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Convolution-Based Forced Detection Monte Carlo Simulation Incorporating Septal Penetration Modeling

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Abstract

In SPECT imaging, photon transport effects such as scatter, attenuation and septal penetration can negatively affect the quality of the reconstructed image and the accuracy of quantitation estimation. As such, it is useful to model these effects as carefully as possible during the image reconstruction process. Many of these effects can be included in Monte Carlo (MC) based image reconstruction using convolution-based forced detection (CFD). With CFD Monte Carlo (CFD-MC), often only the geometric response of the collimator is modeled, thereby making the assumption that the collimator materials are thick enough to completely absorb photons. However, in order to retain high collimator sensitivity and high spatial resolution, it is required that the septa be as thin as possible, thus resulting in a significant amount of septal penetration for high energy radionuclides. A method for modeling the effects of both collimator septal penetration and geometric response using ray tracing (RT) techniques has been performed and included into a CFD-MC program. Two look-up tables are pre-calculated based on the specific collimator parameters and radionuclides, and subsequently incorporated into the SIMIND MC program. One table consists of the cumulative septal thickness between any point on the collimator and the center location of the collimator. The other table presents the resultant collimator response for a point source at different distances from the collimator and for various energies. A series of RT simulations have been compared to experimental data for different radionuclides and collimators. Results of the RT technique matches experimental data of collimator response very well, producing correlation coefficients higher than 0.995. Reasonable values of the parameters in the lookup table and computation speed are discussed in order to achieve high accuracy while using minimal storage space for the look-up tables. In order to achieve noise-free projection images from MC, it is seen that the inclusion of the RT implementation for septal penetration increases the speed of the simulation by a factor of about 7,500 compared to the conventional SIMIND MC program.

Index Terms

Acceleration; modelling; Monte Carlo; SPECT

I. Introduction

Monte Carlo (MC) is a well-known computational tool used to simulate the photon transport in single photon emission computed tomography (SPECT). While accurate, the main drawback of MC is its enormous computational burden which prevents it from being routinely implemented into clinical practice. In recent years, several variance reduction techniques (VRT) have been developed to improve its detection efficiency as well as the computation speed. One of the latest implementations used to significantly accelerated MC modeling is convolution-based forced detection (CFD) [1], [2]. With CFD, photons are tracked through the object under study, and then forced to travel in the direction perpendicular to detector surface. The probability of photon detection is convolved with a Gaussian blur function representative of the collimator response function. This mathematical model can only estimate the geometric response based on the assumption that photons traveling in the collimator material will be completely attenuated. However, when the photon energy exceeds 200 keV, there is high probability that photons are able to penetrate the collimator [3]. Furthermore, some photons may interact with the collimator materials before they penetrate the septa. Therefore, a complete characterization of the collimator response involves geometric response, septal penetration and scatter. Although different collimators are used to minimize septal penetration, thin septa are preferred when high detection efficiency is required. Therefore, the effect of septal penetration is typically significant when high or medium energy radionuclides are used. Overall, the amount of septal penetration is determined by the path length of rays through the septa which depends on the distance between the collimator and radioactive source, hole length, hole pattern and septal thickness [4].

Several studies have been done to model collimator response. Ljungberg, *et al.* have developed a Delta-Scattering technique by sampling a series of random numbers to determine the photon interaction types in the collimator when the traverse of photons are tracked [5]. Du, *et al.* used Monte Carlo N-Particle (MCNP) to simulate photon interaction types in the collimator [6]. Song, *et al.* presented a method using MC code to generate the collimator response models based upon the angular response functions (ARFs), which depends on the photon traveling directions and photon energies [7]. Recently, Staelens, *et al.* extended convolution-based forced detection method (CFD) by convolving the photon transmission probabilities with MC pre-simulated collimator responses when ray tracing method is performed onto the object [8]. However, the MC simulation of a noise-free collimator response image is extremely time consuming. In this work, we will introduce a method using ray tracing (RT) to analytically generate the collimator response model including geometric response and septal penetration. The collimator scatter compartment is not going to be discussed here because of the difficulty in analytically simulating photon scatters. RT has been previously used to evaluate the collimator characters by modeling the collimator response [3], [4], [9]. However, it has not been applied to compensate for collimator response. Eventually, the RT resultant models will be applied into CFD MC code for further acceleration.

