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Magnitude and phase behavior of multiresolution BOLD signal

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Abstract

High spatial resolution fMRI provides a more precise estimate of brain activity than low resolution fMRI. The magnitude and phase parts of the BOLD signals are impacted differently by changes in the scan resolution. In this paper, we report on a numerical simulation to show the impact of spatial resolution upon the complex-valued BOLD signal in terms of magnitude and phase variation. We generate realistic capillary networks in cortex voxels, calculate the BOLD-induced magnetic field disturbance and the complex BOLD signals for the voxel and its subvoxels, and thereby characterize the magnitude and phase behaviors across multiple grid resolutions. Our results show that: 1) at higher spatial resolution there is greater spatial variation in the phase of the BOLD signal as compared to its magnitude; 2) the spatial variation of the phase signal monotonically increases with respect to spatial resolution while for the magnitude the spatial variation may reach a maximum at some resolution level; 3) voxels containing large capillaries have higher phase spatial variation than those with smaller capillaries; 4) the amplitude spatial variation at a resolution level increases with respect to relaxation time whereas the phase variation is generally unaffected.

Keywords

Computational fMRI; BOLD simulation; high spatial resolution; voxel subdivision; BOLD signal; magnitude; phase; contrast; gridel

I. Introduction

For accurate spatial depiction of anatomical structure and physiological functions, there is an ongoing push towards higher spatial resolution functional MRI (fMRI) [1–7]. Higher resolution fMRI will improve the brain activity mapping in terms of spatial localization, topographic characterization, and topological connections of neuron activities. The blood-oxygenation-level-dependent (BOLD) signal is attributed to spin precession of protons in BOLD-induced inhomogeneous magnetic field (called BOLD field henceforth). Since the spin precession of a single proton is technically undetectable with the current MRI technology, what we observe in fMRI is the collective behavior of a tremendous number of protons in individual voxels [8]. The BOLD signal resulting from intravoxel spin dephasing is represented by a complex number consisting of amplitude and phase parts. However, only its amplitude part is widely utilized; the phase part is not found in use so far. Due to the detection of collective spin precession, a voxel BOLD signal conceals the individual behaviors of protons within the voxel. By spatial subdivision, a voxel is partitioned into subvoxels; correspondingly, the voxel signal is decomposed into subvoxel signals thus revealing additional spatial information. In this paper, we show BOLD contrast regeneration by recursively decomposing voxels and BOLD signals.

Currently, the phase part of the fMRI signals is not typically used. One reason for this may be due to unrestricted phase accumulation and phase wrapping associated with the complex voxel signal formation [9–11]. Another reason may be due to the fact that the amplitude pattern

(assuming a nonnegative intensity distribution) is intuitively appreciable because it reproduces the realistic scene, whereas the phase pattern (assuming both positive and negative distributions) manifests itself in an encrypted form (in terms of spatial or temporal shifts) and consequently is not straightforwardly perceivable.^{12,13} Perhaps the primary reason for phase is ignored is due to the lack of fMRI phase mechanisms, as evidenced from the ongoing research^{6,9-11,14-23}. Toward the goal of making use of BOLD signal phase, in this paper, we attempt to understand the BOLD phase behavior at different spatial resolutions by numerical simulation.

II. Methodology

II.1 Voxel partition and gridels

An fMRI image reflects the brain activity with a finite spatial resolution ($\sim$ mm). Brain anatomy shows that the brain cortex consists of capillary vasculature (radius $\in [2, 10]$ micron), which is responsible for BOLD activity (typically causing magnetic field perturbation < 1 ppm: parts per million). We can simulate a cortex voxel by filling it with a random capillary vasculature under the condition of a fixed blood volume fraction. Let V_0 denote a cortex voxel of size $D_0 \times D_0 \times D_0$ ($D_0 \sim$ mm), which can be expressed by a binary volume

$$V_0(x, y, z) = \begin{cases} 1, & (x, y, z) \in \text{vessel} \\ 0, & \text{otherwise} \end{cases} \quad |x| \leq \frac{D_0}{2}, |y| \leq \frac{D_0}{2}, |z| \leq \frac{D_0}{2}, \quad [1]$$

As spatial resolution increases, the voxel size decreases. The multigrid voxel subdivision scheme is depicted in Fig. 1(a), which is an octadic spatial partition by a $2 \times 2 \times 2$ grid, producing 8 subvoxels that can be labeled by indices in an octet set $K_8 = \{1, 2, 3, 4, 5, 6, 7, 8\}$. The recursive voxel partition produces a full octadic tree, octree for short²⁴, as diagramed in Fig. 1(b). In an octree, a node at level j can be uniquely designed by a j -tuple $(^1k, ^2k, \dots, ^jk)$, such as the exemplary illustration of a node at $(^13, ^28, ^37)$ in Fig. 1(b). In the appendix, we provide the notation and formulas for describing an octree.

In principle, a digital image can be reconstructed into (or represented by) a digital array with an arbitrary grid resolution²⁵. We coin the word “gridel” to designate a grid element that is generated by rectilinear gridding partition. The application of an octadic partition to a gridel produces an octet of children gridels in Fig. 1(a), in which V_0 is considered a gridel that has no parents as well. The gridels generated from multigrid partition can be systematically organized by an octree as shown in Fig. 1(b). By assigning each octree node a property and using the j -tuple label, we can construct a multigrid volume (or mask) octree $V_j(^1k, ^2k, \dots, ^jk)$, a multigrid BOLD field octree $B_j(^1k, ^2k, \dots, ^jk)$, and a multigrid BOLD signal octree $S_j(^1k, ^2k, \dots, ^jk)$, and so forth. We also call a multigrid octree a gridel octree because the multigrid decomposition produces a collection of gridels that are used to construct an octree. In particular, the gridel at level j takes on a cubic bin in size of $d_j \times d_j \times d_j$, with $d_j = D_0/2^j$, which can be considered as a local mask in voxel subdivision. As a result, the voxel representation and manipulation for multiresolution BOLD simulation involve three important masks: V_ε , V_j and V_0 , which are governed by a spatial containment relationship: $V_\varepsilon \subseteq V_j \subseteq V_0$. Specifically, V_ε is the building block for both voxel V_0 and gridels V_j ; V_0 is initially supported or defined by as many as N_ε^3 building blocks (fixed number); a gridel V_j consists of a number of N_j^3 building blocks, usually, $1 < N_j \ll 2^{N_\varepsilon}$ for $0 < j \ll N_\varepsilon$; in extreme cases, a gridel V_j could be as large as V_0 at $j=0$ or as tiny as V_ε at $j=N_\varepsilon$. Based on a finest grid array representation (in size of $N_\varepsilon \times N_\varepsilon \times N_\varepsilon$) of a voxel V_0 , our multiresolution voxel and signal decomposition strategy is similar to the “coarse-to-fine” scheme used in wavelet transform²⁶.

