Three-dimensional multiexcitation magnetoacoustic tomography with magnetic induction

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Magnetoacoustic tomography with magnetic induction (MAT-MI) is a hybrid imaging modality proposed to image electrical conductivity contrast of biological tissue with high spatial resolution. This modality combines magnetic excitations with ultrasound detection through the Lorentz force based coupling mechanism. However, previous studies have shown that MAT-MI method with single type of magnetic excitation can only reconstruct the conductivity boundaries of a sample. In order to achieve more complete conductivity contrast reconstruction, we proposed a multiexcitation MAT-MI approach. In this approach, multiple magnetic excitations using different coil configurations are applied to the object sequentially and ultrasonic signals corresponding to each excitation are collected for conductivity image reconstruction. In this study, we validate the new multiexcitation MAT-MI method for three-dimensional (3D) conductivity imaging through both computer simulations and phantom experiments. 3D volume data are obtained by utilizing acoustic focusing and cylindrical scanning under each magnetic excitation. It is shown in our simulation and experiment results that with a common ultrasound probe that has limited bandwidth we are able to correctly reconstruct the 3D relative conductivity contrast of the imaging object. As compared to those conductivity boundary images generated by previous single-excitation MAT-MI, the new multiexcitation MAT-MI method provides more complete conductivity contrast reconstruction, and therefore, more valuable information in possible clinical and research applications. © 2010 American Institute of Physics. [doi:10.1063/1.3526001]

I. INTRODUCTION

The clinical and research value of tissue electrical properties, including conductivity and permittivity, have long been appreciated in many biomedical applications. For example, it has been reported that breast cancer tissue has significantly higher conductivity than its surrounding tissues¹ suggesting the possible use of electrical conductivity as an endogenous breast cancer screening contrast.² In addition, detailed information about tissue electrical properties plays an important role in electromagnetic source imaging³ and specific absorption rate (SAR) estimation in high field magnetic resonance imaging (MRI).⁴ Therefore, noninvasive bioimpedance imaging is of great research interest and has been studied for years.

Electrical impedance tomography (EIT) (Refs. 5 and 6) is an imaging modality that gives impedance image reconstruction from measured voltages due to different current injections through multiple surface electrodes. EIT has benefits of low cost, real-time speed and safety. Its major limitations include low spatial resolution and degraded sensitivity in the center of an object. Magnetic induction tomography (MIT) is another bioimpedance imaging modality that employs both magnetic transmitting and sensing technologies.^{7,8} In MIT, measurements of the secondary magnetic field produced by the induced eddy current in a conductive tissue object are taken by small coils arranged around the object for impedance image reconstruction. However, spatial resolution of

current MIT technique is still quite limited. In order to solve the technical difficulties in EIT and to achieve high spatial resolution and high accuracy in conductivity imaging, magnetic resonance electrical impedance tomography (MREIT) was developed by combining EIT and magnetic resonance current density imaging (MRCDI) technique.^{9,10} In MREIT, a MRI scanner is used to detect the induced magnetic field generated by injected current flowing in biological tissues. Till now, conductivity images with high spatial resolution have been obtained using the MREIT technique in both *in vitro* and *in vivo* experiments.⁹ Currently, MREIT is mainly limited by its requirement of high level current injection to obtain an acceptable signal-to-noise (SNR) level. Recently, some researchers have also proposed to extract electrical properties of human body at Larmor frequency using MRI B1 mapping technique.^{4,11} This modality was named magnetic resonance electrical properties tomography (MREPT). As this technique does not require any current injection to provide high resolution impedance imaging, it holds higher applicability and flexibility in biomedical applications, but more studies are needed to demonstrate its feasibility and medical values.

Besides the aforementioned electromagnetic imaging methods, alternative approaches for noninvasive imaging of electrical current and conductivity exist such as those hybrid modalities utilizing the coupling between electromagnetic field and acoustic field as reported in magnetoacoustic tomography (MAT) (Refs. 12 and 13) and Hall effect imaging (HEI).^{14,15} This kind of coupling is realized through Lorentz force acting on spontaneous or injected current flow in a

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magnetic field. Based on this coupling mechanism, with the existence of a static magnetic field, time varying current flowing in biological tissues can generate acoustic vibrations that are detectable by ultrasound probes. On the other hand, one can also apply ultrasonic energy to a sample sitting in a static magnetic field and record voltage/current signals to obtain the sample's conductivity information.^{14,16,17} However, the use of current injection or voltage/current measurement through surface electrodes still make these methods limited by the "shielding effect"¹⁵ which suggests a degraded sensitivity in those areas surrounded by low conductive layers such as the brain and breast in human body.

