The design of a reflectionless arterial prosthesis

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Abstract We propose a new technique to characterize a reflectionless arterial prosthesis. The corresponding transmission and reflection coefficients are determined from the geometric and the elastic properties of the arterial wall, and the interaction between the latter and the prosthesis are studied accordingly.

Keywords Blood waves · Transmission coefficient · Reflection coefficient · Prosthesis

1 Introduction

Elastic tubes are widely used nowadays in fluid flow, and their importance no longer needs to be demonstrated. From the biological field up to the industrial domain, different types of elastic tubes are encountered and while many studies have been done, there still exist unresolved questions.

The first studies on elastic tubes presented a linear approach of the fluid flow therein with differences arising only in the consideration of the tube wall contribution [1, 2]. The nonlinear studies that came after brought out the importance of the fluid convective term and particularly took into account the dynamical properties of the tube wall where the nonlinear stress–strain relationship is considered [3–6]. Most of the works that followed contributed to improve the modeling, in a more realistic way, of elastic tubes [7–9] and took into account viscosity, tapering aspect of the tubes [5, 10], pathological cases of vessels and prostheses [11].

Some studies on blood waves focused on the study of the behaviour of waves in the regions of discontinuities [12, 13]. In general, any discontinuity zone present in an artery gives rise to the phenomena of wave transmission and reflection. These discontinuities are in

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general due to the change in the elastic properties of the arterial wall, diseases, bifurcations and prostheses.

The objective of this paper is to determine the expression of the transmission and reflection coefficients in the prosthesis using a nonlinear model of blood flow. This theory is based on the separation of pressure waves into their forward and backward running components proximal and distal to the prosthesis. We then use the same approach to investigate the characteristics of a reflectionless prosthesis.

In the study, we consider that the fluid in the vessel is Newtonian, viscous, homogeneous and incompressible. Furthermore, we take into account the variation of the radius and elasticity of the wall. The rest of the paper is therefore organized as follows: in Section 2, the physical and mathematical problems are formulated. In Section 3, the nonlinear model of the flow is used to determine the transmission and reflection coefficients of the prosthesis, and the conditions for a reflectionless prosthesis are derived accordingly. The last section is devoted to concluding remarks.

2 Physical model, governing equations and separation of waves

2.1 Physical model and governing equations

The artery is assumed to be conical, with a nonlinear stress–strain relation. The geometry of the artery with the presence of a prosthesis is illustrated in Fig. 1. The index i ($i = 1, 2$ or 3) represents each portion, where $i = 2$ stands for the prosthesis and $i = 1$ and $i = 3$ represent the leftmost and rightmost portions of the vessel, respectively. For the present discrete case, we apply the averaging procedure, used in the literature, to the Navier–Stokes equations [5] and we derive the following one-dimensional model

$$\frac{\partial R_i^2}{\partial t} + \frac{\partial}{\partial z} (R_i^2 W_i) = 0, \quad (1)$$

$$\frac{\partial W_i}{\partial t} + W_i \frac{\partial W_i}{\partial z} + \frac{1}{\rho} \frac{\partial P_i}{\partial z} = \nu_i \left(-8 \frac{W_i}{R_i^2} + \frac{\partial^2 W_i}{\partial z^2} \right), \quad (2)$$

where z is the axial coordinate, t is time, P_i is the pressure inside the vessel, W_i is the axial component of the fluid velocity, R_i is the radius of the vessel, ρ is the fluid density and ν_i is kinematic viscosity.

Fig. 1 Geometry of the artery with prosthesis, where a stented vessel is shown in a cylindrical coordinate system (r , radial coordinate; z , longitudinal coordinate)

