A new method to combine 3D reconstruction volumes for multiple parallel circular cone beam orbits

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Purpose: This article presents a new reconstruction method for 3D imaging using a multiple 360° circular orbit cone beam CT system, specifically a way to combine 3D volumes reconstructed with each orbit. The main goal is to improve the noise performance in the combined image while avoiding cone beam artifacts.

Methods: The cone beam projection data of each orbit are reconstructed using the FDK algorithm. When at least a portion of the total volume can be reconstructed by more than one source, the proposed combination method combines these overlap regions using weighted averaging in frequency space. The local exactness and the noise performance of the combination method were tested with computer simulations of a Defrise phantom, a FORBILD head phantom, and uniform noise in the raw data.

Results: A noiseless simulation showed that the local exactness of the reconstructed volume from the source with the smallest tilt angle was preserved in the combined image. A noise simulation demonstrated that the combination method improved the noise performance compared to a single orbit reconstruction.

Conclusions: In CT systems which have overlap volumes that can be reconstructed with data from more than one orbit and in which the spatial frequency content of each reconstruction can be calculated, the proposed method offers improved noise performance while keeping the local exactness of data from the source with the smallest tilt angle. © 2010 American Association of Physicists in Medicine. [DOI: [10.1118/1.3484058](https://doi.org/10.1118/1.3484058)]

Key words: computed tomography (CT), cone beam CT, FDK, noise, combination method

I. INTRODUCTION

Over the past few decades, cone beam CT has been steadily improved with the aim of imaging a thick volume with faster scan times, thinner slices, and reduced motion artifacts in a single gantry rotation.^{1–3}

For a circular orbit cone beam CT system, Feldkamp, Davis, and Kress⁴ developed a practical reconstruction method referred to as the FDK algorithm that is a generalization of 2D fan beam filtered backprojection. The FDK algorithm yields an exact image for the central plane. However, for large cone angles and slices away from the central plane, significant cone beam artifacts are produced in the reconstructed image due to the insufficient sampling of the single circular orbit.⁵ According to Tuy's⁶ condition, an exact 3D reconstruction from cone beam projections is possible only when there exists at least one cone beam source point on every plane that intersects the object, and clearly, a circular orbit cone beam CT system does not satisfy this condition.

The data insufficiency of 360° circular orbit cone beam CT has been described in frequency space by several authors.^{7–12} For a brief description, consider a single impulse object on the z -axis but off the central plane [Fig. 1(a)].

Since the impulse object is sampled by one oblique ray, the cone beam projection is analogous to the oblique 2D parallel ray projection shown in Fig. 1(b). According to the central slice theorem (CST),^{13,14} the 2D oblique parallel rays sample one oblique plane in frequency space [Fig. 1(c)]. During a 360° rotation, this plane sweeps through frequency space but fails to sample a missing cone region [Fig. 1(d)] which prevents exact reconstruction. However, for small cone angles, the FDK algorithm produces a tolerable artifact level in the reconstructed image. For example, current clinical systems image 2–4 cm thick volumes with little evidence of data insufficiency.

To acquire a complete data set for even wider volumes, several source orbits have been proposed. For example, orthogonal and oblique dual orbits were proposed in Refs. 6 and 15 and reconstruction algorithms based on the 3D Radon inversion formula^{16,17} and filtered backprojection^{18,19} were developed. The orthogonal circle-and-line,²⁰ saddle,^{21–23} and helical orbits^{24–28} were also investigated by several authors. A rotating line source CT system is another possible geometry to provide a complete data set since it effectively has multiple circular source orbits.^{29–31}

Recently, a multiple source inverse geometry CT (MS-

FIG. 1. (a) An impulse object scanned by a cone beam CT system. (b) The equivalent 2D parallel ray projection. (c) The sampled 2D plane in frequency space. (d) The missing cone region in frequency space after 360° rotation.

FIG. 2. Geometry of the MS-IGCT system.

