Robust stability analysis of delayed Takagi-Sugeno fuzzy Hopfield neural networks with discontinuous activation functions

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Abstract In this paper, the global robust stability problem of delayed Takagi–Sugeno fuzzy Hopfield neural networks with discontinuous activation functions (TSFHNNs) is considered. Based on Lyapunov stability theory and M -matrices theory, we derive a stability criterion to guarantee the global robust stability of TSFHNNs. Compared with the existing literature, we remove the assumptions on the neuron activations such as Lipschitz conditions, bounded, monotonic increasing property or the assumption that the right-limit value is bigger than the left one at the discontinuous point. Finally, two numerical examples are given to show the effectiveness of the proposed stability results.

Keyword Delayed Hopfield neural network · Global robust asymptotical stability · Discontinuous neuron activations · M -matrices theory · T–S fuzzy model

Introduction

Neural networks with discontinuous activation functions, which were first introduced by Forti and Nistri [1], are important and do frequently arise in practice when dealing with dynamical systems possessing high-slope nonlinear elements. Recently, some authors started the dynamical analysis of the neural networks with discontinuous activation functions [2–13]. In [2, 3] some sufficient conditions have been obtained for the global convergence stability of neural networks with discontinuous neuron activations. The global asymptotic stability for periodic solutions of Hopfield neural networks with discontinuous activation functions are derived in [4]. In [5], the dynamical behaviors of a class of recurrent neural networks with discontinuous neuron activations were considered. Moreover, the stability analysis of Cohen-Grossberg neural networks with discontinuous neuron activations was studied in [6, 7]. However, as far as we know, almost all the results on the stability of neural networks with discontinuous activation functions are conducted under some special assumptions on neuron activation functions. These assumptions frequently include those such as Lipschitz conditions, bounded, monotonic increasing property or the fact that the right-limit value is bigger than the left one at the discontinuous point [4, 5, 7, 9, 10]. Based on this motivation, in this paper, we consider the stability analysis of Hopfield neural networks with discontinuous activation functions without these assumptions on the neuron activations.

In practical implementation of neural networks, time delays often arise in the processing of information storage and transmission. Furthermore, uncertainties are inevitable in neural networks due to the existence of modelling errors and external disturbance. It is important to ensure that the designed network is stable in the presence of these

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uncertainties. As a result, the robust stability analysis of neural networks with delays has been investigated by many researchers over the past years.

On the other hand, fuzzy logic theory has been efficiently developed to many applications and is shown to be an effective approach to neural network and its stability. Takagi–Sugeno (T–S) first introduced fuzzy models in [14] and then the T–S fuzzy model is successfully and effectively used in neural network [15–21]. Thus, there has been an increasing interest in the study of this fuzzy neural network model. It is known that, by using the T–S fuzzy model, a nonlinear system can be described as a weighted sum of some simple linear subsystems, and then can be stabilized by a model-based fuzzy control. Many stability and control issues related to the T–S fuzzy neural network have been studied in the past two decades. In [15], some results on the stability analysis of T–S fuzzy cellular neural networks were obtained. The global asymptotic stability problem of T–S fuzzy BAM neural networks with time-varying delays and parameter uncertainties is considered in [20]. However, as far as we know, almost all documents with regard to fuzzy neural network stability analysis have been proposed based on linear matrix inequality (LMI) approach and Lyapunov stability theorem [15–21].

Based on the above discussion, we will generalize the ordinary T–S fuzzy models to express a class of delayed Hopfield neural networks with discontinuous activation functions. The main purpose of this paper is to study the global robust stability results of TSFHNNs in terms of Lyapunov stability theory and the M -matrices theory. The neuron activations constraints in this paper are less compared with the existing literature. we also provide two numerical examples to demonstrate the effectiveness of the proposed stability results.