For the wall dynamics, we use the second law of Newton on a portion of the vessel wall and we obtain the relation [14]:

$$\rho_{0i} H_i \frac{\partial^2 \delta R_i}{\partial t^2} = P_i - \bar{P} + \gamma_i \frac{\partial \delta R_i}{\partial t} + \beta_i \frac{\partial^2 \delta R_i}{\partial z^2} + \varphi_i \frac{\partial^3 \delta R_i}{\partial t \partial z^2} - \frac{h_i}{R_i} \sigma_{ti}. \quad (3)$$

where $\bar{P}$ is the pressure outside the vessel, δR_i denotes the vessel deformation, ρ_{0i} is the wall density, and H_i and h_i are the effective inertial thickness and the thickness of the wall, respectively. γ_i is the viscoelasticity coefficient due to the absorption effects of the material. β_i is the shear modulus, and φ_i is another coefficient of viscoelasticity. σ_{ti} is the approximated exponential function for the stress–strain relationship given in [15, 16]. This approximated relation is widely known as [3, 4, 6, 7]

$$\sigma_{ti} = E_i \frac{\delta R_i}{\bar{R}_i} \left(\frac{1 + a_i \delta R_i}{\bar{R}_i} \right), \quad (4)$$

where $\bar{R}_i(z)$ is the stationary radius of the vessel, E_i is the Young modulus and a_i represents the nonlinear coefficient of elasticity in the portion i .

The vessel radius $R_i(z, t)$ in the portion i is described as

$$R_i(z, t) = \bar{R}_i(z) + \delta R_i(z, t). \quad (5)$$

As vessel segments are tapered elastic tubes with increasing rigidity away from the heart, they represent general cases of thin-walled elastic tubes with variable Young modulus. Many authors [4, 7, 8] showed that the stationary radius and the variation of elasticity along the wall will obey the relations

$$\bar{R}_i(z) = R_{0i}(1 + f(z)), \quad E_i(z) = E_{0i}(1 + g(z)), \quad (6)$$

with $f(z) = -m_i(z - z_{0i})$, $g(z) = n_i(z - z_{0i})$. Here, $f(z)$ denotes the decrease of the tube radius and $g(z)$ the increase of the tube wall rigidity along the tube originating from the heart. R_{0i} is the radius at the entrance of the portion i and z_{0i} the abscissa at the entrance of the vessel segment, in the same portion. The positive coefficients m_i and n_i characterize the rate of decrease and increase of the radius and wall rigidity in the portion i , respectively.

As we consider weak wall displacement, we introduce the approximation:

$$R_i - \bar{R}_i = \delta R_i = \frac{\bar{R}_i}{2R_0} (A_i^* - 1) \quad \text{and} \quad R_i = \frac{\bar{R}_i}{2R_0} (A_i^* + 1) \quad (7)$$

where $A_i^* = S_i/S_{0i}$ and $S_{0i} = \pi \bar{R}_i^2$ are the normalized cross sectional area of the tube and the cross sectional area at the entrance of the i th portions, respectively. R_0 , w_0 and E_0 are, respectively, the reference values of radius, velocity and Young modulus at the entrance of the vessel. We define the viscosity coefficients n'_1 and n'_2 as:

$$n'_1 = 32 \frac{R_0 \nu}{w_0}, \quad n'_2 = \frac{\nu}{w_0 R_0}, \quad (8)$$

and the dimensionless quantities

$$W_i = w_0 w_i^*, \quad P_i - \bar{P} = p_0 p_i^*, \quad t = t^* T, \quad z = z^* R_0, \quad (9)$$

and we also include the following coefficients

$$\alpha_{1i} = \frac{\bar{R}_i}{R_0}, \quad \alpha_{2i} = \frac{H_{0i}}{H_0}, \quad \alpha_{3i} = \frac{E_{0i}h_{0i}}{E_0h_0}, \quad \alpha_{4i} = \frac{\rho_{0i}}{\rho_0}, \quad p_0 = \rho w_0^2. \quad (10)$$

We assume that the first and last portions ($i = 1$ and $i = 3$) are made up of the same material properties.

So, Eqs. 1–3 thus illustrate mass conservation,

$$s_t \frac{\partial}{\partial t^*} (\alpha_{1i}^2 (A_i^* + 1)^2) + \frac{\partial}{\partial z^*} (\alpha_{1i}^2 (A_i^* + 1)^2 w_i^*) = 0, \quad (11)$$

momentum conservation,

$$s_t \frac{\partial w_i^*}{\partial t^*} + w_i^* \frac{\partial w_i^*}{\partial z^*} + \frac{\partial p_i^*}{\partial z^*} + \frac{n_1}{\alpha_{1i}^2 (1 + A_i^*)^2} w_i^* - n_2 \frac{\partial^2 w_i^*}{\partial z^{*2}} = 0, \quad (12)$$