To avoid the "shielding effect" associated with the usage of surface electrodes, magnetoacoustic tomography with magnetic induction (MAT-MI) was proposed.¹⁸⁻²³ MAT-MI utilizes magnetic induction to induce eddy current in a conductive imaging sample and this induced eddy current gives rise to acoustic vibrations through the same Lorentz force coupling mechanism as in MAT/HEI. Ultrasound signals are then collected for image reconstruction. We have conducted theoretical and experiment studies in order to demonstrate the feasibility to produce high spatial resolution images with conductivity related contrast using the MAT-MI approach. However, our previous studies have also shown that both two-dimensional (2D) and three-dimensional (3D) MAT-MI approaches with single type of magnetic stimulation can only yield conductivity boundary images of conductive phantoms or tissue objects.^{19,21}

In order to achieve more complete conductivity contrast reconstruction in MAT-MI, we proposed a new multiexcitation MAT-MI approach.²⁴ In this approach, multiple magnetic excitations using different coil configurations are employed and ultrasound measurements corresponding to all these excitations are collected together to derive the conductivity distribution inside the sample. In this study, we validate the multiexcitation MAT-MI approach for 3D conductivity imaging in both computer simulations and phantom experiments. 3D volume data are obtained by utilizing acoustic focusing and cylindrical scanning under each magnetic excitation. It is shown that with a common ultrasound probe that has limited bandwidth we are able to correctly reconstruct the 3D relative conductivity contrast of the object. As compared to those conductivity boundary images generated by previous single-excitation MAT-MI, the new multiexcitation MAT-MI method provides more complete conductivity contrast reconstruction and therefore more valuable information in possible clinical and research applications.

II. METHODS

A. Imaging problem description

We consider a 3D conductive sample domain Ω with its isotropic conductivity $\sigma(\mathbf{r})$ and boundary $\partial\Omega$. A static magnetic field with flux density $\mathbf{B}_0(\mathbf{r})$ is presented in domain Ω . In the proposed 3D multiexcitation MAT-MI approach, we sequentially apply N , while $N \geq 2$, different time-varying stimulating magnetic fields to the sample domain. Denote the j th magnetic stimulation sent through the j th coil setup as

$\mathbf{B}_1^j(\mathbf{r}, t)$ for $j = 1, \dots, N$. The j th magnetic stimulation induces in the 3D sample volume the j th electrical field $\mathbf{E}^j(\mathbf{r}, t)$ and eddy current density distribution $\mathbf{J}^j(\mathbf{r}, t)$. As in MAT-MI we use around microsecond long current pulse to drive the stimulating coils, the corresponding MHz skin depth in general biological tissue is at the level of meters, so the magnetic induction problem in MAT-MI can be considered quasi-static and magnetic diffusion can be ignored. This leads to separable spatial and temporal function in related electromagnetic variables, i.e., $\mathbf{B}_1^j(\mathbf{r}, t) = \mathbf{B}_1^j(\mathbf{r})f_j(t)$, $\mathbf{E}^j(\mathbf{r}, t) = \mathbf{E}^j(\mathbf{r})f_j'(t)$, and $\mathbf{J}^j(\mathbf{r}, t) = \mathbf{J}^j(\mathbf{r})f_j'(t)$ where the prime denotes first order time derivative. The quasistatic condition also suggests the stimulating magnetic field in the sample can be approximated by the magnetic field generated by the same coil configuration in free space.²⁵ In addition, according to literature reports, in most soft biological tissues the displacement current can be ignored as it is estimated to be 10 to 50 times smaller than the conduction current at megahertz frequency.¹⁸ This estimation was obtained by comparing the values of σ and $\omega\epsilon$ in muscle and fat tissues at 1 MHz, where ϵ is the permittivity and ω is the angular frequency.²⁶ We therefore ignored the displacement current effect in the present study. Of course, for the human body in its *in vivo* wet states, the validity of ignoring the displacement current and the corresponding influence on MAT-MI imaging might need further investigation in the future. Applying those simplifications under the quasi-static condition and ignoring the displacement current, the magnetic induction problem in MAT-MI can be described as in Eq. (1),²⁰

$$\begin{cases} \nabla \cdot \left(\sigma \left(-\frac{\partial \mathbf{A}^j}{\partial t} - \nabla \varphi^j \right) \right) = 0 & \text{in } \Omega \\ \mathbf{J}^j \cdot \mathbf{n} = 0 & \text{on } \partial\Omega \end{cases} \quad j = 1, \dots, N, \quad (1)$$

where $\mathbf{A}^j$ is the magnetic vector potential corresponding to the j th magnetic stimulation with $\mathbf{B}_1^j = \nabla \times \mathbf{A}^j$; $\varphi^j(\mathbf{r})$ is the electrical scalar potential; $\mathbf{n}$ is the unit vector normal to the boundary surface $\partial\Omega$. According to the quasi-static condition, the magnetic vector potential $\mathbf{A}^j$ in Eq. (1) can be directly estimated from the j th stimulating coil geometry and it is considered a known variable. With known σ and $\mathbf{A}^j$, together with a chosen reference position with zero potential, we can obtain a unique solution of φ^j throughout the conductive volume domain Ω using the finite element method (FEM).²⁵ The corresponding electrical field and current density can then be calculated, as $\mathbf{E}^j = -\partial \mathbf{A}^j / \partial t - \nabla \varphi^j$ and $\mathbf{J}^j = \sigma \mathbf{E}^j$.