System description and preliminaries

Consider the following delayed Hopfield neural networks with discontinuous activation functions described by:

$$\dot{x}(t) = -Dx(t) + (A + \Delta A)g(x(t)) + (B + \Delta B)g(x(t - \tau)), \quad (1)$$

where $x(t) = (x_1(t), \dots, x_n(t))^T$ is the state vector of the neural network; $D = \text{diag}\{d_1, \dots, d_n\}$ denotes the real diagonal matrix, and $d_i > 0$, $(i = 1, 2, \dots, n)$ are the neuron self-inhibitions; τ is the unknown time delay; $A = (a_{ij})_{n \times n}$ and $B = (b_{ij})_{n \times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. The matrices ΔA and ΔB represent the parameters uncertainties, respectively. Moreover, $g(x(t)) = (g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t)))^T$ models the output vector of the neural network.

For convenience, first, we shall use the following notation.

Let $\mathbb{R}^n$ ($n \geq 0$) be an n -dimensional real Euclidean space. Let A^T denote the transpose of matrix A and $\text{diag}[\cdot]$ denote a diagonal matrix. $F > 0$ ($F \geq 0$) implies that the matrix F is symmetric positive definite (positive semidefinite). I and 0 denote the identity matrix and zero matrix with compatible dimensions. Moreover, $K[W]$ denotes the closure of the convex hull of W . $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent the minimum and maximum eigenvalues of matrix A , respectively. We say that a column vector $x \in \mathbb{R}^n$ is positive if $x_i \geq 0$, $i = 1, 2, \dots, n$. Given a positive vector $\beta \in \mathbb{R}^n$,

$$\|x\|_{\beta} = \sum_{i=1}^n \beta_i |x_i|,$$

defines a norm in $\mathbb{R}^n$. Furthermore, the norm of matrix $G = (g_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$ is defined as

$$\|G\| = \sup\{\|Gx\| : \|x\| = 1\} = \sqrt{\lambda_{\max}(G^T G)}.$$

Second, let the activations belong to the following set of discontinuous functions.

Assumption 1 Throughout the paper, we assumed that g_i are continuous and only have a finite number of jump discontinuous points in every compact set of $\mathbb{R}$.

If $g(\cdot)$ meets the Assumption 1, the right-hand side of (1) is discontinuous.

We would like to introduce some definitions and preliminary lemmas, which will be used throughout the paper.

Definition 1 (Forti and Nistri [1]) Suppose $E \subset \mathbb{R}^n$. Then $x \mapsto F(x)$ is called a set-value map from $E \hookrightarrow \mathbb{R}^n$ if to each point x of a set $E \subset \mathbb{R}^n$, there corresponds a nonempty set $F(x) \subset \mathbb{R}^n$. A set-value map F with nonempty values is said to be upper semicontinuous at $x_0 \in E$, if for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$. $F(x)$ is said to have a closed (convex, compact) image if for each $x \in E$, $F(x)$ is closed (convex, compact). Graph $(F(E)) = \{(x, y) \mid x \in E, \text{ and } y \in F(x)\}$, where E is subset of $\mathbb{R}^n$.

Definition 2 A matrix $A \in \mathbb{R}^{n,n}$, is said to be Lyapunov diagonally stable (LDS), if there exists a positive diagonal matrix P such that the symmetric part of PA is positive definite, i.e.

$$(PA)^S = \frac{1}{2}(PA + A^T P) > 0.$$

Definition 3 We say that a square matrix is an M -matrix if it has all non-positive elements outside the diagonal and all positive successive principal minors.

Now we introduce the concept of Filippov solution. Consider the following system:

$$\frac{dx}{dt} = f(x), \quad (2)$$

where $f(\cdot)$ is not continuous. We have the following definition.