and wall dynamics,

$$\begin{aligned} & 2\alpha_{01i}\alpha_{2i}\alpha_{4i}a_{1i} \frac{\partial^2}{\partial t^{*2}} (\alpha_{1i}(A_i^* - 1)) \\ &= 2\alpha_{1i}(A_i^* + 1)p_i^* + \alpha_{1i}a_{2i}(A_i^* + 1) \frac{\partial}{\partial t^*} (\alpha_{1i}(A_i^* - 1)) \\ &+ \alpha_{1i}b_{1i}(A_i^* + 1) \frac{\partial^2}{\partial z^{*2}} (\alpha_{1i}(A_i^* - 1)) + \alpha_{1i}b_{2i}(A_i^* + 1) \frac{\partial^3}{\partial t^* \partial z^{*2}} (\alpha_{1i}(A_i^* - 1)), \\ &- \varepsilon_i^{-1} \alpha_{3i}(A_i^* - 1)(2 + a_i A_i^* - a_i), \end{aligned} \quad (13)$$

where

$$a_{1i} = \frac{s_i^2}{\rho w_0}, \quad a_{2i} = \frac{\gamma_i}{\rho w_0} s_t, \quad b_{1i} = \frac{\beta_i}{\rho w_0^2 R_0}, \quad b_{2i} = \frac{\varphi_i}{\rho w_0 R_0^2} s_t,$$

with $\varepsilon_i = \rho w_0^2 R_0 (E_{0i} h_{0i})^{-1}$ being the relative stiffness of the vessel. The flow pulsation has a dimensionless period which is given by the inverse of the Strouhal number $s_t^{-1} = T w_0 R_0^{-1}$.

2.2 The linear theory for separation of forward and backward waves

Following the mathematical procedure of [3, 17], we set

$$w_i^* = \epsilon w_{1i} + \epsilon^2 w_{2i}, \quad (14)$$

$$p_i^* = \epsilon p_{1i} + \epsilon^2 p_{2i}, \quad (15)$$

$$A_i^* = 1 + \epsilon A_{1i} + \epsilon^2 A_{2i}, \quad (16)$$

where ϵ is a small parameter measuring the weakness of dispersion and nonlinearity. Also weak are the parameters modelling the taper effect of the wall, the compliance variation and the blood viscosity which become

$$\gamma_i = \epsilon^{1/2} \gamma_{1i}, \quad \beta_i = \epsilon^{1/2} \beta_{1i}, \quad \varphi_i = \epsilon^{1/2} \varphi_{1i}, \quad n'_1 = \epsilon^{3/2} n_1, \quad n'_2 = \epsilon^{3/2} n_2. \quad (17)$$

Next, the coordinate system is changed to slowly varying coordinates

$$\xi_i = \epsilon_i^{1/2}(z^* - c_i t^*), \quad \eta_i = \epsilon_i^{1/2}(z^* + c_i t^*), \quad \tau_i = \epsilon_i^{3/2} t^*. \quad (18)$$

where c_i represents the wave group velocity and is assumed to be the velocity of the linear wave as it enters the medium. ξ_i is an independent variable representing the forward wave whereas η_i stands for the backward wave.

The approximation of the flow motion at the first order, ϵ^1 , gives:

$$-c_i s_t \frac{\partial A_{1i}}{\partial \xi_i} + \frac{\partial w_{1i}}{\partial \xi_i} + c_i s_t \frac{\partial A_{1i}}{\partial \eta_i} + \frac{\partial w_{1i}}{\partial \eta_i} = 0, \quad (19)$$

$$-c_i s_t \frac{\partial w_{1i}}{\partial \xi_i} + \frac{\partial p_{1i}}{\partial \xi_i} + c_i s_t \frac{\partial w_{1i}}{\partial \eta_i} + \frac{\partial p_{1i}}{\partial \eta_i} = 0, \quad (20)$$

$$\alpha_{01i} p_{1i} - \epsilon_i^{-1} \alpha_{03i} A_{1i} = 0. \quad (21)$$

In the present work, the study will be limited to this first order of perturbation. The second order, useful to obtain the analytic expression of the wave, is not used here. So, we consider the waves to be separated explicitly as forward and backward elements (where f and b are, respectively, used as superscripts) [18, 19].