IGCT) system was proposed (Fig. 2).³² The use of multiple source rows in the axial direction can result in the ability to image a thick volume in a single gantry rotation without cone beam artifacts. The MS-IGCT system has distributed sources in the transverse direction, each collecting data from a portion of the field-of-view (FOV), but the projection data of sources located at the same z -position can be rebinned into a full FOV cone beam data set, so the reconstruction problem reduces to the same one as for multiple parallel circular orbits. Thus, while the work in this paper was motivated by this MS-IGCT system, it is also relevant to systems with multiple parallel circular orbits and the problem will be discussed here in that context. If the MS-IGCT system has rather wide source spacing in the z -direction, it may not fully satisfy Tuy's sufficiency condition. Experience with narrow cone angle data and FDK reconstruction suggests that image quality should be acceptable, but an algorithm based on Radon inversion may not be applicable unless extrapolation methods in Radon space are applied.^{33,34}

In this paper, we present a new reconstruction method for 3D imaging using a multiple 360° circular orbit cone beam CT system, specifically for cases in which regions of the object can be reconstructed with data from multiple sources.

One simple approach to reconstruct values at a particular location is to use only the data from the source that has the smallest tilt angle at that location. The image would have optimum accuracy but this is inefficient from a noise perspective. Another straightforward approach is to average the FDK reconstructions from multiple sources. This is desirable for noise performance but would introduce cone beam artifacts from the higher tilt angle data. As a compromise approach, the reconstructed volumes from different sources can be combined using weighted averaging or ray-based weighting of the raw data could be used, where the weights can be determined by the tilt angle of each source for the overlap volume but still, cone beam artifacts would not be avoidable.

Our approach is related to Kudo's³⁵ work. In that work, the overlap volumes from two orthogonal circular cone beam orbits were combined in frequency space. While Kudo's work handled the redundant region properly, the data combination was performed using the same region in frequency space for the entire volume, which, as we will show, is sub-optimal from a noise perspective, especially for a large cone angle system. Here, we propose a way to combine the 3D volumes to improve the noise performance using a spatially dependent region in frequency space while avoiding cone

FIG. 3. A multiple circular orbit cone beam CT system. (a) A vertex path of each source and (b) an overlap volume.

beam artifacts. We first present the basic principle of the combination method and apply it to small and large cone angle systems which have different source spacing in the z -direction. A method to handle data truncation is also presented. Then, by using computer simulations, we examine the exactness and the noise performance of the combination method.

II. METHODS

The investigated system geometry uses n parallel circular vertex paths [shown in Fig. 3(a)] expressed as

$$S_k(\lambda) = (d \cos \lambda, d \sin \lambda, z_k) \quad \text{if } 0 \leq \lambda < 2\pi, \quad (1)$$

where k is the index of a source ($k=1, \dots, n$) and d is a source radius.

For the acquired projection data from each source, FDK produces a reconstructed image $i_k(\vec{x})$, where $\vec{x}=(x, y, z)^T$. It is known that in any region of the image, FDK correctly reconstructs the measured spatial frequencies (i.e., those outside the missing cone).¹⁰ Since each source has a divergent beam, there is an overlap volume that can be reconstructed from multiple sources as shown in Fig. 3(b). In some cases, one may want to reduce the cone angles to the minimum required to collect sufficient data, but some overlap is likely to remain. Further, all other parameters being equal, a larger cone angle increases the total x-ray flux and the volume of the overlap regions, and therefore can improve SNR for a given acquisition time or achieve a shorter acquisition time for a given SNR. We focus on this overlap volume and describe a combination method in frequency space that can improve the noise performance while maintaining the exactness of the reconstructed volume.

FIG. 4. (a) An on-axis point. (b) The missing cone region in frequency space for sources 2 and 3.

FIG. 5. (a) An off-axis point. (b) The asymmetric shape of the missing cone region in frequency space for sources 2 and 3.