Definition 4 ([22]) Consider the set-value map $\Phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ defined as

$$\Phi(x) = \bigcap_{\delta > 0} \bigcap_{\mu(N)=0} K[f(B(x, \delta) - N)],$$

where $K(E)$ is the closure of the convex hull of set E . $B(x, \delta) = \{y : \|y - x\| \leq \delta\}$ and $\mu(N)$ is Lebesgue measure of set N . A solution of the cauchy problem for (2) with condition $x(0) = x_0$ is an absolutely continuous function $x(t)$, $t \in [0, T]$, which satisfies $x(0) = x_0$, and differential inclusion:

$$\frac{dx}{dt} \in \Phi(x), \quad \text{a.e. } t \in [0, T]. \quad (3)$$

For the good approximations of solutions of actual systems with high-gain nonlinearities [23], the concept of the solution in the sense of Filippov is to find a wide application in engineering applications. Now we extend the concept of Filippov solution to the delayed differential Eq. 1 as follows:

Definition 5 ([2]) A function $x : [-\tau, T) \rightarrow \mathbb{R}^n$, $T \in (0, +\infty]$, is a state solution of (1) on $[0, T)$ if

- (1) x is continuous on $[-\tau, T)$, and absolutely continuous on $[0, T)$;
- (2) there exists a measurable function $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_n)^T : [-\tau, T) \rightarrow \mathbb{R}^n$ such that $\gamma(t) \in K[g(x(t))]$ for a.a. $t \in [-\tau, T)$ and

$$\dot{x}(t) = -Dx(t) + A\gamma(t) + B\gamma(t - \tau), \quad \text{for a. a. } t \in [0, T), \quad (4)$$

where $K[g(x)] = (K[g_1(x)], K[g_2(x)], \dots, K[g_n(x)])^T$, and $K[g_i(x)] = [g_i^-(x), g_i^+(x)]$.

Any function γ satisfying (2) is called an output associated to x and the couple $[x, \gamma]$ will be simply called a solution of (1).

Observe that the above definition of solution to (1) implies that

$$\dot{x}(t) \in -Dx(t) + AK[g(x(t))] + BK[g(x(t - \tau))], \quad \text{for a. a. } t \in [0, T), \quad (5)$$

namely, x is a solution of (1) in the sense of Filippov [22].

In what following, it is convenient to give the following definition of an initial value problem (IVP) associated to (1).

Definition 6 ([2]) (IVP) For any continuous function $\phi : [-\tau, 0] \rightarrow \mathbb{R}^n$ and any measurable selection $\psi : [-\tau, 0] \rightarrow \mathbb{R}^n$, such that $\psi(s) \in K[g(\phi(s))]$ for a.a. $s \in [-\tau, 0]$, by an IVP associated to (1) with initial condition (ϕ, ψ) , we mean the following problem: find a couple of functions $[x, \gamma] : [-\tau, T) \rightarrow \mathbb{R}^n \times \mathbb{R}^n$, such that x is a solution of (1) on $[-\tau, T)$ for some $T > 0$; and γ is an output associated to x ; and

$$\begin{cases} \dot{x}(t) = -Dx(t) + A\gamma(t) + B\gamma(t - \tau), & \text{for a. a. } t \in [0, T), \\ \gamma(t) \in K[g(x(t))], & \text{for a. a. } t \in [0, T), \\ x(s) = \phi(s), & \forall s \in [-\tau, 0], \\ \gamma(s) = \psi(s), & \text{for a. a. } s \in [-\tau, 0]. \end{cases}$$

Definition 7 (Equilibrium) An equilibrium point (EP) of system (1) is a vector $x^* \in \mathbb{R}^n$ that satisfies

$$0 \in -Dx^* + (A + B)K[g(x^*)].$$

Equivalently, $x^* \in \mathbb{R}^n$ is an EP of (1) if there exists $\gamma^* \in K[g(x^*)]$ such that

$$0 = -Dx^* + (A + B)\gamma^*. \quad (6)$$

Any vector $\gamma^* \in K[g(x^*)]$ satisfying (6) is called an output equilibrium point (OEP) of system (1) corresponding to the EP x^* .