With the magnetically induced eddy current $\mathbf{J}^j$ and the static magnetic field $\mathbf{B}_0$, the Lorentz force acting on the eddy current over a unit volume can be described as $\mathbf{J}^j \times \mathbf{B}_0$. In MAT-MI the divergence of Lorentz force acts as distributed acoustic source and the corresponding wave equation governing the wave propagation process can be described as in Eq. (2),¹³

$$\nabla^2 p_j - \frac{1}{c_s^2} \frac{\partial^2 p_j}{\partial t^2} = \nabla \cdot (\mathbf{J}^j \times \mathbf{B}_0) \quad j = 1, \dots, N, \quad (2)$$

where p_j is the acoustic pressure corresponding to the j th magnetic stimulation and c_s is the speed of sound in the

media. Here we assume the imaging sample is acoustically homogeneous and c_s is constant in the media. This assumption however would limit the MAT-MI application to imaging soft tissues which have relatively small acoustic heterogeneity of less than 10%.¹⁸ Depending on the ultrasound measurement scheme, the pressure signal p_j at certain detection location may be considered as a 3D volume integration or 2D surface integration of the acoustic sources multiplied with appropriate 3D or 2D Green's function.^{20,21} With the collected acoustic pressure signals, the MAT-MI imaging problem then concerns how to reconstruct the 3D electrical conductivity distribution in the sample domain.

B. Reconstruction algorithms

The MAT-MI image reconstruction can be divided into two steps. In the first step, we reconstruct the MAT-MI acoustic source maps, i.e., the distribution of the divergence of Lorentz force $\nabla \cdot (\mathbf{J}^j \times \mathbf{B}_0)$. In the second step, we then reconstruct the electrical conductivity contrast from the obtained acoustic source maps.¹⁸

For the first step, depending on the ultrasound data acquisition scheme, there are two ways to reconstruct the 3D MAT-MI acoustic sources. If the ultrasound data acquisition is conducted using unfocused small-aperture detector, ideally point detector, and collected from a large view angle, the 3D MAT-MI acoustic sources can be reconstructed using a 3D time reversal back projection method.^{18,20} Accordingly, if the ultrasound data acquisition is conducted using focused transducer and collected slice by slice in a tomographic way as in Ref. 22, a 2D back projection algorithm may be used to reconstruct the MAT-MI acoustic sources in each slice. In this case, the slice thickness is determined by the focal beam width of the transducer and the in-slice resolution is mainly determined by the central frequency and bandwidth of the transducer. We took the second way in this study by utilizing a focused transducer and a cylindrical scanning scheme because it is easier to implement and evaluate in experiment.

With the reconstructed MAT-MI acoustic source distributions in all slices under the N different magnetic excitations, we can then apply the multiexcitation MAT-MI algorithm, as will be introduced in the following part, to reconstruct the 3D conductivity contrast of the sample. Before that, we need to note a discrepancy between the acoustic source maps reconstructed from the complete acoustic pressure signal p_j and those reconstructed from the bandwidth limited pressure signal $\tilde{p}_j$. Here we regard the ultrasound pressure signal to be complete if it is obtained from an ideal ultrasound probe with unlimited bandwidth. This kind of complete signal, however, is generally not available in practical experiments. What we can acquire in MAT-MI experiment is the bandwidth limited acoustic signal $\tilde{p}_j$ which can be considered as band-pass filtered p_j . The reconstruction algorithms for both these two conditions are discussed below.

We first assume the 3D MAT-MI acoustic source map is accurately reconstructed from the ideal p_j . Assuming that the static magnetic field is uniform and sits in the Z direction, i.e., $\mathbf{B}_0 = B_{0z} \hat{z}$, according to Ohm's law and vector identities

we can expand the MAT-MI acoustic source term as in Eq. (3),²⁵

$$\nabla \cdot (\mathbf{J}^j \times \mathbf{B}_0) = \left(\sigma(-B_{1z}^j) + \left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right) \cdot (E_y^j, -E_x^j) \right) B_{0z} f_j'(t) \quad (3)$$

$j = 1, \dots, N,$

where E_x^j and E_y^j are the x and y components of the induced electrical field vector $\mathbf{E}^j$, respectively. $(\partial \sigma / \partial x, \partial \sigma / \partial y)$ is the partial conductivity gradient vector. B_{1z}^j is the z component of $\mathbf{B}_1^j(\mathbf{r})$. This equation holds for every position inside the 3D sample domain Ω . Denote $AS^j(\mathbf{r}, t) = \nabla \cdot (\mathbf{J}^j \times \mathbf{B}_0)$, where $AS^j(\mathbf{r}, t) = AS^j(\mathbf{r}) f_j'(t)$, Eq. (3) can be rewritten in a matrix form as in Eq. (4)

$$\mathbf{U} \mathbf{x} = \mathbf{b}, \quad (4)$$

where

$$\mathbf{U} = \begin{bmatrix} E_y^1 & -E_x^1 \\ \vdots & \vdots \\ E_y^N & -E_x^N \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \frac{\partial \sigma}{\partial x} \\ \frac{\partial \sigma}{\partial y} \end{bmatrix} \quad \text{and}$$

$$\mathbf{b} = \begin{bmatrix} \frac{AS^1(\mathbf{r})}{B_{0z}} + \sigma B_{1z}^1 \\ \vdots \\ \frac{AS^N(\mathbf{r})}{B_{0z}} + \sigma B_{1z}^N \end{bmatrix}.$$

