely absorbed or escaping. In the case of multiple interactions our algorithm estimates the position of the first interaction for each of the coincidence photons.

The motivation for estimating the depth of interaction (DOI) is to avoid the well-known parallax error. Several methods for DOI estimation have been evaluated for PET detector modules [1], [2], [3], [4], [5], [6], [7], [8], but to our knowledge this use of a Bayesian estimator for the estimation of a line of response from coincidence events is unique.

By providing depth-of-interaction information one can more accurately determine the line of response of a positron annihilation. Depth-of-interaction information can also aid in decoding coincidence data where the photons interact multiple times in the detector module. For a beam of photons normally incident on the center of a detector module using a block of LFS of size $200 \times 200 \times 20 \text{ mm}^3$, approximately 44% of 511 keV photons will interact once, 36% interact twice, 15% interact thrice, and the remaining 5% interaction four or more times in the detector module (either deposits all energy or escapes the module). Thus coincidences where each photon only interacts once in the detector module accounts for approximately 19% of the total coincidences.

If we employ Anger logic (energy-weighted centroid), then multiple interactions would lead to inaccuracies in the estimate a line of response. See figure 2.

II. Statistical Positioning Algorithm

We develop a statistical algorithm to estimate the three-dimensional positions of the first interactions of a coincidence pair. For this task we develop probability density functions (PDFs) to model the photon process and the detection process. All PDFs will be denoted by $f(\cdot)$. We start with the photon process.

A. The Photon Process

In this section we describe our probabilistic model of the process of photon interactions in matter. This is a well-known, but complicated stochastic process. For purposes of speed of computation we use a simplified model of this process. Our model considers the energy-dependent distribution of photoelectric absorption and Compton scatter. Angular distributions of photons that undergo Compton scatter are modeled by the Klein-Nishina formula [9]. We do not consider positron range, non-collinearity of positron annihilation, photon polarization, or coherent (Rayleigh) scatter.

We start by defining a few random variables. Let X_k and Y_k denote the three-dimensional interaction positions of a photon coincidence pair. The random variables N_x and N_y denote the total number of interactions of each photon in the detector module before the photon either becomes completely absorbed or escapes. Let Γ_k denote the energy (keV) of the photon after the k th interaction and $\Delta_k = \Gamma_{k-1} - \Gamma_k$ denote the energy (keV) deposited by the k th interaction. The trajectory of the photon after the k th interaction will be denoted by the random variable R_k . Note that random variables will always be denoted by capital letters and realizations of random variables will be denoted by lower case letters. The random variable triplets (X_0, Γ_0, R_0) represent the position, energy, and trajectory of the photon as it first enters the detector module.

Let $\mu_\tau(\gamma)$ be the photo-electric absorption coefficient and $\mu_\sigma(\gamma)$ be the Compton scatter attenuation coefficient of the scintillation material. We also let $\mu(\gamma) = \mu_\tau(\gamma) + \mu_\sigma(\gamma)$. The probability that a photon of energy γ_{k-1} will have energy γ_k after undergoing Compton scatter is given by

$$s(\gamma_k|\gamma_{k-1}) = \frac{1 + \left(\frac{\gamma_k}{\gamma_{k-1}}\right)^2 + \left(\frac{\gamma_k}{\gamma_{k-1}}\right) \left[\left(\frac{511}{\gamma_{k-1}} - \frac{511}{\gamma_k} + 1\right)^2 - 1 \right]}{\gamma_k \gamma_{k-1}} \\ \times C(\gamma_{k-1}) \mathbf{1}_{\left\{(\gamma_{k-1}, \gamma_k): \frac{511\gamma_{k-1}}{511+2\gamma_{k-1}} \leq \gamma_k \leq \gamma_{k-1}\right\}}(\gamma_{k-1}, \gamma_k)$$

The angle of scatter is then given by

$$\theta(\gamma_k, \gamma_{k-1}) = \cos^{-1} \left(\frac{511}{\gamma_{k-1}} - \frac{511}{\gamma_k} + 1 \right).$$

This is known as the Klein-Nishina formula. Note that $C(\gamma_{k-1})$ is a normalizing factor such that