Definition 8 The equilibrium point x^* of system (1) is said to be globally asymptotically stable, if for any solution $x(t)$ of (1), whose existence interval is $[0, +\infty)$, we have

$$\lim_{t \rightarrow \infty} x(t) = x^*.$$

Lemma 1 ([7], Lemma 2) Let $x : J \rightarrow \mathbb{R}^n$ be a state solution of (1). Then, function $t \mapsto \|x(t)\|_\beta$ is locally Lipschitz continuous and

$$\frac{d}{dt}\|x\|_\beta = v(t)^T \dot{x}(t) = \sum_{i=1}^n v_i(t) \dot{x}_i(t), \quad \text{for a. a. } t \in J,$$

where $v_i(t) = \beta_i \text{sign}(x_i(t))$, if $x_i(t) \neq 0$; and $v_i(t)$ can be arbitrarily chosen in $[-\beta_i, \beta_i]$, if $x_i(t) = 0$.

Lemma 2 ([8], Theorem 1) (The existence of the equilibrium) If there exist a positive definite diagonal matrix $P = \text{diag}\{P_1, P_2, \dots, P_n\}$ and a positive definite symmetric matrix Q such that

$$\begin{bmatrix} -PA - A^T P - Q & -PB \\ -B^T P & Q \end{bmatrix} > 0, \quad (7)$$

then there exists an equilibrium point of system (1).

Based on the above definitions and lemmas, we obtain that the equilibrium point of the systems (1) exist. In this section, we will consider a delayed Hopfield neural networks with discontinuous activation functions, which is represented by a T-S fuzzy model composed of a set of

fuzzy implications. Based on the T–S fuzzy model concept, a general class of T–S fuzzy Hopfield neural networks with discontinuous activation functions is considered here. The model of Takagi-Sugeno fuzzy Hopfield neural networks with time delays is described as follows.

Plant Rule k

IF $\theta_1(t)$ is M_{k1} and ... and $\theta_r(t)$ is M_{kr}

THEN

$$\begin{aligned}\dot{x}(t) &= -D_k x(t) + (A_k + \Delta A_k)g(x(t)) \\ &\quad + (B_k + \Delta B_k)g(x(t - \tau)), \\ x(t) &= \phi_k(t) \in C([- \tau, 0], \mathbb{R}^n),\end{aligned}\quad (8)$$

where $\theta_i(t)$, ($i = 1, 2, \dots, n$) are known variables. M_{kl} ($k \in \{1, 2, \dots, m\}$, $l \in \{1, 2, \dots, r\}$) is the fuzzy set and m is the number of model rules. The matrix $D_k = \text{diag}\{(d_k)_1, \dots, (d_k)_n\}$ is a positive diagonal matrix, $A = (a_k)_{ij}$, $B = (b_k)_{ij} \in \mathbb{R}^{n \times n}$ are known constant matrices with appropriate dimensions. The parameter uncertainties $\Delta A_k, \Delta B_k$ are time varying matrices with appropriate dimensions, which are defined as follows:

$$[\Delta A_k, \Delta B_k] = M_k F(t) [E_{1k}, E_{2k}],$$

where E_{1k}, E_{2k}, M_k are known constant matrices of appropriate dimensions and $F(t)$ is an known time varying matrix with Lebesgue measurable elements bounded by

$$F^T(t)F(t) \leq I.$$

By using the fuzzy inference method with a singleton fuzzification, product inference and centre average defuzzification, the overall fuzzy model is of the following form

$$\begin{aligned}\dot{x}(t) &= \sum_{k=1}^m \eta_k(\theta(t)) [-D_k x(t) + (A_k + \Delta A_k)g(x(t)) \\ &\quad + (B_k + \Delta B_k)g(x(t - \tau))], \\ x(t) &= \sum_{k=1}^m \eta_k(\theta(t)) \cdot \phi_k(t), \quad t \in [- \tau, 0],\end{aligned}\quad (9)$$

where

$$\eta_k(\theta(t)) = \frac{\omega_k(\theta(t))}{\sum_{k=1}^m \omega_k(\theta(t))}, \quad \omega_k(\theta(t)) = \prod_{l=1}^r M_{kl}(\theta(t)),$$

respectively. The term $M_{kl}(\theta(t))$ is the grade membership of $\theta_l(t)$ in M_{kl} .