This matrix equation can be solved at each spatial location using a regularized least square method as in Eq. (5)

$$\mathbf{x} = (\mathbf{U}^T \mathbf{U} + \lambda \mathbf{I})^{-1} \mathbf{U}^T \mathbf{b}, \quad (5)$$

where $\mathbf{U}^T$ is the transpose of $\mathbf{U}$; $\mathbf{I}$ is a 2×2 identity matrix and λ is a regularization parameter which is set to be proportional to the condition number of $\mathbf{U}$. As we can see from Eq. (4) that, the entries in each row of matrix $\mathbf{U}$ are the magnetically induced electric field in the imaging object corresponding to certain excitation pattern. In order to obtain good image quality, it is desired to have the electrical fields induced by different excitation patterns being perpendicular to each other, therefore giving a better condition number of matrix $\mathbf{U}$. With the partial gradient $\nabla \sigma = (\partial \sigma / \partial x, \partial \sigma / \partial y)$ in all the imaging slices, a 2D global integration technique can be used to calculate the conductivity distribution $\sigma(\mathbf{r})$ in every imaging slice. Here we use a layer potential integration technique as in Eq. (6),²⁷

$$\sigma(\mathbf{r}) = - \int_S \nabla_{\mathbf{r}'} \Phi(\mathbf{r} - \mathbf{r}') \cdot \nabla \sigma(\mathbf{r}') d\mathbf{r}' + \int_{\partial S} \mathbf{n}_{\mathbf{r}'} \cdot \nabla_{\mathbf{r}'} \Phi(\mathbf{r} - \mathbf{r}') \sigma_{\partial S}(\mathbf{r}') dl_{\mathbf{r}'}, \quad (6)$$

where $\Phi(\mathbf{r} - \mathbf{r}') = 1/2\pi \log|\mathbf{r} - \mathbf{r}'|$ is the two dimensional Green's function of the Laplacian operator. S denotes the 2D integration region in the slice where the integration is performed and ∂S denotes its boundary. $\sigma_{\partial S}$ is the conductivity value restricted at the boundary ∂S and can be obtained experimentally. In practice, this can be done by applying cer-

tain coupling material with known conductivity value on the sample surface, and letting ∂S reside in the area filled with this coupling material. The integration as in Eq. (6) is then conducted in all the imaging slices in order to give the 3D conductivity distribution reconstruction. However, note here that the entries of matrix $\mathbf{U}$ are components of the induced electrical field, which depends on the unknown conductivity distribution $\sigma(\mathbf{r})$ and the vector $\mathbf{b}$ contains a term related to the conductivity distribution too. Therefore, in order to solve $\mathbf{x}$ in Eq. (4) and to derive the conductivity $\sigma(\mathbf{r})$, an iterative algorithm was proposed as following.

Step 1: Reconstruct the acoustic source map AS^j in all the slices using back projection algorithm with measured p_j for $j=1, \dots, N$

Step 2: Let $i=0$ and assume an initial uniform conductivity distribution σ_0

Step 3: Solve Eq. (1) in the 3D conductive volume for $j=1, \dots, N$.

Step 4: Compute $\mathbf{E}^j$ based on the solution from Step 3.

Step 5: Compute σ_{i+1} using Eqs. (5) and (6) on every imaging slice.

Step 6: If the relative error between σ_{i+1} and σ_i is larger than the given tolerance ε_r , replace i by $i+1$ and go to Step 3. Otherwise finish the procedure and use σ_{i+1} as the solution.

Next, we consider the impact of the bandwidth limitation in acoustic measurements on the reconstruction algorithm. Using similar back projection algorithm with the bandwidth limited acoustic measurements $\tilde{p}_j$, we can only reconstruct part of the acoustic source distribution, denoting it $\tilde{AS}^j$. As shown in Eq. (3) the complete MAT-MI acoustic source AS^j consists of two components. For piecewise homogeneous sample, the component that is related to the conductivity gradient $\nabla\sigma$ can be considered as a wide band component. On the contrary, the component that is related to the conductivity distribution σ itself depends on the object geometry and is considered narrow band. Our previous studies have also demonstrated that the conductivity gradient related component is dominant in the partly reconstructed MAT-MI acoustic source map $\tilde{AS}^j$ obtained from bandwidth limited ultrasound measurements.^{19,21} Consequently, we have the approximation as in Eq. (7)

$$\tilde{AS}^j \approx \left[\left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right) (E_y^j, -E_x^j) \right] B_{0z} f_j'(t) \quad j=1, \dots, N. \quad (7)$$

Defining $\tilde{\mathbf{b}} = [\tilde{AS}^1(\mathbf{r})/B_{0z} \dots \tilde{AS}^N(\mathbf{r})/B_{0z}]^T$ and replacing $\mathbf{b}$ with $\tilde{\mathbf{b}}$ in Eq. (5), similar reconstruction algorithm procedure as described above can then be applied.