II.A. Basic principle

To combine an overlap volume from different sources, we first consider a single on-axis point p , shown in Fig. 4(a), and three sources in the z -direction. In a small neighborhood around p , the cone beam rays from each source can be approximated as 2D parallel rays and therefore the CST holds. For the single point, source 1 has a zero tilt angle while sources 2 and 3 have relatively larger tilt angles. As a result, the reconstructed volumes from sources 2 and 3 have missing cone regions [i.e., $\bar{\Gamma}_2$ and $\bar{\Gamma}_3$ in Fig. 4(b)] in frequency space and the size of the missing cone region is determined by the tilt angle of each source for the point. For any volume, since the outside of the missing cone region is reconstructed correctly,¹⁰ this information can be utilized to improve the noise performance using weighted averaging in frequency space

FIG. 6. An entire overlap volume and segmented volumes.

TABLE I. Simulation parameters.

	Small cone angle system	Large cone angle system
Reconstructed volume size	$6.6 \times 6.6 \times 6.6 \text{ cm}^3$	$20 \times 20 \times 20 \text{ cm}^3$
z -location of source 1	2 cm	6 cm
z -location of source 2	0 cm	0 cm
z -location of source 3	-2 cm	-6 cm
SID		45 cm
DID		40 cm
Detector cell size	$0.5 \text{ mm} \times 0.5 \text{ mm}$ [384 (row) by 256 (column)]	$1.5 \text{ mm} \times 1.5 \text{ mm}$ [384 (row) by 256 (column)]
Source size	$0.5 \text{ mm} \times 0.5 \text{ mm}$	$1.5 \text{ mm} \times 1.5 \text{ mm}$
Number of view		400
The largest tilt angle of source 1	5° (for center slab), 7° (for lower slab)	18° (for center slab), 27° (for lower slab)
The largest tilt angle of source 2	5° (for upper and lower slab)	18° (for upper and lower slab)
The largest tilt angle of source 3	5° (for center slab), 7° (for upper slab)	18° (for center slab), 27° (for upper slab)

$$I_c(\vec{f}) = \begin{cases} I_1(\vec{f}) & \text{if } \vec{f} \in \bar{\Gamma}_2 \\ (I_1(\vec{f}) + I_2(\vec{f})/2) & \text{if } \vec{f} \in \Gamma_2 \cap \bar{\Gamma}_3 \\ (I_1(\vec{f}) + I_2(\vec{f}) + I_3(\vec{f})/3) & \text{otherwise} \end{cases}, \quad (2)$$

where $I_k(\vec{f})$ is Fourier transform of $i_k(\vec{x})$ and $\vec{f} = (f_x, f_y, f_z)$. While Eq. (2) uses abrupt transitions between regions and weighting values, smoother transitions can be used as discussed below.

Effectively, the missing cone region for source 2 is filled in with data from source 1 and the missing cone region for source 3 is filled in using data from sources 1 and 2. Since I_c is a weighted averaging of three data sets and does not have a missing cone region in frequency space, the combined data volume should have better noise performance than a single data volume, but should still have minimal artifacts and preserve the original spatial resolution.

For an off-axis point p , shown in Fig. 5(a), source 1 still has a zero tilt angle over all views while sources 2 and 3 have different tilt angles from view to view. Therefore, the missing cone regions in frequency space from sources 2 and 3 are asymmetric with respect to the f_z axis as shown in Fig. 5(b).¹⁰ The reconstructed volumes from sources 1–3 can still be combined in frequency space using Eq. (2).

Of course, to combine data in frequency space, we need to consider a volume of data (e.g., to take the Fourier transform). The combination concept for a single point can be extended to an overlap volume. The overlap volume is composed of many points, each with a locally varying missing cone region in frequency space.¹⁰ For any region and view angle, we can consider the point with the largest tilt angle

FIG. 7. 1D plot of the segmentation window function for three slabs for a small and a large cone angle system.

and the region of frequency space that is touched by the corresponding plane as the view angle is swept. The inscribed conical region in frequency space may be suspect, but frequencies outside that region should be properly reconstructed. We use this approach to define the frequency space region that needs to be filled in. If we consider the entire overlap volume imaged by three sources as shown in Fig. 6, in general, no single source can accurately portray the complete volume. However, if we subdivide the volume into three slabs as shown, each slab should be reconstructed relatively accurately by one source. The reconstruction from the other sources can provide information for part of frequency space as described.