II.2 Multigrid BOLD field partition

Given the vessel geometry for a cortex voxel, we assume that blood flow is confined in vessels, and the brain activity changes the oxygenation level in the blood stream in the cortex region. This BOLD mechanism allows us to express the 3D blood susceptibility variation pattern by ¹²

$$\begin{aligned}\Delta\chi(x, y, z) &= \Delta\chi_b V_0(x, y, z) \\ &\text{with} \\ \Delta\chi_b &= (1 - Y)\Delta\chi_{do}Hct\end{aligned}\quad [2]$$

where Hct stands for blood hematocrit, Y denotes oxygenation saturation level, $\Delta\chi_{do}$ the susceptibility difference between oxygenated and deoxygenated blood. Eq. [2] serves as the connection between our numerical simulation and biology (vessel network) and physiology (blood susceptibility variation).

From the susceptibility distribution, we can calculate the BOLD field distribution ²⁷⁻²⁹

$$\Delta B(x, y, z) = IFT \left\{ \left(\frac{k_z^2}{k_x^2 + k_y^2 + k_z^2} - \frac{1}{3} \right) FT\{V_0(x, y, z)\} \right\} \Delta\chi_b B_0 \quad [3]$$

where *FT* and *IFT* stands for Fourier transform and inverse Fourier transform, respectively, and (k_x, k_y, k_z) represents the Cartesian coordinates in Fourier domain. In fact, the z-component of $\Delta B(x, y, z)$ is the blood-susceptibility-induced magnetic field, which is responsible for BOLD signal.

In conformation with the support grid of the voxel V_0 , the spatial distribution of ΔB over V_0 can also be discretized on the $N_e \times N_e \times N_e$ array by using the mask bins in Eq. [A2]. Corresponding to $octree_j(V_0)$ in Eq. [A4], we can construct a BOLD field tree by calculating the gridel-averaged BOLD field by

$$\Delta B_j(l^1 k, l^2 k, \dots, l^j k) = \frac{\sum_{i_x} \sum_{i_y} \sum_{i_z} \Delta B(i_x, i_y, i_z) V_j(i_x, i_y, i_z, l^1 k, l^2 k, \dots, l^j k)}{N_j^3} \quad [4]$$

where (i_x, i_y, i_z) denotes the discrete locations inside the voxel (see Eq. [A1]). There are 8^j entries for ΔB_j at level j , we can perform statistics in terms of { ‘standard deviation (std)’, ‘maximum (max)’, ‘minimum (min)’ } to describe the local BOLD field inhomogeneity. Meanwhile, for visualization purposes, we can arrange the 8^j entries into an $M_x \times M_y$ matrix under the condition of $M_x M_y = 8^j$. Specifically, the matrix arrangements for 4-level decomposed entries are as follows: 2×4 ($=8$), 8×8 ($=8^2$), 32×16 ($=8^3$), and 64×64 ($=8^4$).

II.3 Multigrid BOLD signals

Due to the indivisibility of V_e , the magnetic field in V_e is assumed to be locally uniform (with a single value in numerical simulation). In MRI physics, a single spin in the magnetic field ΔB can be expressed by $\exp(-i\gamma\Delta B t)$, where γ is the gyromagnetic ratio and t the relaxation time ¹². Albeit a small macroscopic dimension ($\sim$ micron), V_e may contain a tremendous number of protons (counted in units of mole or Avagadro’s number), which are indistinguishable to signal detection by the current MRI technology. Thus we model the collective precession behavior in V_e at (i_x, i_y, i_z) by ²³

$$S_{\varepsilon}(i_x, i_y, i_z, t) = \exp(-i\gamma \Delta B(i_x, i_y, i_z)t) \quad [5]$$

Although there are numerous protons in V_{ε} , we calculate the signal for V_{ε} in Eq.[5] by a single spin precession in magnetic field $\Delta B(i_x, i_y, i_z)$. The agglomerative spin precessions at a position in BOLD field has been described by a concept of “spin packet”³⁰, for which we will implement it digitally by an illustration in Fig. 2. The “spin packet” model allows us to calculate the BOLD signal resulting from numerous protons in gridel V_j by manipulating a manageable number of N_j^3 spin packets. On the other hand, the “spin packet” model sets the limit of simulation accuracy due to the indivisibility in a spin packet. Exposed to inhomogeneous BOLD field, the spin packets distributed in a gridel V_j precess out of phase, which is described as precession dephasing that is responsible for gridel BOLD signal formation.

The “spin packet” model enables the computation implementation (by Eq. [5]) for the aggregation of numerous protons that render spin precessions in the local magnetic field over the build block V_{ε} . It is a practical approximate model due to its finite size. This model can be justified when V_{ε} is substantially tiny such that the local magnetic field over V_{ε} is uniform and the spins in V_{ε} render in-phase precession. In theory, when ε approaches infinitesimal, $\varepsilon \rightarrow 0$, the “spin packet” model will eventually reduce to the single proton model; nevertheless, the astronomical number of protons are beyond the enumeration by the state-of-the-art computer. In practice, $\varepsilon \ll D_0$, the current MRI technology limits the spatial resolution ($V_0 \gg V_{\varepsilon}$), therefore, we don't need to simulate the spatial resolutions of BOLD signal far beyond the experimental feasibility. Suppose a voxel is in size of $320 \times 320 \times 320$ micron³, for example, the MRI resolution is 20 micron, then 4-level decomposition suffices (see our simulation later: $j = \{1, 2, 3, 4\}$ and $N_{\varepsilon} = 512$). So we only need to render a few levels ($j \ll N_{\varepsilon}$) for the coarse-to-fine multigrid decomposition. As discussed above, the number of gridel nodes in an octree proliferates exponentially (by 8^j , see Eq. [A2]). Let $S_j(t; {}^1k, {}^2k, \dots, {}^jk)$ denote the BOLD signal generated from all spin packets within a gridel at $({}^1k, {}^2k, \dots, {}^jk)$, then it can be calculated by

$$S_j(t; {}^1k, {}^2k, \dots, {}^jk) = \frac{\sum_{i_x} \sum_{i_y} \sum_{i_z} S_{\varepsilon}(i_x, i_y, i_z, t) V_j(i_x, i_y, i_z; {}^1k, {}^2k, \dots, {}^jk)}{N_j^3} \quad [6]$$

where V_j serves as a spatial mask (in Eq. [A2]) that only counts the spin packets within the gridel in question.