As we will show in the simulation and experiment studies, with unlimited bandwidth acoustic measurements we can accurately reconstruct the 3D conductivity distributions. With limited bandwidth acoustic measurements, we are not able to quantitatively reconstruct the absolute conductivity values. Instead, we are only able to reconstruct the 3D relative conductivity contrast.

FIG. 1. (Color online) Diagram of the 3D multiexcitation MAT-MI system used in the computer simulation and experiment studies. Each coil is labeled with its group number (A, B, or C) and coil number (1, 2, 3, or 4) in its group. Coils belonging to the same group are synchronized to generate a specific excitation pattern. Arrow marked on each coil indicates the excitation current flowing direction. A focused transducer is used to scan around the sample to collect ultrasound signals. Each MAT-MI acoustic source image slice is obtained by doing a horizontal scan under certain excitation. Horizontal scans at different vertical locations produce the 3D multislice volume data.

C. Computer simulation methods

Figure 1 shows the diagram of the 3D multiexcitation MAT-MI system setup used in our computer simulation study. The static magnetic field is assumed to be uniform in the imaging area and pointing in Z direction. The flux density B_{0z} is set to 1 Tesla. Three groups of coils are utilized to sequentially send three different magnetic excitations, i.e., $N=3$. Each coil is labeled with its group number (A, B, or C) and coil number (1, 2, 3, or 4) in its group. Coils belonging to the same group are synchronized to generate a specific excitation pattern. Coil group A contains two figure-eight coil pairs, i.e., A-1 with A-2 and A-3 with A-4, located in planes of $Z=4$ cm and $Z=-4$ cm, respectively. Each figure-eight coil pair is arranged along the X axis and every coil in the group has a radius of 8 cm. The distance between the two coil centers in the figure-eight coil pair is 19 cm. In addition, coils A-1 and A-3 are placed in the manner of a Helmholtz coil pair and coils A-2 and A-4 have similar arrangement. Coil group B is similar to group A, but is arranged along the Y axis. Coil group C contains a pair of Helmholtz coils with 8 cm coil radius and its axis is the Z axis. The current flow direction in each coil is marked by a red arrow. We assume each coil has one turn and has the same current flow amplitude. For simplicity we set $f_j'(t) = \delta(t)$ and the maximum current changing rate in every coil is set to be 1e8 A/s. This current changing rate leads to a changing rate of stimulating magnetic field B_{1z} generated by excitation Group C of 1124 T/s at the coordinate center. A focused ultrasound transducer scans around the sample in a cylindrical scheme, with the radius of the cylinder to be 20 cm. Each horizontal scan gives the data for reconstructing the MAT-MI acoustic source in the corresponding slice, while the vertical scan gives the multiple slice volume data for reconstructing the 3D conductivity distribution.

In order to obtain the forward solution for the 3D MAT-MI problem, a FEM forward solver was developed using COMSOL software and MATLAB. A $12 \times 12 \times 6$ cm³ vol-

ume was first meshed to regular hexahedral elements with size of $2 \times 2 \times 6 \text{ mm}^3$. Each hexahedral element was then divided into five linear tetrahedral elements. The final FEM mesh has 180 000 tetrahedral elements with 40 931 nodes. All nodes with the same z coordinate are considered to be located in the same slice with slice thickness to be 6 mm. Magnetic vector potential produced by each current carrying coil in certain excitation group was calculated in each element in terms of elliptic integrals and was added up according the principle of superposition.²⁸ The solution to the magnetic induction problem as in Eq. (1) in the form of electrical potential ϕ^j was obtained on each node and the corresponding electrical field $\mathbf{E}^j$ and current density $\mathbf{J}^j$ were calculated at the center of each element. The current density value was then interpolated to each node for calculation of MAT-MI acoustic source in each slice. In the computer simulation, we assume the focused transducer has a sharp focusing gain profile along Z direction that the acoustic signal it can detect at certain location only arises from sources in a single slice in its focal zone. Therefore, a 2D circular integration was used to simulate the complete acoustic measurements p_j . In addition, in order to simulate the limited bandwidth measurements $\tilde{p}_j$, we used an impulse response function $IR(t)$ that has a central frequency at 500 KHz and around 100% bandwidth to convolve with the pressure signal p_j , i.e., $\tilde{p}_j = p_j \otimes IR(t)$. Here the percentage bandwidth is defined as the ratio of the half strength frequency bandwidth over the transducer's central frequency. For example, the 500 KHz, 100% bandwidth transducer has a half amplitude bandwidth of 500 KHz, i.e., from 250 to 750 KHz.