$$\int_{\mathbb{R}} s(\gamma_k|\gamma_{k-1}) d\gamma_k = 1.$$

Thus the probability density function of the photon process is given by

$$f(\gamma_{x,k}, \mathbf{x}_k, \dots, \mathbf{x}_1, \gamma_{y,l}, \mathbf{y}_l, \dots, \mathbf{y}_1) \\ = [\delta(\gamma_{x,k}) \mu_\tau(\gamma_{x,k-1}) + s(\gamma_{x,k}|\gamma_{x,k-1}) \mu_\sigma(\gamma_{x,k-1})] \\ \times e^{-\mu(\gamma_{x,k-1}) \|\mathbf{x}_{k-1} - \mathbf{x}_k\|} \\ \times \frac{1}{(2\pi)^{k-1}} \prod_{i=1}^{k-1} s(\gamma_{x,i}|\gamma_{x,i-1}) \mu_\sigma(\gamma_{x,i-1}) e^{-\mu(\gamma_{x,i-1}) \|\mathbf{x}_{i-1} - \mathbf{x}_i\|} \\ \times [\delta(\gamma_{y,l}) \mu_\tau(\gamma_{y,l-1}) + s(\gamma_{y,l}|\gamma_{y,l-1}) \mu_\sigma(\gamma_{y,l-1})] \\ \times e^{-\mu(\gamma_{y,l-1}) \|\mathbf{y}_{l-1} - \mathbf{y}_l\|} \\ \times \frac{1}{(2\pi)^{l-1}} \prod_{j=1}^{l-1} s(\gamma_{y,j}|\gamma_{y,j-1}) \mu_\sigma(\gamma_{y,j-1}) e^{-\mu(\gamma_{y,j-1}) \|\mathbf{y}_{j-1} - \mathbf{y}_j\|},$$

where $\|\cdot\|$ denotes the euclidean norm.

B. The Detection Process

Now we develop a stochastic model for the signals measured from a light-sharing crystal pair. Let $\mathbf{M} = (M_a, M_b)^T$ be a random variable for the signals measured by the dMiCE crystal pair. We model this signal pair as a joint Gaussian given by

$$\mathbf{M} = \begin{cases} (M_{\text{primary}}, M_{\text{secondary}})^T, & \text{if } M_a \text{ is primary signal} \\ (M_{\text{secondary}}, M_{\text{primary}})^T, & \text{otherwise.} \end{cases}$$

Let d is the energy resolution, r is the slope of light response, z is the depth of interaction, and δ is the energy deposited by the photon that produces the signal pair. See figure 3. The the mean and variance of M_{primary} are given by $E[M_{\text{primary}}] = (rz + b)\delta$, $\text{Var}[M_{\text{primary}}] =$

$(dE[M_{\text{primary}}])^2$ and the mean and variance of $M_{\text{secondary}}$ are given by $E[M_{\text{secondary}}] = (1-rz-b)\delta$, $\text{Var}[M_{\text{secondary}}] = (dE[M_{\text{secondary}}])^2$. Note that the parameters of the distribution of $\mathbf{M}$ depends on depth of interaction and the energy deposited.

C. Bayes Estimation of Line of Response

Now we are ready to outline our statistical positioning algorithm. The algorithm is designed to estimate the first point of interaction of each photon in a coincidence pair. Since we do not know in which order the signal pairs were produced, we must first estimate the order of the signal pairs. The order of interaction is estimated by

$$(\mathbf{m}_{x1}, \dots, \mathbf{m}_{xk}, \mathbf{m}_{y1}, \dots, \mathbf{m}_{yl}) \equiv \text{argmax} f(\mathbf{m}_{xi_1}, \dots, \mathbf{m}_{xi_k}, \mathbf{m}_{yj_1}, \dots, \mathbf{m}_{yj_l}),$$

where the max is taken over all permutations of the signal pairs.

For convenience, we let

$$\mathbf{x}_{-} \equiv (\mathbf{x}_k, \dots, \mathbf{x}_1)$$

$$\mathbf{y}_{-} \equiv (\mathbf{y}_l, \dots, \mathbf{y}_1)$$

$$\mathbf{m}_{-x} \equiv (\mathbf{m}_{xk}, \dots, \mathbf{m}_{x1})$$

$$\mathbf{m}_{-y} \equiv (\mathbf{m}_{yl}, \dots, \mathbf{m}_{y1}).$$

We estimate the points of first interaction of a coincidence pair by Bayesian estimation given by

$$\begin{bmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{y}}_1 \end{bmatrix} \equiv E \left[(\mathbf{X}_1, \mathbf{Y}_1)^T | \mathbf{m}_{-x}, \mathbf{m}_{-y} \right] = \frac{\int \dots \int \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \end{bmatrix} f(\gamma_{xk}, \mathbf{x}_{-}, \gamma_{yl}, \mathbf{y}_{-} | \mathbf{m}_{-x}, \mathbf{m}_{-y}) d\mathbf{x} d\gamma_{xk} d\mathbf{y} d\gamma_{yl}}{\int \dots \int f(\gamma_{xk}, \mathbf{x}_{-}, \gamma_{yl}, \mathbf{y}_{-} | \mathbf{m}_{-x}, \mathbf{m}_{-y}) d\mathbf{x} d\gamma_{xk} d\mathbf{y} d\gamma_{yl}},$$

where

$$\begin{aligned} & f(\gamma_{xk}, \mathbf{x}_{-}, \gamma_{yl}, \mathbf{y}_{-} | \mathbf{m}_{-x}, \mathbf{m}_{-y}) \\ &= \frac{f(\gamma_{xk}, \mathbf{x}_{-}, \gamma_{yl}, \mathbf{y}_{-}) \prod_{i=1}^k f(\mathbf{m}_{xi} | \mathbf{x}_i, \delta_{xi}) \prod_{j=1}^l f(\mathbf{m}_{yj} | \mathbf{y}_j, \delta_{yj})}{f(\mathbf{m}_{-x}, \mathbf{m}_{-y})} \propto f(\gamma_{xk}, \mathbf{x}_{-}, \gamma_{yl}, \mathbf{y}_{-}) \prod_{i=1}^k f(\mathbf{m}_{xi} | \mathbf{x}_i, \delta_{xi}) \prod_{j=1}^l f(\mathbf{m}_{yj} | \mathbf{y}_j, \delta_{yj}). \end{aligned}$$