FIG. 8. (a) 1D and (b) 2D plot of the segmentation window function for a large cone angle system.

Since the combined data volume is produced based on the largest tilt angle in the overlap volume, as the range of tilt angles increases (e.g., due to a larger segmented volume), more data need to be filled in during the combination. This can increase the variance of the combined data volume due to more data being discarded. To improve the noise performance further, the overlap volume can be segmented into smaller volumes and then combined. Since each segmented small volume has a smaller range of tilt angles, the amount of data used is maximized, and therefore the combined data volume has a reduced variance. We recognize that our method assumes that the cone beam artifacts generated by a region are localized to the segment that contains this region and have a minimal impact to the neighboring regions.

In our study, we compared the noise performance for two segmentation methods. The first method segmented the entire overlap volume into three slabs as shown in Fig. 6. In the second method, the entire overlap volume was segmented into 48 regions using segmentation windows (explained below). After combination, all segmented volumes were merged to produce a final combined image and the exactness and the noise performance of the final combined image were examined.

For a system with n circular x-ray orbits, the reconstruction and combination method for the overlap volume is summarized as follows:

- (1) Perform 3D FDK reconstruction with each source to obtain n image volumes $i_k(\vec{x})$ ($k=1, \dots, n$).
- (2) Segment $i_k(\vec{x})$ into m subvolumes $i_{kj}(\vec{x})$ ($j=1, \dots, m$).
- (3) Take 3D FFT for the segmented subvolume $I_{kj}(\vec{f})$.
- (4) For each source, define a missing cone region in frequency space using the largest tilt angle.
- (5) Perform weighted averaging in frequency space.

$$I_{cj}(\vec{f}) = \sum_{k=1}^n W_{kj}(\vec{f}) I_{kj}(\vec{f}), \quad (3)$$

where $W_{kj}(\vec{f})$ is a weighting function to handle redundant data in frequency space.

- (6) Take 3D IFFT to obtain combined subvolume image $i_{cj}(\vec{x})$.
- (7) Steps 3–6 are performed for all segmented subvolumes.
- (8) Subvolumes $i_{cj}(\vec{x})$ are merged to produce the combined image volume $i_c(\vec{x})$.

Above, we assumed that the entire volume is illuminated by all sources. However, in a real imaging system, axial truncation may not be avoidable due to the finite detector size and long axial extent of the object. In FDK reconstruction, axial truncation produces image artifacts, but reliable reconstruction is still achievable for the volume that sees the x-ray source during the complete rotation.^{36–38} Therefore, to handle axial truncation, the combination algorithm can be modified as follows:

- (1) Segment reliably reconstructed volumes from each source,
- (2) Select the overlap volume,

FIG. 9. Geometry for the simulation of axial truncation.

- (3) Combine the overlap volume in frequency space, and
- (4) Take 3D IFFT.