The inverse reconstruction of MAT-MI acoustic sources in the simulation study was first conducted on each imaging slice with each magnetic excitation pattern. After that, the multiexcitation MAT-MI iterative algorithm was applied. The tolerance value ϵ_t was set to be 0.01. In the simulation using limited bandwidth data, a least-square deconvolution filter²⁹ was applied before doing the image reconstruction. We have also conducted a performance testing under different SNR levels. In this simulation, the SNR was defined as the ratio of the maximum pressure signal amplitude over the standard deviation of the added Gaussian random noise.

D. Experiment methods

The experiment system setup has similar design as shown in Fig. 1. The static magnetic field is generated from two permanent magnets and the field strength was measured to be 0.2 T (Gaussmeter, Alpha Laboratory) at the coordinate center where the object is located. All the coils have radius of 4.45 cm. All the coils in excitation group C have three turns, while all the coils in excitation group A and B have 2 turns. The distance between the upper coils and lower coils in each group is around 6 cm. A home made stimulator was developed to drive these coils. The current flowing in the coils has a waveform approximating a bipolar single cycle sinusoid which lasts $2 \mu\text{s}$. The dynamic magnetic excitation generated by the stimulating coils was measured by a sensing coil with radius of 1.5 cm connected to an oscilloscope. The estimated maximum current changing rate in excitation group

C is $1.5\text{e}8 \text{ A/s}$, which corresponds to a magnetic field changing rate of $6\text{e}3 \text{ T/s}$ at the coordinate center. The maximum dynamic magnetic field strength B_{1z} is estimated to be less than 0.006 T at the coordinate center. Excitation group A and B have similar levels of stimulating current flowing in their coils.

The MAT-MI acoustic signal measurement was conducted using a cylindrical scanning scheme. During experiments, both the sample and the transducer were submerged in distilled water media for acoustic coupling. A 500 KHz flat transducer (Panametrics V301) with 29 mm diameter and around 60% bandwidth was employed in this study. An acoustic lens was placed in front of the transducer to implement ultrasound focusing. Both the lens and the transducer were mounted to a scanning frame that can scan around the sample. Each horizontal scan has 320° view angle and 2.5° scanning step. The scanning radius was about 16 cm. The vertical scan was done with a 5 mm scanning step. The focusing lens gives about 7 mm half strength beam width at the imaging center. Signals collected using this transducer was fed into preamplifiers with 90 dB gain and digitized by a 5 MHz data acquisition card. Signal averaging was used to increase SNR.

With the ultrasound signal data acquired under each of the three magnetic excitations, we can first reconstruct the slice images of the MAT-MI acoustic source using the 2D time reversal back projection algorithm.²¹ In order to build the 3D FEM mesh for applying the proposed multiexcitation reconstruction algorithm, one of the MAT-MI acoustic source slice images was segmented to pick out the outmost boundary of the imaging sample and used to define the conductive object domain.

III. RESULTS

A. Computer simulation results

In order to validate the proposed 3D multiexcitation MAT-MI approach, we first conducted well controlled computer simulation studies. Figure 2 shows the 3D conductivity model used in our computer simulation study. As shown in Fig. 2(a) this 3D model has an overall cylindrical structure with four internal small structures located at different places in the 3D volume. Figure 2(b) shows a multiple axial slice representation of this model. Using the developed FEM forward solver, we can simulate the induced eddy current and MAT-MI acoustic sources in the 3D conductivity model under the three magnetic excitations described above. The result is shown in Fig. 3. Figures 3(a)–3(c) illustrate the induced eddy current distributions in the $Z=0 \text{ m}$ slice corresponding to the magnetic excitations A, B, and C, respectively. The corresponding MAT-MI acoustic source distributions in the same slice are shown in Figs. 3(d)–3(f). We can clearly see in this forward simulation the different current flow patterns and MAT-MI acoustic source patterns induced by different magnetic excitations.

After simulating the MAT-MI acoustic signals, we applied the proposed 3D multiexcitation MAT-MI reconstruction algorithm. An ideal case under the noise free condition and assuming unlimited bandwidth acoustic measurements is

FIG. 2. (Color online) (a) 3D conductivity volume model used in the computer simulation. (b) Multiple conductivity image slices of the model at different locations along the Z direction.

shown in Fig. 4. In this simulation, the algorithm converged after five times of iteration. Figures 4(a) and 4(b) show the 3D structure and multiple axial slice representation of the reconstructed conductivity distribution. The correlation coefficient (CC) and relative error (RE) between the reconstructed 3D conductivity multiple slice data and the target multiple slice data are 97% and 6%, respectively. Figure 4(c) shows a profile comparison between the target and reconstructed conductivity distribution at $Z=0$ m, $Y=0.01$ m. As compared to the target conductivity distribution shown in Fig. 2, it is shown here that under these ideal conditions, we

can accurately reconstruct the 3D conductivity distribution using the proposed multiexcitation MAT-MI approach. In addition, we can also see in Fig. 4(a) the impact of the limited slice thickness which determines the spatial resolution along Z direction in the reconstructed 3D conductivity volume.