We assume that

$$\omega_k(\theta(t)) \geq 0, \quad \sum_{k=1}^m \eta_k(\theta(t)) = 1.$$

For convenience, let $\bar{A}_k = A_k + \Delta A_k = ((\bar{a}_k)_{ij})_{n \times n}$, $\bar{B}_k = B_k + \Delta B_k = ((\bar{b}_k)_{ij})_{n \times n}$, then the system (9) can be expressed as follows:

$$\dot{x}(t) = \sum_{k=1}^m \eta_k(\theta(t)) [-D_k x(t) + \bar{A}_k g(x(t)) + \bar{B}_k g(x(t - \tau))]. \quad (10)$$

In this paper, our objective is to study the robust stability for uncertain fuzzy Hopfield neural networks (10) with discontinuous activation functions and time delays.

To establish the main results for fuzzy Hopfield neural networks (10), it is necessary to make the following assumption:

Assumption 2 We have $(\bar{a}_k)_{ii} < 0$ for $i = 1, 2, \dots, n$, and the matrix $C_k = ((c_k)_{ij}) \in \mathbb{R}^{n \times n}$ is a M-matrix, where

$$(c_k)_{ij} = \begin{cases} |(\bar{a}_k)_{ii}| - |(\bar{b}_k)_{ii}|, & i = j, \\ -|(\bar{a}_k)_{ij}| - |(\bar{b}_k)_{ij}|, & i \neq j. \end{cases} \quad (11)$$

Global robust stability result

In this section, we will derive a stability criterion for delayed fuzzy Hopfield neural networks with discontinuous activation functions.

Theorem 1 Under the Assumptions 1–2, the equilibrium point of systems (10) is globally robust asymptotically stable if the conditions of Lemma 2 hold.

Proof Consider the Lyapunov functional

$$V(t) = \|x(t)\|_\beta + \sum_{i,j=1}^n \beta_i |(\bar{b}_k)_{ij}| \int_{t-\tau}^t |g_i(x(s))| ds. \quad (12)$$

By means of Lemma 1, we have

$$\frac{d}{dt} \|x\|_\beta = v(t)^T \dot{x}(t) = \sum_{i=1}^n v_i(t) \dot{x}_i(t), \quad \text{for a. a. } t \in J,$$

with

$$v_i(t) = \begin{cases} \beta_i \text{sign}(x_i(t)), & x_i(t) \neq 0, \\ \beta_i \text{sign}(g_i(x(t))), & x_i(t) = 0, g_i(x(t)) \neq 0, \\ 0, & x_i(t) = g_i(x(t)) = 0. \end{cases} \quad (13)$$

According to (13), we have

$$v_i(t)x_i(t) = \beta_i |x_i(t)|, \quad \text{and } v_i(t)g_i(x(t)) = \beta_i |g_i(x(t))|. \quad (14)$$