II.B. Simulations

The proposed combination method was tested for small and large cone angle systems. To investigate exactness (i.e., ability to avoid cone beam artifacts), we used noiseless projection data for a Defrise phantom in a water bath and also a FORBILD head phantom (www.imp.uni-erlangen.de/phantoms). The simulation parameters are summarized in Table I. The detector array was large enough to avoid axial truncation (axial truncation is studied below). For the noise simulation, we generated air scan data which had uniform distribution (10 000 photons per ray). The reconstructed image was composed of $256 \times 256 \times 256$ pixels for both systems. The reconstructed volume of each source was segmented into three slabs ($256 \times 256 \times 88$ matrix each) using the segmentation window functions in Fig. 7. Each slab was augmented to a $256 \times 256 \times 256$ matrix by zero padding. Since the small cone angle system has relatively small tilt angles for each slab (e.g., $<5^\circ$ and $<7^\circ$), the segmented regions were combined without further segmentation. For the large cone angle system, the overlap volume was segmented into 48 regions (16 regions in each of the slabs) using the segmentation window functions in Fig. 8. Each window function is composed of a $256 \times 256 \times 88$ matrix (i.e., composed of 88 slices with a 256×256 matrix), with the values at a voxel depending on the maximum of the x and y distances from the center, as shown in Fig. 8. Each slice has the same window function values. By using the smoothly varying function values, spectral leakage that can be caused by abrupt truncation was prevented.³⁹ Sixteen segmentation window functions were applied to each slab, which were then augmented to a $256 \times 256 \times 256$ matrix by zero padding. During the combination, we used a smoothly varying weighting function to avoid image artifacts caused by the abrupt transition in frequency space. For the upper slab, we first defined the following function:

$$T_{kj}(\vec{f}) = \begin{cases} 1 & \text{if } \theta < \theta_{kj} \\ 0 & \text{if } \theta > \theta_0 \text{ for } k=2,3, \\ \exp[\frac{((\cos^2(\theta) - \cos^2(\theta_{kj})) / ((\cos^2(\theta) - \cos^2(\theta_0)))]} & \text{otherwise} \end{cases}$$

$$\theta = \tan^{-1}(\sqrt{f_x^2 + f_y^2}/f_z), \quad \theta_0 = \theta_{kj} + \pi/45, \quad (4)$$

where θ_{kj} is the largest tilt angle of the subvolume $i_{kj}(\vec{x})$.

For any subvolume in the upper slab, the largest tilt angle of source 2 is less than that of source 3. So, we used following weighting function:

$$W_{1j}(\vec{f}) = T_{2j}(\vec{f}) + (T_{3j}(\vec{f}) - T_{2j}(\vec{f}))/2 + (1 - T_{3j}(\vec{f}))/3,$$

$$W_{2j}(\vec{f}) = (T_{3j}(\vec{f}) - T_{2j}(\vec{f}))/2 + (1 - T_{3j}(\vec{f}))/3,$$

(a)

(b)

FIG. 10. Reconstructed images of (a) a Defrise phantom in a water bath and (b) a FORBILD head phantom for the small cone angle system. (a) The coronal plane of each source and combined image and the corresponding vertical profile. (b) Coronal at $y=0.54$ cm, sagittal, axial, and difference sagittal image of source 1 (first row), source 2 (second row), source 3 (third row), and combined data (fourth row). The display windows are $[0-100 \text{ HU}]$ and $[-50 \text{ to } 50 \text{ HU}]$ for the reconstructed image and the difference image, respectively.

$$W_{3j}(\vec{f}) = (1 - T_{3j}(\vec{f}))/3. \quad (5)$$

In a similar manner, the weighting functions were defined for the center and lower slabs.

To test the combination method for axially truncated data, we used the geometry of the small cone angle system except with two sources as shown in Fig. 9. To increase the effect of the axial truncation, we used a 256×256 detector array and increased the source spacing by 4 cm. The FORBILD head phantom was simulated and reconstructed with each source onto a $256 \times 256 \times 256$ matrix. Source 2 can image regions 2 and 3. Therefore, the combination was performed for regions 2 and 3.