$$w_{1i}(\xi_i, \eta_i, \tau_i) = w_{1i}^f(\xi_i, \tau_i) + w_{1i}^b(\eta_i, \tau_i), \quad (22)$$

$$p_{1i}(\xi_i, \eta_i, \tau_i) = p_{1i}^f(\xi_i, \tau_i) + p_{1i}^b(\eta_i, \tau_i), \quad (23)$$

$$A_{1i}(\xi_i, \eta_i, \tau_i) = A_{1i}^f(\xi_i, \tau_i) + A_{1i}^b(\eta_i, \tau_i). \quad (24)$$

Introducing Eqs. 22–24 into Eqs. 19–21 leads to the following relation

$$p_{1i}^f = c_i s_t w_{1i}^f = c_i^2 s_t^2 A_{1i}^f, \quad (25)$$

and

$$p_{1i}^b = -c_i s_t w_{1i}^b = c_i^2 s_t^2 A_{1i}^b, \quad (26)$$

with

$$c_i^2 = \frac{\alpha_{03i}}{\epsilon_i \alpha_{01i}}, \quad (27)$$

3 Transmission and reflection coefficients and conditions for reflectionless arterial prosthesis

3.1 Transmission and reflection coefficients

In this part, we propose a new technique for the estimation of the transmission, T , and the reflection, B , coefficients in the prosthesis proportional to the local transmission, T_i , and reflection, B_i , coefficients at the interface of the prosthesis.

For the computation of the characteristic coefficients, we use the impedance of the system given by the ratio of the pressure and flow at the interfaces [16, 17]

$$Z_c = \frac{P_{1i}^f}{Q_{1i}^f} = -\frac{P_{1i}^b}{Q_{1i}^b}. \quad (28)$$

The local transmission, T_i , and reflection, B_i , coefficients are defined as

$$T_i = \frac{Q_{i+1}^f}{Q_{1i}^f}, \quad B_i = \frac{P_{1i}^b}{P_{1i}^f} = -\frac{Q_{1i}^b}{Q_{1i}^f}, \quad (29)$$

and are expressed in terms of the flows. Of course, in the literature, this is not usual, but from the conservation law $\frac{Q_{i+1}^f + Q_{1i}^b}{Q_{1i}^f} = \frac{Q_{1i}^f}{Q_{1i}^f}$ the relation $T_i + B_i = 1$ is not verified when the pressures are mainly used. This also brings out the difference between the method used in this paper and the one used by Stergiopoulos et al. [18].

If we consider at the interfaces the conservation of mass and energy, it will appear that, at the first interface,

$$P_{11}^f + P_{11}^b = P_{12}^f, \quad \pi \bar{R}_1^2(z_{02})(w_{11}^f + w_{11}^b) = \pi \bar{R}_2^2(z_{02})w_{12}^f, \quad (30)$$

while at the second interface

$$P_{12}^f + P_{12}^b = P_{13}^f, \quad \pi \bar{R}_2^2(z_{03})(w_{12}^f + w_{12}^b) = \pi \bar{R}_3^2(z_{03})w_{13}^f. \quad (31)$$

Inserting Eqs. 25–27 into Eqs. 30 and 31 helps to obtain the local transmission coefficient T_1 (resp. T_2) and the local reflection coefficient B_1 (resp. B_2), respectively, at the first and second interfaces

$$T_1 = \frac{2}{1 + \left(\frac{E_{02}h_{02}R_{01}^5(1-m_1L_v)^5}{E_{01}h_{01}R_{02}^5(1+n_1L_v)} \right)^{1/2}}, \quad T_2 = \frac{2}{1 + \left(\frac{E_{01}h_{01}R_{02}^5(1+n_1(L_v+L_p))(1-m_2L_p)^5}{E_{02}h_{02}R_{01}^5(1+n_2L_p)(1-m_1(L_v+L_p))} \right)^{1/2}}, \quad (32)$$

