Fibonacci, Littler, and the Hand: A Brief Review

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Abstract In a landmark paper published in 1973, the eminent hand surgeon J. William Littler, MD, proposed two mathematical relationships between the anatomic and functional geometry of the hand. His proposal that the motion of the tips of the fingers follow an equiangular spiral has been experimentally supported. Studies have not supported his other idea that the lengths of the phalanges follow a Fibonacci series. This review, after providing the necessary mathematical background, reexamines Littler's claims, presents the associated studies, and re-evaluates their conclusions. Our analysis shows that the functional lengths of the phalanges of the little finger actually do follow a Fibonacci series and that the functional lengths of the index, long, and ring fingers follow a mathematical relative of the Fibonacci series.

Keywords Littler · Fibonacci · Hand · Phalanges · Mathematics · Golden ratio

The adaptability of human hands, especially for activities such as tool making, differentiates us from our primate relatives [14]. Although the mathematical relationships underlying biological forms have long been an active area of study, relatively few studies have addressed the properties of our hands in basic mathematical terms [3, 4, 11, 19, 20, 24]. In a landmark paper published in 1973, the eminent hand surgeon

J. William Littler, MD, proposed two mathematical relationships between the anatomic and functional geometry of the hand [12, 15]. His proposal that the motion of the tips of the fingers follow an equiangular spiral has been experimentally supported [7]. Studies have not supported his other idea that the lengths of the phalanges follow a Fibonacci series [8, 16]. This review, after providing the necessary mathematical background, reexamines Littler's claims, presents the associated studies, and re-evaluates their conclusions. With re-analysis of previously published experimental data, we show that the functional lengths of the phalanges of the little finger actually do follow a Fibonacci series and that the functional lengths of the index, long, and ring fingers follow a mathematical relative of the Fibonacci series, which we have called the Littler series in Dr. Littler's honor.

Mathematical Background [6, 9, 10, 13, 23]

The Fibonacci series, discovered in 1202 by the Italian Leonardo di Pisa, or Fibonacci, is a sequence of numbers for which, beginning with 0 and 1, the successive term is the sum of every two previous consecutive terms. The Fibonacci series begins 0, 1, 1, 2, 3, 5, 8, 13, 21, 34... Originally used to model rabbit reproduction, the Fibonacci series has proven useful in the modeling of biological and financial systems as well as in electronics and music [5]. It is the most basic summative sequence.

The general class of summative sequences, which can begin with any two numbers, are known as Lucas sequences, named after the nineteenth century French mathematician Edouard Lucas. For example, Lucas studied a series, related to the Fibonacci series that begins with the terms 2 and 1 (2, 1, 3, 4, 7...). Likewise, a Lucas sequence could begin with the numbers 1 and 1.3, for example, 1, 1.3, 2.3, 3.6, 5.9... All Lucas sequences are united by the property that as the series

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reaches an infinite number of terms, the ratio between a term and the term preceding it in the series approaches Φ :1 or approximately 1.618:1 (Fig. 1). This is an essential point to understand. With regard to the phalangeal lengths predicted by Dr. Littler, only as the series of adjacent lengths progresses will the ratio of adjacent lengths become close to Φ :1.

The ratio Φ :1 has been known since antiquity as the Golden Ratio or the Golden Mean or Number, for its apparent esthetic pleasure. The Golden Ratio is the ratio between two lines of unequal length, where the ratio of the lengths of the shorter to the longer is the same as the ratio between the lengths of the longer and the sum of the lengths, $a/b = b/(a + b)$ (Fig. 2). The Golden Ratio is also linked, through golden geometric shapes like triangles, rectangles, and pentagrams, to the equiangular spiral. Golden geometric shapes have the property that the ratio between the lengths of their sides is the Golden Ratio. The Golden Ratio has been used extensively in ancient and modern architecture, painting, and music, and is manifest in many forms in nature (Fig. 3) [1, 2, 17, 18, 22].