Note that

$$f\left(\begin{smallmatrix} \mathbf{m} \\ -x \quad -y \end{smallmatrix}\right) = \int \cdots \int f\left(\gamma_{xk}, \mathbf{x}, \gamma_{yl}, \mathbf{y}\right) \prod_{i=1}^k f\left(\mathbf{m}_{xi} | \mathbf{x}_i, \delta_{xi}\right) \times \prod_{j=1}^l f\left(\mathbf{m}_{yj} | \mathbf{y}_j, \delta_{yj}\right) d\mathbf{x} d\gamma_{xk} d\mathbf{y} d\gamma_{yl}.$$

Thus we see that all of the above calculations can be done with knowledge of the photon process and detection process PDFs outlined above.

III. Numerical Experiments

We tested our algorithm with data simulated from the PET system outlined in figure 1. Photon interactions within the detector modules were simulated with our Monte Carlo-based INTERACT software [10]. The output of this software was fed through a program that simulated the noisy detector signals.

First we tested the accuracy of our algorithm for different values of energy resolution

$\left(d = \frac{0.15}{2\sqrt{2\ln 2}}, \frac{0.2}{2\sqrt{2\ln 2}}\right)$, slope of light response $\left(r = \frac{0.1}{H}, \frac{0.2}{H}, \frac{0.3}{H}, \frac{0.4}{H}\right)$, and LOR angle (0° , 10° , 20° , 30° , 40° , 50° , 60°). For each experiment we simulated one thousand coincidence events from a point source at the center of the scanner. The LOR accuracy was measured by computing the distance from the point source to the estimated LOR, i.e.,

$$\|\widehat{\mathbf{x}}_1 - \langle \widehat{\mathbf{x}}_1 - \widehat{\mathbf{y}}_1, \widehat{\mathbf{x}}_1 \rangle (\widehat{\mathbf{x}}_1 - \widehat{\mathbf{y}}_1)\|,$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product. The LOR error was then calculated by a sample mean of the above calculation over estimations of all one thousand events. The coincidence events were not filtered by total energy deposited in the detection system. Results are shown in figure 4.

We also evaluated the performance of our LOR estimator with focal plane tomography. Ten thousand coincidence pairs were simulated with an isotropic distribution. For each coincidence pair, the LOR was estimated and backprojected onto a plane parallel to the face of the detector modules and through the point source located at the center of the PET system. The (normalized) distribution of the backprojected LORs is displayed in figure 5. Table I lists the first and second order statistics of these distributions.

IV. Conclusion and Discussion

In this work we have developed a three-dimensional statistical positioning algorithm for a high-resolution dMiCE PET detector. Initial simulated results show that this algorithm performs very well under a variety of parameters. The dMiCE crystal pairs are currently under evaluation.

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Fig 1.
 (left) Simulated PET system utilized for initial testing of the DOI reconstruction algorithm.
 (right) Diagram showing dMiCE light sharing crystal pair with triangular-shaped light reflector. Individual crystals are of size $2 \times 2 \times 20 \text{ mm}^3$.

Fig 2.
Anger logic leads to LOR inaccuracies.

Fig 3.

Model for the mean light response of dMiCE crystal pair as a function of the depth of interaction.

Fig 4.

Plot of distance of estimated line of response (LOR) to the point source where the LORs were estimated with the Anger and Bayes estimates. Red lines indicate results of simulations with 15% energy resolution and blue lines indicate results of simulations with 20% energy

resolution. (left) LOR error with slope of light response equal to $\frac{0.3}{H}$. (right) LOR error with LOR angle equal to 50° .

Fig 5.

Focal plane tomography results. Estimated LORs are back-projected onto a plane containing the point source and parallel to the detector modules. The plots show the resulting 2D

distributions with slope of light response set to $\frac{0.3}{H}$ and 15% energy resolution. (left) Bayes Estimate. (right) Anger Estimate.

TABLE I
Focal Plane Tomography Statistics

	Bayes Estimate	Anger Estimate
Bias	(0.0008, -0.0018)	(-0.0078, -0.0326)
Covariance Matrix	$\begin{bmatrix} 1.5509 & 0.0002 \\ 0.0002 & 1.6279 \end{bmatrix}$	$\begin{bmatrix} 6.5640 & 0.0484 \\ 0.0484 & 6.8706 \end{bmatrix}$