We have also conducted simulation to test the performance of the 3D multiexcitation MAT-MI approach with limited bandwidth acoustic measurements and under different SNR levels. Figure 5(a) shows the simulated transducer's impulse response. Figures 5(b)–5(d) show the reconstructed conductivity image slices at $Z=0$ m under SNR level of

FIG. 3. (Color online) (a)–(c) are the simulation of the eddy current distribution (displaying x and y components) at the $Z=0$ m slice in the 3D volume induced by the magnetic excitation group A, B, and C, respectively. (d)–(f) are the simulation of MAT-MI acoustic source distributions corresponding to the eddy current distributions shown in (a)–(c).

FIG. 4. (Color online) (a) Reconstructed 3D conductivity distribution under noise free condition and with the assumption that the acoustic measurements are bandwidth unlimited. (b) Corresponding conductivity image slices at different locations along the Z direction. (c) Conductivity profile comparison between the target and the reconstructed conductivity distribution at $Z=0$ m, $Y=0.01$ m.

100, 30, and 10, respectively. The CCs between the reconstructed 3D conductivity multiple slice data and the target conductivity multiple slice data under these SNR levels are 86%, 85%, and 82%, respectively. The corresponding REs are 18.18%, 18.19%, and 18.26%. It is observed that the major reconstruction error comes from the bandwidth limitation in the acoustic measurements instead of the added white noise. Note here we used different color scales as compared to Fig. 2. The fact is that with the limited bandwidth acoustic measurements, we can only reconstruct the relative conductivity contrast instead of the absolute conductivity values. In addition, as shown in Fig. 5, some ringing artifacts around those conductivity boundaries can be observed in the reconstructed image slices. Figure 6 shows more comprehensive simulation results with limited bandwidth acoustic measurements and SNR of 30. Figures 6(a) and 6(b) show the 3D structure and multiple axial slice representation of the reconstructed conductivity distribution and Fig. 6(c) shows a pro-

file comparison between the target and reconstructed conductivity distribution at $Z=0$ m, $Y=0.01$ m. As shown in this figure, the conductivity distribution reconstructed using limited bandwidth data has a scaling factor as compared to the target distribution. This scaling factor contributes a large amount of conductivity reconstruction errors calculated above. In addition, this effect is believed to be caused by the multiexcitation iterative algorithm with the presence of the reconstruction errors in the reconstructed MAT-MI acoustic source $\tilde{A}\tilde{S}^i$ which is derived from limited bandwidth pressure measurement data. However, in spite of some errors and artifacts, the 3D relative conductivity distribution is well reconstructed.

B. Experiment results

Using the developed 3D multiexcitation MAT-MI system, we have conducted a phantom experiment study. Fig-

FIG. 5. (Color online) (a) Simulated impulse response of the ultrasound transducer that centers at 500 KHz. (b), (c), and (d) are the reconstructed conductivity image slices at $Z=0$ m with limited bandwidth data and SNR to be 100, 30, and 10, respectively.

ures 7(a) and 7(b) show the top and side view photos of the gel phantom we used in this study. Figure 7(c) shows a diagram of the phantom geometry and the locations of the transducer's central focus along Z direction where different image slices are obtained. In order to keep most part of the transducer and the acoustic lens in water, we only collected acoustic signals from the bottom part of the phantom. For the top part of the phantom, where no image slice is collected, we assume it has the same conductivity distribution as that in slice 5. The background material of the phantom is made from 15% salinity gel. All the white columns are made from tissue fat. The embedded gel column marked by the red circle in Fig. 7(a) is made from 20% salinity gel.

Figure 8 shows the reconstructed MAT-MI acoustic source images at the bottom slice, i.e., slice 1, under three different magnetic excitations. It is shown that most significant MAT-MI acoustic sources are located at conductivity boundaries. Three different acoustic source patterns can be clearly seen that corresponds to the three different magnetic excitations. Figure 9 shows the five image slices from bottom to top under the magnetic stimulation C. Those two small fat columns are reconstructed in image slice 1, 2, and 3, but not shown in the top two image slices. The big column on the contrary is consistently reconstructed in all the five image slices. The conductivity images obtained using the previous single excitation MAT-MI method would look exactly the same as images shown in Figs. 8 and 9 except with reversed color scale.^{19,21} With all the data collected under three different magnetic excitations, we applied the 3D multiexcitation MAT-MI algorithm and the reconstructed relative conductivity contrast image slices are shown in Fig. 10. Despite some imaging noise and artifacts, the 3D relative conductivity contrast can be clearly identified and agrees well with the phantom geometry. Two small low conductive regions are reconstructed in slice 1, 2, and 3, but are not visible in slice 4 and slice 5. On the contrary, the big fat column

with an embedded high conductive gel column is consistently reconstructed in all the image slices. As compared to the MAT-MI acoustic source images (similar to the conductivity images obtained using single excitation method), which emphasize the conductivity boundaries, the reconstructed conductivity image slices using the 3D multiexcitation algorithm give a more informative conductivity contrast map and enable us to better differentiate the material types in the phantom based on their electrical conductivity properties.