Usually, we need to examine the amplitude and phase of the BOLD signals at level j by performing a nonlinear operation on $S_j(t)$, that is,

$$\begin{aligned} A_j(t; {}^1k, {}^2k, \dots, {}^jk) &= |S_j(t; {}^1k, {}^2k, \dots, {}^jk)| \\ \Phi_j(t; {}^1k, {}^2k, \dots, {}^jk) &= \arg S_j(t; {}^1k, {}^2k, \dots, {}^jk) \\ &\text{with} \\ S_j(t; {}^1k, {}^2k, \dots, {}^jk) &= A_j(t; {}^1k, {}^2k, \dots, {}^jk) \exp(-i\Phi_j(t; {}^1k, {}^2k, \dots, {}^jk)) \end{aligned} \quad [7]$$

where the complex BOLD signal is expressed by amplitude $A_j(t)$ and phase $\Phi_j(t)$.

II. 4. Characterization of multiresolution amplitude and phase behavior

In appendix B, we depict the multiresolution BOLD signal tree and its properties. Based on the octree notations, we can characterize the amplitude variation and phase variation behavior not only at a resolution level but also across multiple resolution levels.

The multigrid BOLD signals possess global and local rebinning invariance (see Eqs. [B3] and [B4]) if we do not interfere with them nonlinearly. In practice, we must get the amplitude phase parts separately from a complex-valued BOLD signal by Eq. [7]. It is the nonlinear operation that brings out the inequalities in Eqs.[B5] and [B6], which are responsible for regenerating the signal differences across gridels (at the same level j). The number of spatially decomposed signal is dependent upon the resolution level, and the dispersion among these entries at a level depends on the relaxation time as well. The dispersion behavior can be statistically characterized by the coefficient of variation (defined by a ratio of std/mean)³¹. With the reference to non-decomposed voxel signal, $S_0(t)=A_0(t)\exp(-i\Phi_0(t))$, we characterize the amplitude and phase dispersions respectively by their coefficients of variations (denoted by cv), that is

$$cvA_j(t)=\frac{std\{A_j(t;^1k,^2k,\dots,^jk)|^1k,^2k,\dots,^jk\in K_8\}}{A_0(t)} \quad [8]$$

$$cvP_j(t)=\frac{std\{\Phi_j(t;^1k,^2k,\dots,^jk)|^1k,^2k,\dots,^jk\in K_8\}}{|\Phi_0(t)|} \quad [9]$$

It is noted that the dimensionless ratios permits the comparison of cvA and cvP with the same measure (dimensionless scale).

At each level j , there are as many as 8^j entries associated with a parameter such as the gridel signal. For visualization purposes, we can arrange them into an $M_x \times M_y$ matrix under the layout of $M_x M_y = 8^j$, in the same way done for ΔB_j display. When the amplitude entries are rearranged into an $M_x \times M_y$ matrix, we can examine its non-uniformity statistically in terms of { 'std', 'max', 'min' } in the sense of image analysis. In order to include the comparison with the non-decomposed voxel signal $S_0(t)$ in the matrix-formatted dispersion characterization, we define the amplitude contrast ($conA$) and phase contrast ($conP$) respectively by

$$conA_j(t;^1k,^2k,\dots,^jk)=\frac{A_j(t;^1k,^2k,\dots,^jk)-A_0(t)}{A_0(t)} \quad [10]$$

$$conP_j(t;^1k,^2k,\dots,^jk)=\frac{\Phi_j(t;^1k,^2k,\dots,^jk)-\Phi_0(t)}{|\Phi_0(t)|} \quad [11]$$

Statistically, "std" can represent the statistical variation, and "max" the extremity (or outlier) distribution. In order to characterize the unpredictable variation of the entries generated at multiresolution levels with both "std" and "max", we define a dimensionless parameter, in terms of "wildness"³², by

$$wildA_j(t)=\frac{\max\{A_j(t;^1k,^2k,\dots,^jk)-\bar{A}_j(t)|^1k,^2k,\dots,^jk\in K_8\}}{std\{A_j(t;^1k,^2k,\dots,^jk)|^1k,^2k,\dots,^jk\in K_8\}} \quad [12]$$

$$\bar{A}_j(t)=\frac{\sum_{^1k\in K_8}\sum_{^2k\in K_8}\dots\sum_{^jk\in K_8}A_j(t;^1k,^2k,\dots,^jk)}{8^j}$$

$$\begin{aligned} wildP_j(t) &= \frac{\max\{\Phi_j(t; {}^1k, {}^2k, \dots, {}^jk) - \bar{\Phi}_j(t) | {}^1k, {}^2k, \dots, {}^jk \in K_8\}}{\frac{std\{\Phi_j(t; {}^1k, {}^2k, \dots, {}^jk) | {}^1k, {}^2k, \dots, {}^jk \in K_8\}}{\sum_{{}^1k \in K_8} \sum_{{}^2k \in K_8} \dots \sum_{{}^jk \in K_8} \Phi_j(t; {}^1k, {}^2k, \dots, {}^jk)}} \\ \bar{\Phi}_j(t) &= \frac{\sum_{{}^1k \in K_8} \sum_{{}^2k \in K_8} \dots \sum_{{}^jk \in K_8} \Phi_j(t; {}^1k, {}^2k, \dots, {}^jk)}{8^j} \end{aligned} \quad [13]$$

Again, the dimensionlessness of wildA and wildP allows us to compare the outlier excursions of amplitude and phase with the same measure.