The equiangular spiral, also known as a logarithmic spiral or the proportional spiral, was first described by Descartes in 1638. It has the property that any radius drawn from the center of the spiral intersects the tangent of the spiral at the same angle. Therefore, it grows at a consistent rate without changing shape. It occurs frequently in nature, most notably in the shape of a nautilus shell. It is also evident in the shape of galaxies, the attack path of a hawk, and in the growth pattern of the mandible [17, 21] (Fig. 4). Equiangular spirals can be created based on the relation-

$$b/a = (a+b) / b = \Phi \approx 1.6180339887\dots$$

Figure 2 The Golden Ratio is a proportion of lengths of line segments, where the ratio between longer and shorter line lengths equals the ratio of the sum of the two line lengths and the longer line length. The ratios equal the Golden Ratio, Φ .

ships inherent in golden rectangles, triangles, and summative sequences (Fig. 5).

Analysis

In 1973, Dr. J. William Littler proposed in his essay “On the Adaptability of Man's Hand (with reference to the equiangular curve)” that the movement of the digits create an equilateral spiral due to the fact that the ratio of the lengths of the phalanges is related to the Fibonacci sequence. In the legend to his first figure, he states, “The primary finger flexor–extensor sweep executes a curve congruent with an equiangular spiral. Carpo-metacarpal-phalangeal and interphalangeal interaxial links determine this curve; their relationships approach closely the ratio 1.1.618...(The Golden Mean), approximated by the Fibonacci series (Fig. 4).” Note that the drawing alludes to the functional lengths around the joints axes and not to the actual bone lengths. Also note that Littler was not expecting the ratio of the adjacent lengths to equal Φ . He mentions that the relationships of the lengths approach the ratio 1:1.618... In the figure, he lists phalangeal lengths that correspond to the series 2.2, 3.3, 5.5, 8.8... As he gives no explanation for the lengths he assigns the phalanges, it is unknown whether the numbers are a theoretical example or are actual measured lengths. Also, in the footnote legend, he lists other phalangeal

Summative Series and Ratios x_n / x_{n-1}	
<i>Fibonacci</i>	0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55...
Ratio	1, 2, 1.5, 1.66, 1.60, 1.625, 1.615, 1.62 ... Φ
<i>Lucas</i>	2, 1, 3, 4, 7, 11, 18, 29, 47...
Ratio	0.5, 3, 1.33, 1.75, 1.57, 1.64, 1.61 ... Φ
<i>Littler</i>	1, 1.3, 2.6, 5.9, 8.5, 14.4, 22, 9.37, 3, 60.2...
Ratio	1.3, 2.2, 2.27, 1.44, 1.69, 1.59, 1.63, 1.61 ... Φ

Figure 1 Examples of summative series and the ratios of successive terms. Taken to the limit, the ratios of successive terms of summative series converges on limit; the ratios of successive terms of summative series converges on Φ .

Figure 3 The design of the Parthenon is based on golden rectangles.

Figure 4 The primary figure from Littler's article. Note the measurements from the centers of joint rotation, and the equiangular spirals that follow the path of the fingertip and the form of the nautilus shell. (*Hand* 5(3):188, 1973; [12]).

lengths. In the paper, he does not explain or discuss the origin of the data. Without an explanation, the validity of these lengths cannot be assured.

Since the numbers in Littler's paper cannot be interpreted as valid, reproducible scientific data, subsequent studies have tried to experimentally test his ideas.

In 1998, Gupta et al. confirmed, as Littler predicted, that the motion of the digits does correspond with an equiangular spiral, excluding the fifth digit, which differed slightly on extension [7]. To explain the divergence of the fifth digit from the equiangular spiral during extension, Gupta proposed that the abduction of the digit during extension is responsible (Fig. 6).

Figure 5 An example of how an equiangular spiral can be constructed from a series of related golden rectangles.

Figure 6 Littler's prediction of the equiangular spiral formed by the path of the fingertips (*Hand* 5(3):189, 1973; [12]).

Studies have also tested Littler's assertions regarding the lengths of the bones. Park et al. concluded that the anatomical bone lengths of the digits failed to correspond to the Fibonacci sequence [16]. Their study looked at the anatomic lengths of the bones measured by X-rays. A subsequent review of their paper by Markley revealed flaws in their analysis [13]. Park speculated, however, that the functional lengths of the digits may correspond to the Fibonacci sequence, but did not pursue the subject further.