IV. DISCUSSION AND CONCLUSION

The MAT-MI imaging approach was proposed to do non-invasive conductivity imaging with high spatial resolution. However, previous computer simulation and phantom experiment studies have shown that with single type of magnetic stimulation, the reconstructed MAT-MI images only show the conductivity boundaries instead of the real conductivity contrast.^{19,21} This is because the collected MAT-MI pressure signal is mainly determined by the conductivity gradient especially when using limited bandwidth ultrasound probes. Consequently, the reconstructed conductivity image using the single excitation MAT-MI method would mainly reflect the projection component of the conductivity gradient in the direction perpendicular to the induced electric field. In this study, we propose the 3D multiexcitation MAT-MI imaging approach, while multiple magnetic excitations with different patterns are applied sequentially to induce different patterns of eddy current and MAT-MI acoustic sources in the imaging object. The corresponding MAT-MI signals are then collected and combined for conductivity reconstruction. As the multiple magnetic excitations in the proposed 3D multiexcitation MAT-MI method utilizes different patterns through different coil configurations, in comparison to the single excitation method, this method allows us to obtain more information about the conductivity gradient, i.e., its cross projection in different directions corresponding to the different induced electric fields. Therefore, using the multiexcitation method we can get a more complete reconstruction of the conductivity gradient and in turn improve conductivity contrast reconstruction in MAT-MI images. It is shown in our computer simulations and phantom experiments that by combining the acoustic measurements under several well designed magnetic excitations, we are able to reconstruct the 3D relative conductivity contrast with measurements obtained from practical bandwidth limited ultrasound probes. The reconstructed relative conductivity contrast information would then allow us to better differentiate tissue types and would significantly benefit potential MAT-MI applications such as cancer detection.

As we discussed, the purpose of using multiple excitation patterns is to generate different patterns of induced electric field in the imaging object and help collect information related to the different projection components of the conductivity gradient. From the perspective of solving the inverse problem, multiple excitations using different patterns can help get a better conditioned matrix U as in Eq. (4), i.e., each rows of it is different or ideally perpendicular to each other. On the other hand, excitations using only one coil configu-

FIG. 6. (Color online) (a) Reconstructed 3D conductivity distribution with limited bandwidth data and SNR to be 30. (b) Corresponding conductivity image slices at different locations along the Z direction. (c) Conductivity profile comparison between the target and the reconstructed conductivity distribution at $Z=0$ m, $Y=0.01$ m.

ration would just give an average effect, which corresponds to a matrix $\mathbf{U}$ with N similar rows and the conductivity gradient vector becomes unsolvable. In addition, further exploration and optimization on the coil designs in the future should be done in terms of improving the solving condition of matrix $\mathbf{U}$.

FIG. 7. (Color online) (a) Top view photo of the 3D gel phantom. (b) Side view photo of the 3D gel phantom. (c) Diagram of the 3D structure of the gel phantom. Arrows on the right indicate the central positions where image slices from slice 1 to slice 5 are obtained.

As shown in our computer simulation, if ideal unlimited bandwidth acoustic measurements are available, we can reconstruct the absolute 3D conductivity distribution. Even though it is very hard to acquire unlimited bandwidth measurements in practice, a well calibrated transducer with wider bandwidth is expected to give better conductivity image reconstruction.

In the present experiment study, we used a 500 KHz MAT-MI system with the central frequencies of both magnetic stimulations and ultrasonic detections to be 500 KHz. About 3 mm spatial resolution is achieved in each image slice because of the tomographic reconstruction we used in each horizontal scan. The spatial resolution of 3 mm can be estimated from the 500 KHz system frequency, i.e., single cycle period of $2 \mu\text{s}$ and the speed of sound of 1500 m/s in most soft biological tissues. It is further verified from the observation that in all the MAT-MI acoustic source images every sharp conductivity boundary expands to 3 mm wide.

Therefore, the imaging resolution of the current multiexcitation MAT-MI system in the XY plane is estimated to be 3 mm. Accurate calibration on the conductivity imaging resolution can be done using a well controlled phantom experiment study which is beyond the scope of this study. However, the effective slice thickness, in another word, the spatial resolution along the Z direction, is determined by the focusing beam width and is estimated to be 7–8 mm in the present system. As the spatial resolution of MAT-MI image is mainly determined by the corresponding spatial resolution that can be achieved in the MAT-MI acoustic source image reconstructed by back projecting the ultrasound signals, MAT-MI systems with higher frequencies and wider bandwidth would in turn give better in-slice spatial resolution and better ultrasound focusing would then give better spatial resolution in Z direction. Furthermore, as the electrical properties of biological tissue are frequency dependent, the conductivity images obtained using the MAT-MI approach would indicate the tissue conductivity contrast at the corresponding frequency range determined by the MAT-MI system. Therefore, the reconstructed MAT-MI image slices obtained in the present experiment study represent conductivity properties of the phantom at around 500 KHz. Higher system frequency thus would provide not only better spatial resolution, but different information about the sample's electrical conductivity contrast.

In summary, we have validated the multiexcitation MAT-MI approach for 3D conductivity imaging. Computer simulation and phantom experiment studies have been conducted to demonstrate the promise of the proposed method in reconstructing the conductivity contrast using ultrasound measurements.

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