In a summary, we calculate the time course signals (with a relaxation time $t \in [0, 120]$ ms) for all the gridels, thereby to reveal the amplitude and phase variation behaviors across multiple resolution levels by $\{cvA_j(t), cvP_j(t)\}$ and $\{wildA_j(t), wildP_j(t)\}$ with $j = \{1, 2, 3, 4\}$, and the behaviors at a resolution level by $\{conA_j(t; {}^1k, {}^2k, \dots, {}^jk), conP_j(t; {}^1k, {}^2k, \dots, {}^jk)\}$ with a fixed j . All the parameters reduce to zeros for the non-decomposed voxel, that is, $cvA_0 = 0$, $cvP_0 = 0$, $conA_0 = 0$, $conP_0 = 0$, $wildA_0 = 0$, $wildP_0 = 0$ because of the singleton of $S_0(t)$. For most cases, the following inequalities are observed:

$$\begin{aligned} cvP_j(t) &> cvA_j(t) \\ wildP_j(t) &> wildA_j(t) \\ std\{conP_j(t; {}^1k, {}^2k, \dots, {}^jk) | {}^1k \in K_8, \dots, {}^jk \in K_8\} &> std\{conA_j(t; {}^1k, {}^2k, \dots, {}^jk) | {}^1k \in K_8, \dots, {}^jk \in K_8\} \end{aligned} \quad [14]$$

$$\begin{aligned} cvP_{j+1}(t) &> cvP_j(t) \\ wildP_{j+1}(t) &> wildP_j(t) \\ std\{conP_{j+1}(t; {}^1k, {}^2k, \dots, {}^jk) | {}^1k \in K_8, \dots, {}^jk \in K_8\} &> std\{conP_j(t; {}^1k, {}^2k, \dots, {}^jk) | {}^1k \in K_8, \dots, {}^jk \in K_8\} \end{aligned} \quad [15]$$

Specifically, the inequalities in Eq.[14] depict the amplitude and phase variations compared at the same resolution level, and those relations in Eq.[15] reveal the across-resolution-level behavior for phase part, which are not necessarily valid for magnitude part. Although, these relations are beyond analytic proof, which are of practical value for noise characterization or high contrast fMRI pursuit. The goal of this paper is to quantitatively characterize these inequalities by numeric simulation, as reported in next section.

III. Numeric simulation

The BOLD fMRI simulation requires a collection of parameters and their settings^{21,23,33,34}. Table 1 lists the parameter settings used in our simulation.

We generated cortex voxels ($D_0 = 320$ micron) in a support grid of $512 \times 512 \times 512$ with capillaries (radius = $\{2.5, 6, 9\}$ micron) under the condition of 2% blood volume fraction. Three voxel geometries are shown in Fig. 3.

For each voxel V_0 , we performed 4-level multigrid decomposition to generate an octree, $octree_4(V_0)$, as a realization of Eq.[A4]. Table 2 collects the octree properties: each gridel at level j assumes a size of $d_j \times d_j \times d_j$, and contains a number of N_j ³ spin packets; the total number of gridels at level j is 8^j . The level $j = N_\epsilon$ is included in the rightmost column as the simulation limit ($\epsilon = 0.625$ micron), which could not be reached by the current MRI technology.

Given a voxel geometry, we simulated the BOLD activity with the parameter setting in Table 1, calculated the BOLD-induced magnetic field by Eq.[3], and performed multigrid decomposition on the BOLD field with the grid masks in $octree_4(V_0)$. Figs. 4 through 11 only demonstrate the results done for the cortex voxel of 6-micron capillary. The simulation results

for vessel networks of three capillaries (radius={2.5, 6, 9}micron) are collected in tables and are presented in Figs 12 through 14.

Fig. 4 shows a slice of the BOLD field pattern under multigrid partition. In particular, Fig. 4 (d) shows a gridel at level $j=3$, as well as a blowup revealing its support grid mesh. Fig. 5 displays the gridel-averaged multigrid magnetic field patterns in image format (by arranging 8^j entries into an $M_x \times M_y$ matrix). It is seen the inhomogeneity of the BOLD field emerges and enhances as the grid resolution refines. The numeric characteristics {‘mean’, ‘std’, ‘max’, ‘min’} for the multigrid BOLD field inhomogeneity for three voxels are collected in Table 3. It shows that the extreme values {‘max’, ‘min’} differ drastically (in magnitude) during multiresolution refinement, which are eventually bounded by the support grid level ($j=N_e$). This BOLD signal refinement behavior implies that there exist considerable cancellation of BOLD field superimposition within gridels.

Before launching multigrid decomposition, we generated the BOLD signal for V_0 , that is $S_0(t)=A_0(t)\exp(-i\Phi_0(t))$. Fig. 6 shows the amplitude decay ($A_0(t)$) and phase angle accumulation ($\Phi_0(t)$) for relaxation time $t \in [0, 120]$ ms. It is seen that the phase accumulates linearly for the overall voxel V_0 .

We started to decompose the voxel recursively and calculate the gridel BOLD signals. Based on voxel partition and BOLD field partition, we calculated the gridel BOLD signal $S_j(t)$ by Eq. [6], then extracted amplitude and phase parts by nonlinear operations in Eq. [7]. For 4-level decomposition, the amplitude parts are shown in Fig. 7 and the phase parts in Fig. 8. It is noted that the BOLD signal decay (amplitude) tends to oscillate in Fig. 7(d), and the phase angle accumulates curvedly in Figs. 8(a) and (b) and some entries dance (due to local fast phase accumulation) in Figs 8(c) and (d).

Based on Eqs. [10] and [11], we calculated conA and conP for the multigrid BOLD signals at each level for a relaxation time range of $[0, 120]$ ms. At relaxation times of $t=\{30, 60, 90\}$ ms, we examined the variations of phase and magnitude across multigrid levels ($j=\{0,1,2,3,4\}$) in terms of standard deviation with respect to relaxation time and resolution level. The results are shown in Fig. 9, which shows that the amplitude variation at a resolution level increases with respect to the relaxation time, while the phase variation seems not to be affected as the relaxation time lengthens. Fig. 9 also shows that the amplitude variation at a relaxation time tends to be saturated after level $j=3$, while the phase variation constantly increases with respect to the resolution level. To inspect the contrast regeneration due to multiresolution spatial decomposition, we show conA and conP in matrix format in Figs. 10 and 11 for 4-level multigrid decomposition and $t=30$ ms.

The BOLD signal refinement behaviors for three capillary networks are presented in Figs. 12 through 14, in terms of {‘std’, ‘cvA’, ‘cvP’, ‘wildA’, ‘wildP’} with respect to three parameters: resolution level j , relaxation time t , and capillary size. Meanwhile, more numerical results for cortex voxels of capillaries of {2.5, 6, 9}micron are collected in Table 4.

Looking at Figs. 9, 12 through 14, and Table 4, we observe the following behaviors.

1. Phase variation is much higher than amplitude variation. Specifically, based on the setting of {voxel: $320 \times 320 \times 320$ micron³, support grid array: $512 \times 512 \times 512$, capillary radius: 6 micron, relaxation time: 60ms, decomposition resolution levels: {1,2,3}} we found an amplitude variation of 0.25 (dimensionless) and a phase angle variation of 0.47 rad, and found that the regenerated phase contrast is about 4 times higher than the amplitude counterpart, and extremely, the phase may excure more wildly than the amplitude (up to ~ 20 times larger, in terms of max/std).

