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Effects of Lewis number on coupled heat and mass transfer in a circular tube subjected to external convective heating

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Abstract

Heat and mass transfer in a circular tube subject to the boundary condition of the third kind is investigated. The closed form of temperature and concentration distributions, the local Nusselt number based on the total external heat transfer and convective heat transfer inside the tube, as well as the Sherwood number were obtained. The effects of Lewis number and Biot number on heat and mass transfer were investigated.

1 Introduction

Coupled heat and mass transfer with sublimation or vapor deposition are often encountered in the lyophilization process, sublimation dehydration, cryopreservation industry, and food engineering processes [1–6]. For example, intracellular ice formation (IIF) is generally believed to be fatal to cells due to mechanical damage of the cellular ultrastructure either by the direct action or by the associated volumetric expansion during cell cryopreservation [7,8]. Vitrification can be used to avoid IIF and potentiate the cell survival rate. Recently, the ultrafast cooling rate system is being developed in order to keep cell vitrification directly and avoid IIF damage when its cooling rate is higher enough [7]. Coupled heat and mass transfer is encountered in the design of such ultrafast cooling system. Another application for sublimation of materials is the preparation of specimens using freeze-drying for a scanning electronic microscope (SEM) or a transmission electronic microscope (TEM) [9]. Coupled forced convective heat and mass transfer have also been widely used in the field of heat and mass transfer enhancement. Therefore, better understanding the mechanisms of the coupled forced convective heat and mass transfer is important in the optimal design of high-efficient heat transfer system.

The theoretical solution of coupled forced heat and mass transfer between two thermally insulated parallel plates can be traced back to later 1960s by Sparrow and his co-workers [10,11]. Kurosaki [12] obtained numerical solution of coupled forced convective heat and mass transfer between two uniformly heated parallel plates. Since heat and mass transfer in a circular tube is more useful than that between two parallel plates, Zhang and Chen [13] obtained an analytical solution of coupled laminar heat and mass transfer in a circular tube with uniform heat flux. Zhang [14] further analyzed the coupled forced convection heat and mass transfer in tube with the boundary condition of the third kind, which is a generic boundary condition because the boundary conditions of the first and second kind can be readily achieved by setting the Biot number to infinity ($Bi \rightarrow \infty$) or zero ($Bi \rightarrow 0$), respectively. Their results show that the Nusselt number based on the convective heat transfer inside the tube is identical to Sherwood number when the Lewis number is unity. In order to better understand the mechanisms of the coupled heat and mass transfer with external convections heating, it is necessary to further investigate the effects of Lewis number on the Nusselt and Sherwood Numbers. Therefore, coupled heat and mass transfer process in a circular tube at different Lewis, Biot number with the boundary condition of the third kind is theoretically investigated in this paper.

2 Governing equations and analytical solution

Figure 1 shows the physical model of the coupled heat and mass transfer problem under consideration. A circular tube with radius R is subject to external convective heating with a heat transfer coefficient, h_e , and temperature, T_e . The internal flow of the tube is assumed to be a slug flow for which velocity is uniform and equal to the entrance velocity U . The concentration and temperature at the entrance are C_o and T_o , respectively. The inner surface of the tube is coated by a thin layer of sublimable material, which can be sublimated under heating. In order to solve the problem analytically, the following assumptions are made:

1. The saturation concentration can be expressed as a linear function of the corresponding temperature [7,12,13].
2. The entrance concentration is the saturation concentration at the entry temperature.
3. The entrance concentration and temperature are uniform.
4. The thermal resistance of the tube wall and the velocity components contributed by the sublimation can be neglected.

The governing equations of heat and mass transfer at cylindered coordinates can be expressed as

$$Ur \frac{\partial T}{\partial x} = \alpha \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \quad (1)$$

$$Ur \frac{\partial C}{\partial x} = D \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) \quad (2)$$

subject to the following boundary conditions

$$\begin{aligned}
T &= T_o; \quad C = C_o \text{ at } x=0 \\
\frac{\partial T}{\partial r} &= \frac{\partial C}{\partial r} = 0 \text{ at } r=0 \\
\rho h_{sg} D \frac{\partial C}{\partial r} + k \frac{\partial T}{\partial r} &= h_e (T_e - T) \text{ at } r=R
\end{aligned} \tag{3}$$

In reference to assumption #1, the concentration at the inner wall depends on the inner wall temperature which can be expressed as [7,12,13]:

$$C = AT + B \tag{4}$$

where A and B are constant.