$$B_1 = \frac{1 - \left(\frac{E_{01}h_{01}R_{02}^5(1+n_1L_v)}{E_{02}h_{02}R_{01}^5(1-m_1L_v)} \right)^{1/2}}{1 + \left(\frac{E_{01}h_{01}R_{02}^5(1+n_1L_v)}{E_{02}h_{02}R_{01}^5(1-m_1L_v)} \right)^{1/2}}, \quad B_2 = \frac{1 - \left(\frac{E_{02}h_{02}R_{01}^5(1+n_2L_p)(1-m_1(L_v+L_p))}{E_{01}h_{01}R_{02}^5(1+n_1(L_v+L_p))(1-m_2L_p)} \right)^{1/2}}{1 + \left(\frac{E_{02}h_{02}R_{01}^5(1+n_2L_p)(1-m_1(L_v+L_p))}{E_{01}h_{01}R_{02}^5(1+n_1(L_v+L_p))(1-m_2L_p)} \right)^{1/2}}, \quad (33)$$

with L_v as the length of the left portion of the vessel and L_p as the length of the prosthesis.

In this work, we suppose that the distal forward wave, P_{12}^f , is the sum of two waves: the part of the proximal forward wave that became transmitted through the prosthesis and the part of the distal backward wave that became reflected by the prosthesis. And we obtain a similar equation for the proximal backward running wave, P_{11}^b . These equations are given by

$$P_{12}^f = TP_{11}^f + BP_{12}^b, \quad \text{and} \quad P_{11}^b = BP_{11}^f + TP_{12}^b, \quad (34)$$

where T and B are the global transmission and reflection coefficients due to the overall prosthesis. Using Eq. 34, we obtain the expressions of the transmission and reflection coefficients, as

$$T = \frac{P_{11}^f P_{12}^f - P_{11}^b P_{12}^b}{P_{11}^{f^2} - P_{12}^{b^2}}, \quad \text{and} \quad B = \frac{P_{11}^f P_{11}^b - P_{12}^f P_{12}^b}{P_{11}^{f^2} - P_{12}^{b^2}}. \quad (35)$$

Inserting relation Eq. 29 into Eq. 35 helps to derive the transmission, T , and reflection, B , coefficients in the prosthesis depending on the local transmission, T_i , and reflection, B_i , coefficients at the interfaces

$$T = T_1 \left(\frac{E_{02} h_{02} R_{01}^5 (1 - m_1 L_v)^5}{E_{01} h_{01} R_{02}^5 (1 + n_1 L_v)} \right)^{1/2} \frac{(1 - B_1 B_2)}{1 - \left(\frac{E_{02} h_{02} R_{01}^5 (1 - m_1 L_v)^5}{E_{01} h_{01} R_{02}^5 (1 + n_1 L_v)} \right) T_1^2 B_2^2}, \quad (36)$$

$$B = B_1 \frac{1 - \left(\frac{E_{02} h_{02} R_{01}^5 (1 - m_1 L_v)^5}{E_{01} h_{01} R_{02}^5 (1 + n_1 L_v)} \right) \frac{T_1^2 B_2}{B_1}}{1 - \left(\frac{E_{02} h_{02} R_{01}^5 (1 - m_1 L_v)^5}{E_{01} h_{01} R_{02}^5 (1 + n_1 L_v)} \right) T_1^2 B_2^2}. \quad (37)$$

3.2 Condition for a reflectionless arterial prosthesis

From Eq. 37, we get the conditions for a reflectionless arterial prostheses. The reflectionless arterial prosthesis is obtained when the reflection coefficient is equal to 0 and the transmission coefficient equal to 1. This gives us the following condition (with $B_1 \neq 0$):

$$\frac{E_{02} h_{02}}{E_{01} h_{01}} = \frac{1}{75ab} \left[f(a, b) + \frac{25b^2 + 630ab + 81a^2}{f(a, b)} + 10b + 9a \right], \quad (38)$$

with

$$a = \frac{R_{01}^5 (1 - m_1 L_v)^5}{R_{02}^5 (1 + n_1 L_v)}, \quad b = \frac{R_{01}^5 (1 + n_2 L_p)(1 - m_1 (L_v + L_p))^5}{R_{02}^5 (1 + n_1 (L_v + L_p))(1 - m_2 L_p)^5} \quad (39)$$

and

$$f(a, b) = \left[-125b^3 + 16875ab^2 + 8505ba^2 + 729a^3 + 600\sqrt{-3 \frac{(5b^2 - 234ab - 27a^2)}{ab}} ab^2 \right]^{\frac{1}{3}}. \quad (40)$$