III. RESULTS

Figure 10(a) shows the central coronal plane of a Defrise phantom in a water bath and the corresponding vertical profile for the small cone angle system. Each source shows different cone beam artifacts due to the location of the source. For any location in the volume, one source produces a relatively artifact free image, but none of the sources can produce an accurate reconstruction of the entire volume. To compare the local accuracy of the combined image, the central vertical profile of the combined image was compared to one generated using only the relatively artifact free region of each source. This result demonstrates that the proposed

FIG. 11. Reconstructed images of (a) a Defrise phantom in a water bath and (b) a FORBILD head phantom for the large cone angle system. (a) The coronal plane of each source and the combined image with different segmentation and corresponding vertical profile. (b) Coronal at $y=1.63$ cm, sagittal, axial, and difference sagittal image of source 1 (first row), source 2 (second row), source 3 (third row), and combined data with three segmentations (fourth row) and 48 segmentations (fifth row). The display windows are $[0-100 \text{ HU}]$ and $[-50 \text{ to } 50 \text{ HU}]$ for the reconstructed image and the difference image, respectively.

FIG. 12. Reconstructed images of a FORBILD head phantom with an axial data truncation. Coronal at $y=0.54$ cm, sagittal, axial, and difference sagittal image of source 1 (first row), source 2 (second row), and combined data (third row). The display windows are $[0-100 \text{ HU}]$ and $[-50 \text{ to } 50 \text{ HU}]$ for the reconstructed image and the difference image, respectively.

method can preserve the local exactness of data from the source with the smallest tilt angle. Figure 10(b) shows the reconstructed images of the FORBILD head phantom for a small cone angle system. To show the cone beam artifacts clearly, we calculated the difference between the reconstructed sagittal images and object definition. The difference images show cone beam artifacts in the images from single source orbits but good local exactness in the combined image.

To evaluate the noise performance, the variances of nine spatial regions (center, middle, and edge of each slab within FOV) were measured from the combined reconstruction volume and each FDK reconstruction volume. The variances of the upper (center and lower) slab for the combined image were normalized by the variances of the upper (center and lower) slab of source 1 (2 and 3). Table II shows the normalized variance ratio in different reconstruction regions. This result shows that the combined image achieves three times lower variance than that of a single FDK image.

Figure 11(a) shows the central coronal plane of a Defrise phantom in a water bath and the corresponding vertical profile, while Fig. 11(b) shows the results for the FORBILD head phantom for the large cone angle system. Because of the increased cone angles, the Defrise phantom images from

TABLE II. Normalized variance ratio for the small cone angle system.

Spatial region	Center	Middle	Edge
Upper slab	0.34	0.34	0.34
Center slab	0.33	0.33	0.33
Lower slab	0.34	0.34	0.34

each source shows severe cone beam artifacts and intensity drop. The residual errors in the combined image are due to the fact that in some regions, none of the single FDK images is exact. The increased cone beam artifacts and intensity drop are observed in the reconstructed and difference images in Fig. 11(b). However, the combined images with either three segmented slabs or 48 segmented regions still preserve the local accuracy of the optimal single source.

The variances of the upper (center and lower) slab for the combined image with 16 segmented regions and without segmentation were normalized by the variances of the upper (center and lower) slab of source 1 (2 and 3). Table III shows the normalized variance ratio at three spatial regions (center, middle, and edge) within FOV for each slab. Because of large maximum tilt angles (e.g., 27° and 18°) for each slab, the combined image with three segmented slabs shows a higher variance ratio than that of a small cone angle system. The combined image with 48 segmented regions shows further improved noise performance for the center and middle regions.

Figure 12 shows the reconstructed images of the FORBILD

TABLE III. Normalized variance ratio for the large cone angle system.

Spatial region		Center	Middle	Edge
Upper slab	Without segmentation	0.38	0.38	0.38
	With 16 segmented regions	0.35	0.35	0.38
Center slab	Without segmentation	0.36	0.37	0.37
	With 16 segmented regions	0.34	0.35	0.37
Lower slab	Without segmentation	0.38	0.38	0.38
	With 16 segmented regions	0.35	0.35	0.38

head phantom for the two source orbit system with axial truncation. Because of the finite detector size, the reconstructed image of source 2 shows severe truncation artifacts, but the lower conical region (i.e., regions 2 and 3 in Fig. 9) is reconstructed reliably. As shown in the difference image of Figs. 10–12, the combined image shows slight artifacts since there are regions where neither source is artifact free. However, relatively good local exactness is still observed in the combined image. Compared to the case of fully overlapped volumes, the combined image with axial truncation will show different noise levels (i.e., lower noise variance by a factor of 2 in regions 2 and 3 of Fig. 9). Even though there cannot be noise reduction in region that is reconstructed with only one source, the improved noise performance in regions 2 and 3 of Fig. 9 can help to improve object detectability.