By defining the following dimensionless variables:

$$\begin{aligned}
\eta &= \frac{r}{R}, \quad X = \frac{x}{PeR}, \quad Pe = \frac{2uR}{\alpha}, \quad Bi = \frac{h_e R}{k}, \\
L &= \frac{Ah_{sg}}{c_p}, \quad Re = \frac{uR}{\nu} \\
\theta &= \frac{T_e - T}{T_e - T_o}, \quad \varphi = \frac{C_e - C}{C_e - C_o}, \quad C_e = AT_e + B, \\
Lew &= \frac{\alpha}{D}, \quad \alpha = \frac{k}{\rho c_p}
\end{aligned} \tag{5}$$

Eqs. 1–3 become:

$$\frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \theta}{\partial \eta} = \frac{1}{2} \frac{\partial \theta}{\partial X} \tag{6}$$

$$\frac{\partial^2 \varphi}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \varphi}{\partial \eta} = \frac{Lew}{2} \frac{\partial \varphi}{\partial X} \tag{7}$$

and

$$\theta = 1; \quad \varphi = 1 \text{ at } X = 0 \tag{8}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{\partial \varphi}{\partial \eta} = 0 \text{ at } \eta = 0 \tag{9}$$

$$\frac{L}{Lew} \frac{\partial \varphi}{\partial \eta} + \frac{\partial \theta}{\partial \eta} = -Bi\theta \text{ at } \eta = 1 \tag{10}$$

$$\theta_w = \varphi_w, \text{ at } \eta = 1 \tag{11}$$

The analytical solutions of Eqs. 6 and 7 with the boundary conditions 8–11 are obtained by separation of variables as shown in following

$$\theta(\eta, X) = \sum_{m=1}^{\infty} \frac{2J_1(\beta_m) J_0(\beta_m \eta)}{\beta_m [J_0^2(\beta_m) + J_1^2(\beta_m)]} e^{-2\beta_m^2 X} \quad (12)$$

$$\begin{aligned} \varphi(\eta, X) &= \\ &= \sum_{m=1}^{\infty} \frac{2J_1(\sqrt{\text{Lew}}\beta_m) J_0(\sqrt{\text{Lew}}\beta_m \eta)}{\sqrt{\text{Lew}}\beta_m [J_0^2(\sqrt{\text{Lew}}\beta_m) + J_1^2(\sqrt{\text{Lew}}\beta_m)]} e^{-2\beta_m^2 X} \end{aligned} \quad (13)$$

The detailed derivation of above solution is presented in the “Appendix”.

3 Heat and mass transfer

The heat and mass transfer can be evaluated by Nusselt and Sherwood number, respectively. There are two different Nusselt numbers: one of them calculated based on the total amount of heat transferred into the fluid and the other is based on the convective heat transfer coefficient inside the tube. Nusselt number based on the total heat supplied by the external heat transfer is:

$$Nu_e = \frac{2 \bar{h} R}{k} = \frac{2R h_e (T_e - T_w)}{k T_w - T_m} = \frac{2Bi\theta_w}{\theta_m - \theta_w} \quad (14)$$

Based on the heat transferred to the fluid inside the tube by convection, the Nusselt number can be obtained by

$$Nu_i = \frac{2R}{k} \frac{-k \frac{\partial T}{\partial r} \Big|_{r=R}}{T_w - T_m} = \frac{2}{\theta_w - \theta_m} \frac{\partial \theta}{\partial \eta} \Big|_{\eta=1} \quad (15)$$

Similarly, the Sherwood number of mass transfer inside the tube can be calculated by

$$Sh = \frac{2R}{D} \frac{-D \frac{\partial C}{\partial r} \Big|_{r=R}}{C_w - C_m} = \frac{2}{\varphi_w - \varphi_m} \frac{\partial \varphi}{\partial \eta} \Big|_{\eta=1} \quad (16)$$