2. At the first few low resolution levels ($j=1,2$), both amplitude and phase variations increase. As the resolution level ever increases ($j>3$) and as the relaxation time lengthens ($t>30\text{ms}$), the magnitude variation may go down, whereas the phase variation increases consistently and drastically.
3. For a fixed blood volume fraction (2% in this simulation), large capillaries incur more phase variation, however, this observation is not always true for amplitude variation. In comparison, phase variation is higher than amplitude variation in terms of cvP/cvA and wildP/wildA , especially these ratios increase drastically for large capillaries as resolution refines.

Above all, we should point out that, in principle, the BOLD signal for a gridel should be calculated based on the “spin packet” model in Eq.[6], rather than from gridel-averaged BOLD field in Eq. [4]. The similarity among the matrix displays in Figs. 5 and 11 suggests an empirical approximation: the BOLD image phase pattern may approximate the gridel-averaged BOLD field. Extremely, Fig. 5 assumes the same pattern as Fig. 11 if $V_j=V_\varepsilon$.

IV Discussion

Currently, the spatial resolution fMRI is typically larger than 1 millimeter. For numerical simulation, we can adopt an arbitrary grid resolution (the limit of support grid) provided that the computation workload is feasible. To provide meaningful simulation results, we start with a cortex voxel in size of $320\times320\times320$ micron³ cortex and multigridding it down to gridels in size of $20\times20\times20$ micron³. The support grid in the “spin packet” model is a main factor that determines the simulation accuracy and validity. By using the multigrid masks and adopting the “spin packet” model, we can carry out the numerical calculation of multigrid BOLD signals by manipulating only a handful of spin packets in a gridel. Nevertheless, accurate simulation pursues high-grid-resolution depiction of vasculature or tiny spin packets. Due to limitation on computation facility and time, we represent the vortex voxel in a support grid by a $512\times512\times512$ array. For highlighting the multigrid BOLD signal behavior, we only consider static BOLD inhomogeneous field and stationary spin packet precession. The effect of dynamic BOLD field and spin diffusion will accentuate the intra-gridel dephasing, which deserves further investigation.

The reason for the regeneration of amplitude contrast and phase contrast due to multigrid voxel decomposition lies in the nonlinear operations of extracting the amplitude and phase parts from a complex-valued fMRI signal. If we do not perform nonlinear operations, all the spin packets are just conceptually rebinned (regrouped) by gridels, and the associative addition (for scalars or vectors) is just a trivial linear operation, therefore the mean values are invariant to the multigrid decompositions (see Table 3). We describe this phenomenon in terms of rebinning invariance (see Appendix B) and suggest use this property to check the algorithm implementation. By the way, the nonlinear operations associated with the magnitude and phase contrast regenerations provide an instance for the physical principle: a physical state may be changed upon detection.

Both amplitude and phase in the complex BOLD signal evolve with the relaxation time. In general, the amplitude decays due to spin precession dispersion, and it may damply oscillate for long relaxation time; meanwhile the phase angle accumulates, which manifests a linear accrual for low resolution levels ($j=1,2$) and develops into sublinear or nonlinear progression for high resolution levels ($j=3,4$). The spatial resolution refinement brings about more details and irregularities to the distribution of the BOLD field perturbation. An emerging peak in the refined BOLD field map accounts for the fast phase accumulation and phase wrapping, thus for the contrast regenerations of multiresolution magnitudes and phases. Statistically, the phase

variation is always higher than the amplitude variation³⁵, and this behavior is expected to be exaggerated for large vessels at fine resolutions.

Due to bipolar BOLD field distribution (assuming both positive and negative values), the phase angles diverse in positive and negative directions during multiresolution decomposition (see Fig.8). We observed that, even for a linear accumulative phase angle for a voxel signal (see Fig. 6), the spatial multiresolution decomposition could spawn a multiple of phase angles that could excure about $\Phi_0(t)$, straying in two opposite directions (positive up or negative down). In most practical case, the BOLD signal $S_0(t)$ for a cortex voxel produces a non-negative amplitude part ($A_0(t) \in (0,1]$) and non-zero phase part ($\Phi_0(t) \neq 0$ for $t > 0$). It is quite unlikely for a cortex voxel to produce a time series of $\Phi_0(t)$ that stays at or closes to zero during intravoxel dephasing. Furthermore, the time series of $A_0(t)$ and $\Phi_0(t)$ stay compromisingly and smoothly amidst the dispersions of $A_j(t)$ and $\Phi_j(t)$, respectively. Therefore, we suggest to use $S_0(t)$ as the reference for gridel signal analysis at all resolution levels. For the purposes of comparing amplitude and phase dispersions, we always use the dimensionless measures, such as { 'cvA', 'cvP' } and { 'wildA', 'wildP' }.

V. Conclusion

As spatial grid resolution refines, local inhomogeneity of the BOLD-induced magnetic field, which was once degenerated due to voxel average at a coarse resolution, now becomes detectable. Examining the amplitude and phase parts of the BOLD signal separately, we observed that phase variation is consistently higher than the amplitude variation as the resolution refines. For particular voxels filled with micron capillaries, with the setting of resolution level $j=1$ and relaxation time $t=30\text{ms}$, we found that the phase variation is larger than the amplitude variation by a factor of about 4, and this ratio continues to rise as the resolution refines. At a resolution level, the amplitude variation increases with respect to relaxation time, whereas the phase variation is basically unaffected in absence of diffusion as the relaxation time lengthens ($<120\text{ms}$). We also found that the amplitude variation during multiresolution refinement may assume a maximum at a certain resolution level for some vascular geometry. Therefore, it is not recommended to pursue higher resolution levels (e.g., $j > 3$ for the capillary vasculature used in this study) when working only with the amplitude signal. These findings show that magnitude and phase behave differently with respect to { 'spatial resolution', 'relaxation time', 'capillary size' }. The fact that the phase variation increases with respect to spatial resolution much more than magnitude variation may explain why the phase part in fMRI is usually considered too noisy to be useful, thus remains unexploited so far. On the other hand, if the phase mechanism is well understood and the factors that cause phase contrast can be controlled, we can make use of it for high-contrast BOLD imaging.