Using this condition (see relation 38), the optimal thickness of the prosthesis is derived as

$$h_{02} = \frac{1}{75ab} \left[f(a, b) + \frac{25b^2 + 630ab + 81a^2}{f(a, b)} + 10b + 9a \right] \frac{E_{01} h_{01}}{E_{02}}. \quad (41)$$

The manufacturing of reflectionless arterial prostheses being one of the primary objectives of research in this field, our result gives a new mathematical condition for the reflectionless arterial prosthesis. This condition takes into account the geometric and elastic properties of the vessel wall.

For numerical calculation, we have used the characteristic parameters for the femoral artery of a dog [6]: $L_v = 4.5$ cm, $E_{01} = 14.1 \times 10^6$ dyn/cm², $n_1 = 0.067$ cm⁻¹, $m_1 = 0.080$ cm⁻¹ and the thickness of the vessel is 0.018 cm. We consider the prosthesis parameters to be $L_p = 4.0$ cm, $n_2 = 0.069$ cm⁻¹ and $m_2 = 0.089$ cm⁻¹.

Figure 2 shows that the transmission coefficient increases to the value 1 on the positive side of the ordinate and the reflection coefficient increases to the value 0 on the negative side, as the ratio of the product of the Young modulus and the wall thickness is increased and for $R_{01}/R_{02} = 0.86$. This behaviour is comparable with the effects of the severity of an

Fig. 2 The dependence of the transmission and reflection coefficients on the ratio of the product of Young modulus and wall thickness of the stented vessel for $R_{01}/R_{02} = 0.86$

extended stenosis as obtained by Stergiopoulos et al. [18]. The negative values obtained in the case of the reflection coefficient are due to the fact that the reflected waves velocity is negative (see Eq. 26).

In Fig. 3, the ratio of thickness (Eq. 41) is plotted as the ratio of the radius varies. When the ratio of the radii increases, the thickness ratio decreases. This figure illustrates a reflectionless arterial prosthesis. For example, in the case of a Palmaz stent with Young modulus $E_{02} = 2.0 \times 10^6$ dyn/cm² [20], inserted in the femoral artery, the ratio of the thickness of the stent to that of the vessel is $\frac{h_{02}}{h_{01}} \simeq 9.63$. Using this condition, the value of the transmission coefficient is approximately equal to 1 while the reflection coefficient is equal to 0, when the ratio of the radius is $R_{01}/R_{02} = 0.94$. Taking into account the thickness of the femoral artery ($h_{01} = 0.018$ cm), the thickness of the stent is obtained as $h_{02} = 0.173$ cm.

Fig. 3 Dependence of the ratio of thickness on that of the radius in the case of the reflectionless prosthesis

Fig. 4 Influence of the left vessel length on the reflectionless prosthesis condition

In Fig. 4, we show that the variation of the length of the left portion of the vessel brings about modifications in the characteristics of the perfect prosthesis. When the length of the vessel increases, the characteristics of the reflectionless prosthesis decrease. As a whole, these results present the interactions between the vessel wall and the prosthesis. Such a behaviour would also be observed for the variation of the elasticity of the vessel. These results are further confirmed by the case of Fig. 5, where the variation of the prosthesis length is taken into account. So, when the length of the prosthesis increases, the characteristics of the reflectionless prosthesis increase. In fact, if there is a perturbation at the entrance of the prosthesis, that prosthesis, due to its characteristics will overcome the perturbation for a better blood flow. That is why increasing the length of the prosthesis deeply modifies the condition for a reflectionless prosthesis as depicted in Fig. 5.

Fig. 5 Influence of the prosthesis length on the reflectionless prosthesis condition

4 Conclusion

In this study, the concept of wave separation has been used to derive the reflection and transmission coefficients of blood waves created by a prosthesis inserted in an artery. This has led to the derivation of conditions for a reflectionless prosthesis. These conditions are useful for the design of a reflectionless arterial prosthesis. In our methodology, we take explicitly into account the interactions between the vessel wall and the prosthesis at the entrance and the exit of the prosthesis.

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