The following relationship between the Nusselt and Sherwood numbers can be obtained from Eq. 10

$$Nu_e = Nu_i + \frac{L}{\text{Lew}} \frac{\varphi_w - \varphi_m}{\theta_w - \theta_m} Sh \quad (17)$$

The dimensionless wall temperature and concentration can be calculated by substituting $\gamma = 1$ into Eqs. 12 and 13, i.e.,

$$\theta_w = \sum_{m=1}^{\infty} \frac{2J_1(\beta_m) J_0(\beta_m)}{\beta_m [J_0^2(\beta_m) + J_1^2(\beta_m)]} e^{-2\beta_m^2 X} \quad (18)$$

$$\varphi_w = \sum_{m=1}^{\infty} \frac{2J_1(\sqrt{\text{Lew}}\beta_m) J_0(\sqrt{\text{Lew}}\beta_m)}{\sqrt{\text{Lew}}\beta_m [J_0^2(\sqrt{\text{Lew}}\beta_m) + J_1^2(\sqrt{\text{Lew}}\beta_m)]} e^{-2\beta_m^2 X} \quad (19)$$

The mean dimensionless temperature and concentration can be obtained by

$$\theta_m = \int_0^1 2\eta \theta d\eta = \sum_{m=1}^{\infty} \frac{4J_1^2(\beta_m)}{\beta_m^2 [J_0^2(\beta_m) + J_1^2(\beta_m)]} e^{-2\beta_m^2 X} \quad (20)$$

$$\varphi_m = \sum_{m=1}^{\infty} \frac{4J_1^2(\sqrt{\text{Lew}}\beta_m)}{\text{Lew}\beta_m^2 [J_0^2(\sqrt{\text{Lew}}\beta_m) + J_1^2(\sqrt{\text{Lew}}\beta_m)]} e^{-2\beta_m^2 X} \quad (21)$$

The dimensionless temperature and concentration gradient at the tube wall can be obtained by

$$\left. \frac{\partial \theta}{\partial \eta} \right|_{\eta=1} = - \sum_{m=1}^{\infty} \frac{2J_1^2(\beta_m)}{J_0^2(\beta_m) + J_1^2(\beta_m)} \beta_m^2 e^{-2\beta_m^2 X} \quad (22)$$

$$\left. \frac{\partial \varphi}{\partial \eta} \right|_{\eta=1} = - \sum_{m=1}^{\infty} \frac{2J_1^2(\sqrt{\text{Lew}}\beta_m)}{J_0^2(\sqrt{\text{Lew}}\beta_m) + J_1^2(\sqrt{\text{Lew}}\beta_m)} e^{-2\beta_m^2 X} \quad (23)$$

4 Results and discussion

Figure 2 shows the effects of the Biot number on the dimensionless wall temperature for $\text{Lew} = 1.4$. For very large Biot number, the dimensionless wall temperature drops drastically in a very short distance to zero, after which the inner wall temperature is identical to the temperature of the external flow. The dimensionless wall temperature gradually increases with decreasing Biot number. When $Bi = 0.1$, the dimensionless wall temperature become a linear function of X , which is similar to the characteristic of the boundary condition of the second kind. The mean dimensionless temperature profiles along the X -direction at different Biot numbers are shown in Fig. 3. For large Biot number, the dimensionless mean temperature decreases dramatically along the X -direction. When $Bi = 0.1$, the dimensionless mean temperature linearly change with X , which is consistent with the characteristics of the boundary condition of the second kind.

Figures 4 and 5 depict the effects of the Biot number on the variations of dimensionless inner wall concentration and mean concentrations along the X -direction. When $Bi = 0.1$, the concentration decreases very quickly in the entrance region ($X \leq 0.1$). The dimensionless wall and mean concentration become linear functions of X for $X > 0.1$, which is consistent with the

results obtained by boundary condition of the second kind. For large Biot number, the dimensionless wall concentration is almost uniformly equal to zero, and the dimensionless mean concentration also decreases rapidly. This means that the dimensionless inner wall concentration is equal to the saturation concentration of the local temperature. Comparing Figs. 2 and 4, one can conclude that the effects of the Biot number on the distributions of dimensionless inner wall concentration are more sensitive than that on the dimensionless inner wall temperature. It can also be concluded from Figs. 3 and 5 that the effect of Biot number on the dimensionless concentration is similar to that on the dimensionless mean temperature.