Hamilton et al., also in response to Littler but without reference to Park, evaluated the functional lengths [8]. In this anatomic and radiographic study, the relative functional

Figure 7 Functional lengths of the phalanges as determined by Hamilton (*Journal of Hand Surgery* (Edinburgh, Scotland) 27:546, 2002; [8]).

lengths of the bones of the fingers were determined. These functional lengths correspond to the lengths between the transverse axes of joint rotation or the joint centers. Their average data from 197 subjects shows that the ratio of the distance from the metacarpophalangeal joint to the proximal interphalangeal joint (MCP-PIP) and the distance from PIP to phalangeal tip is 1:1 in all the fingers. The ratio of the distance from the PIP to the distal interphalangeal joint (PIP-DIP) and the distance from the DIP to the phalangeal tip (DIP-tip) is 1.3:1 for the index, middle, and ring fingers and 1:1 for the little finger. In other words, the ratios of the DIP-tip/PIP-DIP/MCP-PIP distances were 1:1:2 for the little finger and 1:1.3:2.3 for the other fingers (Fig. 7).

Hamilton et al. concluded that their data neither agreed with the ratio of 1:1.618 ($1:\Phi$) that they claim Littler predicted nor with the ratio of 1:1.5 that is shown in the first figure of Littler's paper that relates the ratio between the DIP-tip distance and the PIP-DIP distance (given by Littler as 2.2 and 3.3, respectively). Unfortunately, Hamilton et al. misinterpreted the nature of the mathematical relationship that Littler proposed. Specifically, they analyzed whether the ratios of the lengths of adjacent bones equaled $1:\Phi$ or 1:1.5 rather than examining the data as a summative series. They incorrectly concluded that their data did not comply with Littler's proposition. Most importantly, they failed to notice that their data did correspond with the trademark relationship of the Fibonacci series as seen in any Lucas sequence, that the sum of the first two functional lengths equaled the third functional length. That is, the sum of the DIP-tip distance and the PIP-DIP distance is equal to the MCP-PIP distance. While they were correct that their data did not match either of Littler's ratios, they did not see that their data still supported Littler's basic idea using a series akin to the Fibonacci series. Their data for the index, middle, and ring digits does correspond with a related series, referred to as the Littler series, where the starting values are x and $1.3x$, and the next term in the series is $2.3x$. Their data for the fifth digit, however, corresponds with a series where the starting values are y and y , and the next term in the series is $2y$. This does in fact correspond with a beginning portion of the Fibonacci series: 0, 1, 1, 2, 3, 5, 8... (Fig. 7) Their data actually confirms the relationship between the phalangeal lengths and the equiangular spiral path of the fingers that Littler predicted.

The mathematical relationships described in this paper are important for several reasons. While there is inherent beauty in the intersection of math and nature, these mathematical relationships can better help us understand the basic anatomical framework and corresponding function of the hand. They may be useful in establishing functional phalangeal lengths for hand reconstruction or for the design or placement of prostheses. Through their connection with equiangular spirals, they could aid in individualized

monitoring of injured hands during therapy or with identifying the anatomic source of functional problems.

Additional study is needed to determine if the functional lengths of the metacarpals also follow a summative series. Note that in Littler's diagram of the functional length of the metacarpal, he defined the proximal axis within the capitate, not at the base of the metacarpal.

In his landmark 1973 paper, Littler demonstrated amazing insights into the anatomic and functional geometry of the hand. While previous analyses incorrectly interpreted and tested Littler's claims, this review shows that Littler correctly intuited important mathematical relationships of the hand. Based on published radiological and anatomical studies, his prediction that the lengths of the phalanges for the little finger follow a Fibonacci series was accurate. The lengths for the index, long, and ring fingers follow a newly identified, specific mathematical pattern, the Littler series, which is closely related to the Fibonacci series. The spirals that the tips of the fingers follow during normal motion are a direct result of the mathematical relationship of the phalangeal lengths. These relationships, which help illustrate the basic anatomical and functional foundations of the hand, are a tribute to the insight of Dr. Littler.

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