Now let us evaluate the derivative of $V(t)$ along the trajectories of (10), that is

$$\begin{aligned}
\dot{V}(t)|_{(10)} &= \sum_{i=1}^n v_i(t) \dot{x}_i(t) + \sum_{i,j=1}^n \beta_i |(\bar{b}_k)_{ij}| [|g_j(x(t))| \\
&\quad - |g_j(x(t-\tau))|] \\
&= \sum_{i=1}^n v_i(t) \left\{ \sum_{k=1}^m \eta_k(\theta(t)) [-(d_k)_i x_i(t) \right. \\
&\quad + \sum_{j=1}^n (\bar{a}_k)_{ij} g_j(x(t)) + \sum_{j=1}^n (\bar{b}_k)_{ij} g_j(x(t-\tau))] \Big\} \\
&\quad + \sum_{i,j=1}^n \beta_i |(\bar{b}_k)_{ij}| [|g_j(x(t))| - |g_j(x(t-\tau))|] \\
&= \sum_{k=1}^m \eta_k(\theta(t)) \left\{ - \sum_{i=1}^n (d_k)_i \beta_i |x_i(t)| \right. \\
&\quad + \sum_{i=1}^n [\beta_i \cdot (\bar{a}_k)_{ii} |g_i(x(t))| \\
&\quad + \sum_{i \neq j} v_i(t) (\bar{a}_k)_{ij} g_j(x(t))] \\
&\quad + \sum_{i,j=1}^n v_i(t) (\bar{b}_k)_{ij} g_j(x(t-\tau))] \Big\} \\
&\quad + \sum_{i,j=1}^n \beta_i |(\bar{b}_k)_{ij}| [|g_j(x(t))| - |g_j(x(t-\tau))|].
\end{aligned} \tag{15}$$

From (13) and (14), we know $\sum_{i \neq j} v_i(t) (\bar{a}_k)_{ij} g_j(x(t)) = 0$, if $g_i(x(t)) = 0$; if $g_i(x(t)) \neq 0$, we have

$$\begin{aligned}
\sum_{i \neq j} v_i(t) (\bar{a}_k)_{ij} g_j(x(t)) &= \sum_{i \neq j} v_i(t) (\bar{a}_k)_{ij} g_i(x(t)) \frac{g_j(x(t))}{g_i(x(t))} \\
&= \sum_{i \neq j} \beta_i (\bar{a}_k)_{ij} |g_i(x(t))| \frac{g_j(x(t))}{g_i(x(t))} \\
&\leq \sum_{i \neq j} \beta_i (\bar{a}_k)_{ij} |g_i(x(t))| \frac{|g_j(x(t))|}{|g_i(x(t))|} \\
&= \sum_{i \neq j} \beta_i (\bar{a}_k)_{ij} |g_j(x(t))|.
\end{aligned} \tag{16}$$

Thus, substituting (16) into (15), we have

$$\begin{aligned}
\dot{V}(t)|_{(10)} &= \sum_{k=1}^m \eta_k(\theta(t)) \left\{ - \sum_{i=1}^n (d_k)_i \beta_i |x_i(t)| \right. \\
&\quad + \sum_{i=1}^n [\beta_i \cdot (\bar{a}_k)_{ii} |g_i(x(t))| \\
&\quad + \sum_{i \neq j} \beta_i \cdot (\bar{a}_k)_{ij} |g_j(x(t))|] \\
&\quad + \sum_{i,j=1}^n \beta_i \cdot (\bar{b}_k)_{ij} |g_j(x(t-\tau))| \Big\}
\end{aligned}$$