Figure 6 presents the variation of the local Nusselt number based on the total heat supplied by the external flow at different Biot number. For low Biot number ($Bi = 0.1$ and 1), the Nusselt number decrease quickly when $X \leq 0.02$, but when $X > 0.02$, the Nusselt number increase with X till they becomes constant, which are 15.9 for $Bi = 0.1$ and 15.4 for $Bi = 1$ when $X \geq 0.25$. For high Biot number such as $Bi = 10$ and 100 , the Nusselt number decrease dramatically at the entrance region and reaches constant when $X \geq 0.1$. Figure 7 shows the variation of the local Nusselt number based on the convective heat transfer inside the tube. Comparing Fig. 7 with Fig. 6, although the Nusselt number based on the external heat transfer is much high than that based on the convective heat transfer inside the tube, the effects of Biot number on Nusselt number based on the convective heat transfer inside the tube is similar to that on the Nusselt number based on the external heat transfer. The entrance effect on the Nusselt number of internal flow is very sensitive and the fully developed Nusselt number decrease with increasing Biot number—their values are 8.0 , 7.7 , 6.8 , and 6.5 for $Bi = 0.1$, 1 , 10 , and 100 , respectively. The effect of Biot number on the Sherwood number is illustrated in Fig. 8. Due to the entrance effect, the local Sherwood Number drop dramatically when $X < 0.1$ and they reach constant after $X > 0.1$. For fully developed conditions, The Sherwood number decrease with Biot number increase and the values of Sh are 8.0 , 7.6 , 6.3 , and 5.8 when $Bi = 0.1$, 1 , 10 , and 100 , respectively.

Figure 9 shows the effect of Lewis number on the Nusselt number based on the heat transfer of the external flow at $Bi = 1$ and $L = 1$. As shown in Fig. 9, the Nu_e decrease very quickly and reach constant when $X > 0.1$ for low Lewis Number ($Lew = 0.4$ and 1), however, with Lewis number increase, the Nu_e drop when $X < 0.05$ and then increase till reach constant (the constant value is around 15) for any Lewis number in the range of $0.4 \sim 5$. The effect of the Lewis number on the Nu_e mainly concentrates on the entrance region. For $Bi = 1$ and $L = 1$, the Lewis number effect on the Nusselt number based on the convective heat transfer inside the tube has been illustrated in Fig. 10. The local Nusselt number decrease dramatically at the entrance region ($X < 0.1$) and become constant in the fully developed region when $Lew = 0.4$ and 1 . The different Lewis number effect on the Nu_i mainly focus on the entrance region, and the effect of Lewis number on Nu_i in developed region can be neglected.

The effect of Lewis number on the Sherwood number is shown in Fig. 11. For the $Lew = 0.4$, the Sherwood number decreases when $X < 0.02$, and increases till reach constant at $X > 0.15$. For the $Lew = 1$, 2 and 5 , the Sherwood number drop quickly at the entrance region and arrives constant at the fully developed region. The Sherwood number decrease with increasing Lewis number in the fully developed region. Comparing Figs. 10 and 11, it can be found that the values of Nu_i are identical to the values of Sh when $Lew = 1$ since Eq. 12 is identical to Eq. 13 when $Lew = 1$.

5 Conclusions

Coupled heat and mass transfer in a circular tube subject to external convection was analytically studied and the closed form solutions for temperature and concentration distribution, the local Nusselt number and local Sherwood numbers were obtained. When $Lew = 1.4$ and $X < 0.2$, the Nu_e decreases at first ($X < 0.02$) and then increases till reaches constant along the X -direction

at lower Bi while it decreases when Bi is high ($Bi \geq 10$). When $X \geq 0.2$, Nu_e keeps constant for all case and the Nu_e decreases with the increasing Bi . The effect of Bi on the Nu_i is similar to that on the Nu_e . For any Biot number, Sh decreases dramatically along the X -direction in the entrance region ($X < 0.1$), while they keep constant and decrease with Bi increase in the fully developed region ($X \geq 0.1$). The results also show that the effects of the Lewis number on the Nu_e , Nu_i , and Sh mainly concentrate on the entrance region.