Appendix A

Octree notation for multigrid voxel decomposition

Based on recursive octadic subdivision, the multigrid voxel partition can be represented by a regular octave tree (octree), in which each node can be uniquely designated by its pathway from the tree root. We use $K_8 = \{1,2,3,4,5,6,7,8\}$ to denote the octet, and a j -tuple $(^1k, ^2k, \dots, ^jk)$ to label a node at level j . It is helpful to look at the illustration for node $(^13, ^28, ^37)$ in Fig. 1(b). For numerical simulation, we need to discretize the voxel V_0 in Eq. [1] into an array, called the support grid for V_0 , as large as possible in the sense of simulation rationale. Suppose V_0 be a cube of $D_0 \times D_0 \times D_0$ in physical size (e.g., $D_0 = 320$ micron), and we discretize it into $N_\epsilon \times N_\epsilon \times N_\epsilon$ array ($N_\epsilon \gg 1$) with the grid resolution ϵ by

$$\begin{cases} x=i_x\varepsilon \\ y=i_y\varepsilon, \forall i_x, i_y, i_z \in [-N_\varepsilon/2, N_\varepsilon/2] \\ z=i_z\varepsilon \end{cases}$$

$$N_\varepsilon = \lg(D_0/\varepsilon)/\lg 2$$

$$0 < \varepsilon < D_0$$
[A1]

then the voxel V_0 can be programmably manipulated via its elements indexed by (i_x, i_y, i_z) . The $N_\varepsilon \times N_\varepsilon \times N_\varepsilon$ array for V_0 is its support grid. A support grid element takes on a tiny bin in size of $\varepsilon \times \varepsilon \times \varepsilon$, which is denoted by V_ε . Being the smallest grid element, any property (such as proton spin precession, BOLD field, BOLD signal, etc) associated with a V_ε is indivisible and therefore must be manipulated as a whole or by a single entry. In particular, the indivisibility of spin precessions of protons in V_ε is described as a “spin packet” model in this paper. In principle, the validity of the “spin packet” model demands an as-small-as-possible V_ε as long as the support grid array can be managed by a computer program.

Let V_j denote a j -level gridel with the label of $(^1k, ^2k, \dots, ^jk)$ in the octree, it occupies a volume space that contains as many as N_j^3 spin packets. For a V_j centered at (i_x, i_y, i_z) , we can express its spatial occupancy by

$$V_j(i_x, i_y, i_z; ^1k, ^2k, \dots, ^jk) = \begin{cases} 1, & |i'_x - i_x| < N_j/2, |i'_y - i_y| < N_j/2, |i'_z - i_z| < N_j/2 \\ 0, & \text{otherwise} \end{cases}$$

$$N_j = \text{round}\left(\frac{d_j}{\varepsilon}\right) = \text{round}\left(\frac{D_0}{\varepsilon 2^j}\right) = \frac{N_\varepsilon}{2^j}$$

$$\text{card}\{V_j(i_x, i_y, i_z; ^1k, ^2k, \dots, ^jk) | ^1k, ^2k, \dots, ^jk \in K_8\} = 8^j$$

$$0 < j < N_\varepsilon$$
[A2]

where (i'_x, i'_y, i'_z) denotes the running index for traversing elements in the support grid, and $\text{round}(\cdot)$ the rounding operation of taking integer, $\text{card}\{\cdot\}$ counting the number in a set. We see from Eq. [A2] that there are as many as 8^j gridels at level j during octadic decomposition.

We can consider the gridel V_j in Eq. [A2] as spatial masks, or mask bins, which follow the spatial containment and disjoint partition relationships, as expressed by

$$V_j \subset V_{j-1} \subset \dots \subset V_0$$

$$V_{j-1}(i_x, i_y, i_z; ^1k, ^2k, \dots, ^{j-1}k) = \bigcup_{^jk \in K_8} V_j(i_x, i_y, i_z; ^1k, ^2k, \dots, ^{j-1}k, ^jk)$$

$$\text{card}\{V_j\} = 8 \text{card}\{V_{j-1}\} = \dots = 8^j$$
[A3]

The cardinality relationship indicates the tree growth by 8^j . A J -level octadic partition on voxel V_0 produces an octree, as notated by

$$\text{octree}_J(\text{root}=V_0) = \{V_0; V_1(^1k); V_2(^1k, ^2k); \dots V_J(^1k, ^2k, \dots, ^Jk) | ^jk \in K_8, j=1, 2, \dots, J\}$$
[A4]

The octree is totally determined by a root and the ultimate resolution level J . It is noted that $\text{octree}_J(V_0)$ provides a collection of masks for us to systematically partition a voxel region. For example, we use the multigrid masks to calculate the BOLD fields and signals over multilevel gridels, thereby constructing octrees for multiresolution BOLD fields and signals, as reported in the text.

Appendix B

Multiresolution BOLD signal tree: octree_J(S₀)

Multigrid decomposition upon voxel V₀ produces a spatial mask tree, octree_J(V₀) in Eq. [A4]. For each gridel node, we calculate the BOLD signal. Accordingly, we construct an octree to systematically represent the multiresolution analysis of BOLD signal, that is

$$\text{octree}_J(S_0(t)) = \{S_0(t), S_1(t;^1k), S_2(t;^1k, ^2k), \dots, S_J(t;^1k, ^2k, \dots, ^Jk) | k \in K_8, j=1, 2, \dots, J\} \quad [\text{B1}]$$

In comparison with octree_J(V₀), which reveals the spatial containment relationships for multigrid voxel partition, octree_J(S₀(t)) depicts the multiresolution spatial decomposition behavior of BOLD signal. Specifically, octree_J(S₀(t)) offers the following properties.

1. Each node signal is spatially decomposed into 8 subnode signals (indexed by K₈), which are governed by an additive identity

$$S_{j-1}(t;^1k, ^2k, \dots, ^{j-1}k) \equiv \sum_{k \in K_8} S_j(t;^1k, ^2k, \dots, ^{j-1}k, ^jk), \forall j \in [1, N_\varepsilon]. \quad [\text{B2}]$$

where N_ε represents the maximum resolution level that is determined by the support grid resolution ε (see Eq. [A1]).

2. The summation of BOLD signals over all nodes at any level j is invariant, that is

$$S_0(t) \equiv \sum_{k \in K_8} \sum_{k \in K_8} \dots \sum_{k \in K_8} S_j(t;^1k, ^2k, \dots, ^jk), \forall j \in [1, N_\varepsilon]. \quad [\text{B3}]$$

This property is called global rebinning invariance because all spin packets (in a number of N_j³) at level j are conceptually rebinned into gridels, but not subject to any nonlinear operation, thus remaining intact.

3. Any node is invariant to its multilevel decomposition (to any level), as expressed by

$$S_j(t;^1k, ^2k, \dots, ^jk) \equiv \sum_{j+1k \in K_8} \sum_{j+2k \in K_8} \dots \sum_{j+j'k \in K_8} S_j(t;^1k, ^2k, \dots, ^1k, ^2k, \dots, ^jk, ^{j+1}k, \dots, ^{j+j'}k), \forall j' > j \quad [\text{B4}]$$

This property is called local rebinning invariance, as it applies to a gridel at level j > 0.