In our simulation, the reconstruction time (in MATLAB) of each source was 1824 s, while the time spent to combine three slabs was 17 s. Therefore, even with using 48 segments, the total combination time was less than 1/7 of the total reconstruction time.

IV. DISCUSSION AND CONCLUSIONS

A new method to combine reconstructed overlap volumes of a multiple circular orbit cone beam CT system was developed. Although our work was motivated by the MS-IGCT system, the proposed combination method can be used for any CT systems which have overlap volumes from multiple sources (e.g., orthogonal or oblique dual circles). In a cone beam CT system with multiple circular orbits in the axial direction, each source at a different z -location has a different tilt angle for a particular location in the reconstructed volume, and therefore the artifact levels are also different. Our method relies on knowing whatever areas in the frequency space are accurately contained in the image volumes from each orbit, which is possible with 360° rotation and FDK reconstruction. Instead of using only the data from the single source with the smallest tilt angle, reconstructed volumes from all sources were combined by performing a weighted averaging in frequency space. The method may be similarly applicable in other situations where the frequency content in various image estimates is different but known.

The noiseless simulation showed that the combination method preserved the local accuracy of the optimal single source for the small and the large cone angle systems. Since the combination is performed in frequency space, the resolution of the optimal single source is preserved (cf., the MTF of the combined image in Fig. 4). The noise simulation for the three orbit small cone angle system demonstrated that the combined image showed three times lower variance than that of a single FDK reconstruction. The combined noise image for the large cone angle system showed lower variance than that of a single FDK reconstruction, but higher than that of the small cone angle system. This higher variance was caused by the need to fill in more data during the combination. To reduce this effect, the entire overlap volume was segmented into 48 small volumes and then combined. Future works could consider segmenting each slab into 16 squares.

The squares toward the outer portions of the FOV would have asymmetric missing cones, but proper handling of those may allow further reduction in the variance.

While the proposed method can improve the overall noise performance, the effect on object detectability can be different for different objects since the averaging of noise power varies in frequency space. For example, the noise power outside the missing cone region is reduced by the weighted averaging, and therefore the combined image will show better detectability for an object that has high energy outside the missing cone region. In contrast, for an object containing energy only within the missing cone region, the detectability of the combined image is the same with that of the optimal single source image.

The noise improvement with our method is, in general, also spatially dependent based on the amount and location of overlap between the individual reconstructions. If n individual reconstructed volumes have comparable noise in a region, the local variance is expected to decrease by n in the combination. If the overlap between volumes is small and relatively widely spaced, the local reduction in noise could improve detection performance in those regions but could be distracting. The method is therefore most likely best suited for cases where the amount of overlap (that is, n) is relatively uniform in the FOV.

To handle the axial truncation, we used only reconstructed regions illuminated by the x-ray source during a complete rotation. Reconstruction in regions illuminated by a source less than 360° is important and ray weighting⁴⁰ may be an effective way to use data optimally. The use of a combination method such as the one proposed here for the overlap volumes illuminated by less than 360° is a subject of future research.

Katsevich's FBP algorithm could be an alternative to the method presented here. However, Katsevich's reconstruction requires more computation and memory than other FBP algorithms^{41–44} and becomes more sensitive to discretization errors as the helical pitch increases (analogous to larger source spacing in the z -direction).²⁷ In contrast, our method is computationally efficient and relatively insensitive to discretization errors.

Our method assumes that the cone beam artifacts generated in a region are primarily contained within the region. This probably sets a limit on how small a region can be. Our preliminary work has not shown effects from this assumption.

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