Appendix

The general solution of equation θ (Eq. 6) can be obtained by separation of variables [14]

$$\theta = F_1 e^{-2\beta_n^2 X} J_0(\beta \eta) \quad (24)$$

where, F_1 is an unspecified constant and β is the eigenvalue.

By using separation of variables method, the solution of φ can be written as

$$\varphi = H(X) W(\eta) \quad (25)$$

Substituting Eq. 25 into Eq. 7, we have

$$\frac{1}{2} \frac{H'}{H} = \frac{1}{Le_w W_\eta} (\eta W'' + W') = -\lambda^2 \quad (26)$$

where λ is the eigenvalue. Equation 26 can be rewritten as

$$H' + 2\lambda^2 H = 0 \quad (27)$$

$$W'' + \frac{1}{\eta} W' + (\sqrt{Le_w} \lambda)^2 W = 0 \quad (28)$$

The general solution of Eq. 27 is

$$H = C_1 e^{-2\lambda^2 X} \quad (29)$$

Equation 28 is a zeroth order of Bessel equation, based on the boundary condition specified by Eq. 9, its solution can be expressed as

$$W = C_2 J_0(\sqrt{Le_w} \lambda \eta) \quad (30)$$

Therefore, the dimensionless concentration solution becomes

$$\varphi = F_2 e^{-2\lambda^2 X} J_0(\sqrt{Le_w} \lambda \eta) \quad (31)$$

where, F_2 is another unspecified constant.

Substituting Eqs. 24 and 31 into Eqs. 10 and 11, one obtains

$$\lambda = \beta \quad (32)$$

$$F_1 J_0(\beta) = F_2 J_0(\sqrt{Lew}\beta) \quad (33)$$

Obviously, $F_1 = F_2$ when $Lew = 1$, thus,

$$Bi J_0(\beta) = \beta(L+1) J_1(\beta) \quad (34)$$

When $Lew \neq 1$, Eq. 34 should be replaced by

$$J_0(\sqrt{Lew}\beta) Bi J_0(\beta) = \beta \left[\frac{L}{\sqrt{Lew}} J_1(\sqrt{Lew}\beta) J_0(\beta) + J_0(\sqrt{Lew}\beta) J_1(\beta) \right] \quad (35)$$

There will be many different eigenvalues which satisfy Eqs. 34 or 35, thus, eigenvalues β_m ($m = 1, 2, 3, \dots$) obtained by the eigen equations can be expressed by

$$Bi J_0(\beta_m) = \beta_m(L+1) J_1(\beta_m) \text{ at } Lew=1 \quad (36)$$

$$\begin{aligned} \beta_m \left[\frac{L}{\sqrt{Lew}} J_1(\sqrt{Lew}\beta_m) J_0(\beta_m) + J_0(\sqrt{Lew}\beta_m) J_1(\beta_m) \right] \\ = Bi J_0(\sqrt{Lew}\beta_m) J_0(\beta_m) \text{ at } Lew \neq 1 \end{aligned} \quad (37)$$

Thus, θ and φ can be written as

$$\theta(\eta, X) = \sum_{m=1}^{\infty} A_m e^{-2\beta_m^2 X} J_0(\beta_m \eta) \quad (38)$$

and

$$\varphi(\eta, X) = \sum_{m=1}^{\infty} B_m e^{-2\beta_m^2 X} J_0(\sqrt{Lew}\beta_m \eta) \quad (39)$$

The equation should satisfy the boundary conditions of Eq. 8, i.e.,

$$1 = \sum_{m=1}^{\infty} A_m J_0(\beta_m) \quad (40)$$

Thus,

$$A_m = \frac{\int_0^1 \eta J_0(\beta_m \eta) d\eta}{\int_0^1 \eta (J_0(\beta_m \eta))^2 d\eta} = \frac{2J_1(\beta_m)}{\beta_m [J_0^2(\beta_m) + J_1^2(\beta_m)]} \quad (41)$$