4. The amplitudes and phases of gridel BOLD signals at level j follow the inequality relationships

$$\begin{aligned} \sum_{k \in K_8} \sum_{k \in K_8} \dots \sum_{k \in K_8} A_j(t;^1k, ^2k, \dots, ^jk) &= \sum_{k \in K_8} \sum_{k \in K_8} \dots \sum_{k \in K_8} |s_j(t;^1k, ^2k, \dots, ^jk)| \leq A_0(t) = |S_0(t)|, \forall j \in [1, N_\varepsilon] \\ \sum_{j+1k \in K_8} \dots \sum_{j+j'k \in K_8} A_j(t;^1k, ^2k, \dots, ^jk, ^{j+1}k, \dots, ^{j+j'}k) &= \sum_{j+1k \in K_8} \dots \sum_{j+j'k \in K_8} |s_j(t;^1k, ^2k, \dots, ^jk)| \leq A_j(t) \\ &= |\sum_{j+1k \in K_8} \dots \sum_{j+j'k \in K_8} S_j(t;^1k, ^2k, \dots, ^jk)|, \forall j' > j \end{aligned} \quad [\text{B5}]$$

and

$$\sum_{k=K_8} \sum_{k=K_8} \cdots \sum_{j=K_8} \Phi_j(t; {}^1k, {}^2k, \dots, {}^jk) \neq \Phi_0(t), \forall j \in [1, N]$$

$$\sum_{j+1} \cdots \sum_{j+j'} \Phi_j(t; {}^1k, {}^2k, \dots, {}^jk, {}^{j+1}k, \dots, {}^{j+j'}k) \neq \Phi_j(t; {}^1k, {}^2k, \dots, {}^jk), \forall j' > j$$
[B6]

where the equality relationship holds for perfect uniform magnetic field distribution over the grid in question, implying no contrast.

The properties 1-3 are useful for program verification during algorithm implementation, and the property 4 is responsible for spatial contrast regeneration of magnitude and phase due to multiresolution BOLD signal decomposition.

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Fig 1.

Recursive octadic voxel partition, a gridel is a subvoxel (or a bin in the voxel), which can be used as a mask. (b) Graphic octree. A node at resolution level j is uniquely labeled by a j -tuple $(^1k, ^2k, \dots, ^jk)$, which traces its path to the root. An example of $(^13, ^28, ^37)$ is illustrated with thick lines.

Fig 2.

Illustration of the “spin packet” model and “gridel”. The collective spin precession of numerous protons in V_ϵ is modeled as a single spin packet; a gridel (marked in red box) assumes a bin within a voxel and it only consists of a handful of spin packets.

Fig. 3.
Cortex voxels filled with three capillaries: radius={2.5, 6, 9}micron, with the same blood volume fraction of 0.02.

Fig. 4. Demonstration of multigrid BOLD magnetic field partition at a slice $\Delta B_j(i_x, i_y, i_z=20)$ for : partitions of (a) 2×2 for $j=1$, (b) 4×4 for $j=2$, (c) 8×8 for $j=3$. The support grid for a gridel at level $j=3$ is magnified in (d). All panels share the same colorbar in units of ppmT.

Fig. 5. Matrix presentation of multigrid BOLD magnetic field (spatial subdivision of blood-susceptibility-induced magnetic field inhomogeneity over voxel V_0), calculated by gridel average by Eq. [4] for: (a) $j=1$, (b) $j=2$, (c) $j=3$, (d) $j=4$. The unit of the common grayscale bar is ppmT.

Fig. 6.

The amplitude and phase of the voxel BOLD signal: $S_0(t)=A_0(t)\exp(-i\Phi_0(t))$ (defined in Eq. [7] with $j=0$) which is generated from the plenary of spin packets (in a number of $1.34\times 10^8=512\times 512\times 512$).

Fig. 7. The amplitude decay behavior of multigrid BOLD signal refinement in terms of $A_j(t)$ (calculated by Eq. [7]) for $j=\{1,2,3,4\}$, in which $A_0(t)$ (in thick broken line) is provided as a reference (calculated by [7] with $j=0$). The curves are colored for visual distinction.

Fig. 8. The phase accumulation behavior of multigrid BOLD signal refinement in terms of $\Phi_j(t)$ (calculated by Eq. [7]) for $j=\{1,2,3,4\}$, in which $\Phi_0(t)$ (in thick broken line) is provided as a reference (calculated by [7] with $j=0$). The curves are colored for visual distinction. Notice the different angle ranges (in units of rad) at the ordinates.

Fig. 9.

Amplitude and phase variation behaviors of multigrid BOLD signal refinement in terms of $\text{std}\{\text{conA}_j(t)\}$ and $\text{std}\{\text{conP}_j(t)\}$, with respect to relation time (t) and multigrid level (j). The inset in lower right panel is the close-up of $\text{std}\{\text{conP}_j(t)\}$ at $j=3$.

Fig. 10. Matrix representation of amplitude contrasts (conA_j defined in Eq.[10]) of multigrid BOLD signals. The entry value range is indicated in the grayscale bar (unit: dimensionless).

Fig. 11. Matrix representation of phase contrasts (conP_j defined in Eq.[11]) of multigrid BOLD signals. The entry value range is indicated in the grayscale bar (unit: dimensionless).

Fig. 12.

Amplitude and phase variation behaviors due to multiresolution BOLD signal refinement ($t=30\text{ms}$). The upper row shows the variation behaviors in terms of standard deviation (std), the middle row in coefficient of variation (cvA and cvP in Eqs. [8] and [9]), and lower row in wildness (wildA and wildP in Eqs.[12] and [13]). All plots assume the same legend in left upper panel for three different vessel geometries. Note the ordinate scales for behavior strength.

Fig. 13.

Comparison of amplitude and phase variations due to multiresolution spatial refinement of voxel BOLD signal. The comparison was done with three voxel vessel geometries: radius= {2.5, 6, 9}micron, and 30ms relaxation time. The upper row shows the comparison in terms of cvA and cvP (defined in Eqs. [8] and [9]) and lower row in wildA and wildP (defined in Eqs.[12] and [13]), with respect to resolution level j.

Fig. 14.

Effect of vessel capillary size on magnitude and phase variations during multiresolution BOLD signal refinement ($t=30\text{ms}$). The resolution levels are indicated by the legend in the left upper panel. Three capillary radiuses ($\{2.5, 6, 9\}$ micron) are provided as abscissas. Note the different ordinate scales for comparison strength.