$$\begin{aligned}
&\quad + \sum_{i,j=1}^n \beta_i |(\bar{b}_k)_{ij}| [|g_j(x(t))| - |g_j(x(t-\tau))|] \\
&\leq \sum_{k=1}^m \eta_k(\theta(t)) \left\{ - \sum_{i=1}^n (d_k)_i \beta_i |x_i(t)| \right. \\
&\quad + \sum_{i=1}^n [\beta_i \cdot (\bar{a}_k)_{ii} |g_i(x(t))| \\
&\quad + \sum_{i \neq j} \beta_i \cdot (\bar{a}_k)_{ij} |g_j(x(t))| + \beta_i |(\bar{b}_k)_{ii}| \cdot |g_i(x(t))| \\
&\quad + \sum_{i \neq j} \beta_i |(\bar{b}_k)_{ij}| \cdot |g_j(x(t))|] \Big\} \\
&\leq - \sum_{k=1}^m \eta_k(\theta(t)) D_k \beta \|x(t)\| + \sum_{k=1}^m \eta_k(\theta(t)) \\
&\quad \times \left\{ \sum_{i=1}^n [-\beta_i |(\bar{a}_k)_{ii}| \cdot |g_i(x(t))| \right. \\
&\quad + \beta_i |(\bar{b}_k)_{ii}| \cdot |g_i(x(t))| + \sum_{i \neq j} \beta_i |(\bar{a}_k)_{ij}| \cdot |g_j(x(t))| \\
&\quad + \sum_{i \neq j} \beta_i |(\bar{b}_k)_{ij}| \cdot |g_j(x(t))|] \Big\} \\
&= - \sum_{k=1}^m \eta_k(\theta(t)) D_k \beta \|x(t)\| \\
&\quad - \sum_{k=1}^m \eta_k(\theta(t)) \beta C(|g_1(x(t))|, \dots, |g_n(x(t))|)^T \\
&\leq - \sum_{k=1}^m \eta_k(\theta(t)) D_k \beta \|x(t)\|.
\end{aligned} \tag{17}$$

Therefore, $\dot{V}(t)$ is negative definite. Based on the Lyapunov stability theory, one can see that the equilibrium point of (10) is globally asymptotically stable. This completes the proof. $\square$

Remark 1 From Theorem 1, the uniqueness of the equilibrium is also obtained. Moreover, the fuzzy neural network model (10) contains the parametric uncertainties ΔA and ΔB , so our results are robust.

Remark 2 To our knowledge, almost all documents with regard to fuzzy neural network stability analysis have been proposed based on linear matrix inequality (LMI) approach and Lyapunov stability theorem [15, 16, 17, 18, 19, 20, 21]. In this paper, we discuss the global robust stability problem of uncertain T–S fuzzy delayed Hopfield neural networks with discontinuous activation functions in view of Lyapunov stability theory and M -matrices theory. Thus, the outcomes are brand-new and original compared to existing results.

Remark 3 As far as we know, in the existing literature, almost all results on the stability of neural networks with

discontinuous activation functions are conducted under some special assumptions on neuron activation functions. These assumptions frequently include those such as Lipschitz conditions, bounded, monotonic increasing property or the fact that the right-limit value is bigger than the left one at the discontinuous point [4, 5, 7, 9, 10, 12]. Therefore, our results are more general.

Numerical simulations

In this section, we present two numerical examples to show the effectiveness of the designed controllers.

Example 1 Consider the T–S fuzzy model (10) of delayed fuzzy Hopfield neural network with discontinuous activation function when $m = 1$ and the parameters are as follows:

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} -4 & 4 \\ 2 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 \\ 5 & -3 \end{bmatrix},$$

$$\Delta A = \begin{bmatrix} -0.2\sin(t) & 0.1\cos(t) \\ 0.05\cos(t) & -0.5\sin(t) \end{bmatrix},$$

$$\Delta B = \begin{bmatrix} -0.4\sin(t) & 0.3\cos(t) \\ 0.05\cos(t) & -0.1\sin(t) \end{bmatrix}.$$

The discontinuous activation function is described by

$$g(s) = \begin{cases} 1 + s, & \text{if } s \geq 20, \\ \tanh(s), & \text{if } -20 < s < 20, \\ -1 + s, & \text{if } s \leq -20. \end{cases}$$

It is easy to see that the conditions of Theorem 1 are satisfied. Indeed, we can deduce that

$$C = \begin{bmatrix} 2 - 0.2 | \sin(t) | & -7 - 0.4 | \cos(t) | \\ -7 - 0.1 | \cos(t) | & 3 + 0.4 | \sin(t) | \end{bmatrix}$$

is a M -matrix. Hence, by Theorem 1, we conclude that the equilibrium of (10) is globally robust asymptotically stable. The trajectories of system (10) are shown in Fig. 1.