Similarly:

$$B_m = \frac{\int_0^1 \eta J_0(\sqrt{Le} \beta_m \eta) d\eta}{\int_0^1 \eta (J_0(\sqrt{Le} \beta_m \eta))^2 d\eta} = \frac{2J_1(\sqrt{Le} \beta_m)}{\sqrt{Le} \beta_m [J_0^2(\sqrt{Le} \beta_m) + J_1^2(\sqrt{Le} \beta_m)]} \quad (42)$$

Therefore, Eqs. 38 and 39 can be rewritten as

$$\theta(\eta, X) = \sum_{m=1}^{\infty} \frac{2J_1(\beta_m) J_0(\beta_m \eta)}{\beta_m [J_0^2(\beta_m) + J_1^2(\beta_m)]} e^{-2\beta_m^2 X} \quad (43)$$

$$\varphi(\eta, X) = \sum_{m=1}^{\infty} \frac{2J_1(\sqrt{Le} \beta_m) J_0(\sqrt{Le} \beta_m \eta)}{\sqrt{Le} \beta_m [J_0^2(\sqrt{Le} \beta_m) + J_1^2(\sqrt{Le} \beta_m)]} e^{-2\beta_m^2 X} \quad (44)$$

where β_m can be obtained by Eqs. 36 and 37.

List of symbols

Greek symbols	Subscripts
A	constant in Eq. 4
A_m	coefficient in Eq. 38
B	constant in Eq. 4
B_m	coefficient in Eq. 39
Bi	Biot number
C	mass fraction (dimensionless)
C₁	constant in Eq. 29
C₂	constant in Eq. 30
c_p	specific heat at constant pressure (J/kg K)
D	mass diffusivity (m ² /s)
F₁	unspecified constant in Eq. 24
F₂	unspecified constant in Eq. 31
h	heat transfer coefficient (W/m ² K)
h_{sg}	latent heat of sublimation (J/kg)
J₀	Bessel function of zero order
J₁	Bessel function of first order

k	thermal conductivity (W/m K)
L	dimensionless latent heat defined by Eq. 5
Lew	Lewis number
Nu_e	Nusselt number based on the heat transfer of external flow
Nu_i	Nusselt number based on the heat transfer of the internal tube flow
Pe	Peclet number
R	radius of the tube (m)
Re	Reynolds number
r	radius coordinate
Sh	Sherwood number
T	temperature (K)
U	fluid velocity (m/s)
u	fluid velocity (m/s)
X	dimensionless axial coordinate
x	axial coordinate
W	eigen function
α	thermal diffusivity (m^2/s)
β	eigenvalue
ϕ	dimensionless concentration
η	dimensionless radial coordinate
λ	eigenvalue
θ	dimensionless temperature
ρ	density (kg/m^3)
o	entrance
e	external
i	internal
m	mean value control variable
w	wall

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Fig. 1.
Physical model

Fig. 2.
Effect of Biot number on the dimensionless wall temperature ($Lew = 1.4$, $L = 1$)

Fig. 3.
Effect of Biot number on the dimensionless mean temperature ($Lew = 1.4$, $L = 1$)

Fig. 4.
Effect of Biot number on the dimensionless wall concentration ($Lew = 1.4$, $L = 1$)

Fig. 5.
Effect of Biot number on the dimensionless mean concentration ($Le_w = 1.4$, $L = 1$)

Fig. 6. Effect of Biot number on the Nusselt number based on the external heat transfer ($Lew = 1.4$, $L = 1$)

Fig. 7.
Effect of Biot number on the Nusselt number based on convective heat transfer inside the tube
($Lew = 1.4$, $L = 1$)

Fig. 8.
Effect of Biot number on the Sherwood number ($Lew = 1.4$, $L = 1$)

Fig. 9. Effect of Lewis number on the Nusselt number based on external heat transfer ($Bi = 1, L = 1$)

Fig. 10.
Effect of Lewis number on the Nusselt number based on convective heat transfer inside the tube ($Bi = 1$, $L = 1$)

Fig. 11.
Effect of Lewis number on the Sherwood number ($Bi = 1$, $L = 1$)