Table 1

Parameters for numerical simulation

Nomenclature	Parameter	Remark
Blood susceptibility	$\Delta\chi_b = 1$ ppm	Determined from blood physiology
Main static field	$B_0 = 3$ Tesla	Magnetic field of MRI scanner
Voxel size	$320 \times 320 \times 320 \mu\text{m}^3$	physical size of V_0
Voxel array	$N_\epsilon \times N_\epsilon \times N_\epsilon$ ($N_\epsilon=512$)	grid size of V_0
Multigrid dimensions	$d_j \in \{160, 80, 40, 20\} (\mu\text{m})$ $j=1, 2, 3, 4$	Octadic subdivision for 4 levels
Capillary radius	$\{2.5, 6, 9\} \mu\text{m}$	Typical capillary sizes
Blood volume fraction	2%	Vessel occupancy in a voxel
Spin packet dimension	$V_\epsilon: \epsilon \times \epsilon \times \epsilon, \epsilon=320/N_\epsilon (\mu\text{m})$	Indivisible tiny finite cubicle
Spin packet precession	$S_\epsilon(t) = \alpha \exp(-i\gamma Bt)$	Constant amplitude
Multigrid masks	$V_j \subset V_{j-1} \subset \dots \subset V_0$	Spatial containment
Contrast at level j	$\text{con}A_j(t), \text{con}P_j(t)$	Definition in Eqs. [10] and [11]
Coefficient of variation	$\text{cv}A_j(t), \text{cv}P_j(t)$	Definition in Eqs. [8] and [9]
Dispersion wildness	$\text{wild}A_j(t), \text{wild}P_j(t)$	Definition in Eqs. [12] and [13]

Table 2

Properties and values of an octree.

	j=0	j=1	j=2	j=3	j=4	j=N _e
d _j (μm)	320	160	80	40	20	0.625
card{V _j }	1	8	8 ²	8 ³	8 ⁴	512 ³
N _j ³	512 ³	256 ³	128 ³	64 ³	32 ³	1

Table 3

Inhomogeneity characteristics of multigrid BOLD fields (unit:10⁻³ ppmT). The means are j-invariant, and other statistics are j-dependent.

	12.5μm			6μm			9μm		
	mean±std	max	min	mean±std	max	min	mean±std	max	min
j=1	5.3±0.7	6.3	4.4	5.4±1.3	7.4	3.7	5.5±3.0	10.0	1.3
j=2	5.3±2.3	10.7	1.2	5.4±4.5	14.2	-5.6	5.5±4.6	14.2	-5.4
j=3	5.3±3.7	17.0	-5.9	5.4±7.6	27.8	-24.3	5.5±9.9	52.8	-27.7
j=4	5.3±6.5	32.5	-19.2	5.4±13.0	59.7	-63.0	5.5±16.8	140.9	-87.7
j= N _c	5.3±29.9	614.6	-457.4	5.4±28.7	610.9	-488.6	5.4±28.5	502.0	-528.7

Table 4

Numeric simulation results of multigrad BOLD signal refinement.

	j=1		j=2		j=3		j=4	
	t=30ms	t=60ms	t=30ms	t=60ms	t=30ms	t=60ms	t=30ms	t=60ms
2.5 μ m	max{A _j }	0.90	0.80	0.95	0.89	1.00	1.00	1.00
	std{A _j }	0.03	0.04	0.05	0.08	0.09	0.15	0.23
	cvA _j	0.03	0.06	0.06	0.10	0.11	0.20	0.31
	wildA _j	1.38	1.47	1.78	1.77	1.45	1.62	0.96
	max{ Φ_j } (rad)	0.19	0.39	0.29	0.58	0.51	1.00	1.82
	std(Φ_j) (rad)	0.03	0.05	0.07	0.15	0.11	0.21	0.35
	cvP _j	0.19	0.19	0.49	0.50	0.73	0.73	1.21
	wildP _j	1.87	1.84	2.11	2.04	3.49	3.40	4.37
	wildP _j /wildA _j	1.40	1.96	2.67	3.93	3.06	4.17	10.49
	cvP _j /cvA _j	6.03	3.19	8.89	4.70	6.69	3.60	6.71
6 μ m	max{A _j }	0.91	0.82	0.99	0.98	0.99	1.00	1.00
	std{A _j }	0.05	0.08	0.11	0.17	0.17	0.25	0.26
	cvA _j	0.06	0.12	0.13	0.25	0.20	0.36	0.37
	wildA _j	1.31	1.31	1.34	1.46	0.79	0.93	0.51
	max{ Φ_j } (rad)	0.20	0.42	0.41	0.80	0.74	2.78	3.12
	std(Φ_j) (rad)	0.04	0.08	0.13	0.25	0.22	0.47	0.79
	cvP _j	0.25	0.24	0.82	0.77	1.38	1.40	2.77
	wildP _j	1.09	1.15	1.95	1.89	2.65	5.25	6.69
	wildP _j /wildA _j	0.53	0.71	2.20	2.74	5.80	19.85	63.10
	cvP _j /cvA _j	4.18	1.89	6.47	3.07	6.92	3.95	11.6
9 μ m	max{A _j }	0.99	0.88	0.99	0.99	1.00	0.99	1.00
	std{A _j }	0.07	0.11	0.12	0.18	0.20	0.24	0.19
	cvA _j	0.08	0.15	0.15	0.24	0.22	0.23	0.19
	wildA _j	1.22	1.18	1.09	1.28	0.59	0.76	0.43

	$j=1$		$j=2$		$j=3$		$j=4$	
	$t=30ms$	$t=60ms$	$t=30ms$	$t=60ms$	$t=30ms$	$t=60ms$	$t=30ms$	$t=60ms$
$\max\{\Phi_j\}$ (rad)	0.24	0.49	0.47	0.91	2.67	3.09	3.11	3.08
$\text{std}(\Phi_j)$ (rad)	0.07	0.14	0.13	0.25	0.30	0.51	0.55	0.73
$\text{cv}P_j$	0.55	0.52	0.95	0.93	2.19	1.87	4.02	2.66
$\text{wild}P_j$	1.35	1.48	2.59	2.51	8.43	5.45	5.43	3.87
$\text{wild}P_j/\text{wild}A_j$	1.28	1.98	2.59	3.92	32.92	32.34	118.66	68.61
$\text{cv}P_j/\text{cv}A_j$	6.70	3.43	6.55	3.85	9.55	5.78	19.18	8.40