Example 2 Consider the Plant Rule Eq. (10) with $m = 2$, the T–S fuzzy model of delayed fuzzy Hopfield neural network with discontinuous activation function is of the following form:

Plant Rules

Rule 1: IF $\theta_1(t)$ is M_{k1} , THEN

$$\dot{x}(t) = -D_1x(t) + (A_1 + \Delta A_1)g(x(t)) + (B_1 + \Delta B_1)g(x(t - \tau)), \quad (18)$$

Rule 2: IF $\theta_2(t)$ is M_{k2} , THEN

Fig. 1 State variations of x_1 and x_2 in Example 1 when $m = 1$

$$\dot{x}(t) = -D_2x(t) + (A_2 + \Delta A_2)g(x(t)) + (B_2 + \Delta B_2)g(x(t - \tau)), \quad (19)$$

where the discontinuous activation function is described by

$$g(s) = \begin{cases} 1 + 0.2s, & \text{if } s \geq 50, \\ \tanh(s), & \text{if } -50 < s < 50, \\ -1 + 0.2s, & \text{if } s \leq -50. \end{cases}$$

The membership functions for rules 1 and 2 are $M_{k1} = \frac{1}{e^{-2\theta_1(t) - \theta_2(t)}}$, $M_{k2} = 1 - M_{k1}$, let $\tau = 5$.

$$D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.6 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -6 & 2 \\ 3 & -4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -4 & 4 \\ 2 & -3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -2 & 1 \\ 1.2 & -2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 2 \\ 0.3 & 0.6 \end{bmatrix},$$

$$\Delta A_1 = \begin{bmatrix} -0.5\sin(t) & 0.1\cos(t) \\ 0.2\cos(t) & -0.3\sin(t) \end{bmatrix},$$

$$\Delta A_2 = \begin{bmatrix} -0.2\sin(t) & 0.02\cos(t) \\ 0.2\cos(t) & -0.2\sin(t) \end{bmatrix},$$

$$\Delta B_1 = \begin{bmatrix} -0.1\sin(t) & 0.3\cos(t) \\ 0.4\cos(t) & -0.2\sin(t) \end{bmatrix},$$

$$\Delta B_2 = \begin{bmatrix} -0.2\sin(t) & 0.01\cos(t) \\ 0.01\cos(t) & 0.2\sin(t) \end{bmatrix}.$$

It is easy to see that the conditions of the Theorem 1 are satisfied. Indeed, we can deduce that

$$C_1 = \begin{bmatrix} 4 + 0.4 | \sin(t) | & -3 - 0.4 | \cos(t) | \\ -4.2 - 0.6 | \cos(t) | & 2 + 0.1 | \sin(t) | \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 3 & -6 - 0.3 | \cos(t) | \\ -2.3 - 0.21 | \cos(t) | & 2.4 \end{bmatrix}$$

Fig. 2 State variations of x_1 and x_2 in Example 2 when $m = 1$

Fig. 3 State variations of x_1 and x_2 in Example 2 when $m = 2$

are M -matrix. Hence, by Theorem 1, we conclude that the equilibrium of (10) is globally asymptotically stable.

The trajectories of system (10) are shown in Fig. 2 when $m = 1$, accordingly, the trajectories of system (10) are shown in Fig. 3 when $m = 2$.

Conclusions

This paper discusses the stability problem for a class of T–S fuzzy delayed Hopfield neural networks with discontinuous activation functions. Different from existing results, we obtain the stability criterion to guarantee the global robust stability of TSFHNNs. Moreover, we remove

some assumptions on the neuron activations such as bounded, monotonic increasing property or the fact that the right-limit value is bigger than the left one at the discontinuous point. Finally, two numerical examples are given to verify the theoretical results.

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