II. Methods

A. Models

The fundamental idea of the RT method is shown in Fig. 1(a). Consider a parallel hole collimator and a point source located at the distance, z , above the collimator surface. The gray regions denote the collimator septa and the white represent the collimator hole. A ray passing through the gray region thus contributes to the septal penetration, otherwise, it contributes to the geometric response. Photons are simulated as emitting isotropically from the point source $O(O_1, \dots, O_i, \dots, O_n)$, thus striking onto the collimator back plane. Suppose one of the emitted rays interacts with the collimator at point $A(A_1, \dots, A_i, \dots, A_n)$ on the front plane and $B(B_1, \dots, B_i, \dots, B_n)$ on the back plane, where the subscript i corresponds to i^{th} ray. The direction of

each ray can be specified by the polar angle $\gamma(\gamma_1, \dots, \gamma_i, \dots, \gamma_n)$ and azimuth angle $\theta(\theta_1, \dots, \theta_i, \dots, \theta_n)$. The intensity of the photons striking point B depends on the path length of OB in the septal region, as denoted by P_{OB} . It is easy to see that $P_{OB} = 0$ for geometric response (i.e., no septal penetration). The equation of intensity can be written as:

$$I_B = I_0 \exp(-P_{OB} \cdot \mu) \quad (1)$$

where, I_0 is the initial photon interaction probability at a certain photon scatter location, and μ is the linear attenuation coefficient of the collimator material at a specific photon energy (e.g., 3.12 cm^{-1} for lead at 364.5 keV.)

Fig. 1(b) and (c) simplify Fig. 1(a) and specify two interesting ray characteristics which stress the fundamental ideas of the RT method as presented in this paper. The first point is that the projections of rays from the source along the axis OO'' to a certain point B on the collimator are the same no matter where the point source O is located. Thus, projection line of ray OB is always $O''B$ when point source O is moved along the axis OO'' . In addition, the location of point A' ($A'_1, A'_2, \dots, A'_n$), which is the projection of point A , is moved along $O''B$ with respect to the movement of O , as shown in Fig. 1(b). The second point is that the three dimensional (3D) septal path length P_{OB} can be obtained by the calculation of the two dimensional (2D) path length $P_{A'B}$, which denotes the path length of $A'B$ in the gray region. We then have:

$$P_{OB} = \frac{P_{A'B}}{\sin\theta} = \frac{P_{O''B} - P_{O''A'}}{\sin\theta} \quad (2)$$

where, $P_{O''B}$ and $P_{O''A'}$ represent the path length of $O''B$ and $O''A'$ in the gray region, respectively. It is noted that (2) can be only used for the case of a parallel hole collimator.

If the location of B is denoted as $(L_{O''B}, \gamma)$ ($L_{O''B}$ is the length of segment $O''B$, and γ is the angle of $O''B$ away from the horizontal direction.), and defined by polar coordinates as shown in Fig. 1(d), the value of θ and location of point A' in Fig. 1 can be derived from the collimator parameters and location of the point source O . Suppose the point source is placed above collimator at a distance z , the collimator length is l , and the angle of ray $O''A'$ to the horizontal direction is γ , the same as ray $O''B$. θ can be derived from the trigonometric functions:

$$\theta = \arctan\left(\frac{L_{O''B}}{z+l}\right) \quad (3)$$

where, the L represents the length of the segment denoted by the appropriate subscript, then $L_{O''A'}$ is:

$$L_{O''A'} = z \cdot \tan(\theta) \quad (4)$$

and so the position of A' can be written as $(L_{O''A'}, \gamma)$ on polar plane.

From the above discussion, it is easy to see that the intensity of photons at point B can be calculated by (1) using P_{OB} once the locations of B and O are known. For different O positions, the corresponding values of $P_{O''B}$ are the same while $P_{O''A'}$ may not. However, $P_{O''A'}$ may be the same for different B . The calculation of $P_{O''B}$ and $P_{O''A'}$ are significantly time consuming, therefore, it would be helpful to produce a lookup table LUT_{PL} to store the cumulative septal

thickness $P_{O''B}$ and $P_{O''A}$. In this study, hexagonal hole parallel collimators are used, although this method is applicable for all types of parallel collimators.

Several rays are traced in the collimator back plane from one center point O'' in different directions, with each ray modeled by a large number of equally spaced segments denoted D ($D_1, \dots, D_i, \dots, D_n$), as seen from Fig. 2(a). A region including a collimator hole and the surrounding septa is equally divided into n sub-regions, and the center location of each sub-region is set as the center point O'' . n is a positive integer number which should at least be greater than the ratio of the hole-septa area to a projection image pixel area. In the following discussion, O'' is chosen as the center of a collimator hole as an example.

After $P_{O''D}$, the septal path lengths of segment $O''D$ are calculated and stored in LUT_{PL} together with their polar coordinates, a second lookup table consists of the resultant collimator response functions (denoted as LUT_{CR}) based upon LUT_{PL} and is constructed by the projection images of a point source located at different distances from the collimator at a variety of photon energies. LUT_{PL} is incorporated into SIMIND MC program in order to accelerate it. The Section II-B discusses the creation of LUT_{PL} and LUT_{CR} and their subsequent implementation in CFD-MC.

B. Path Length Lookup Table

The 360° circle centered at point O'' is equally divided into N_r evenly spaced rays, as illustrated in Fig. 2(a). The location of points D on the rays are computed to see whether they are within the septa or inside the collimator hole, which can be seen in Fig. 2(b).

Suppose the center of a hexagonal hole is given by location (C_x, C_y) with width and height denoted as L_x and L_y , respectively, and the distance between two adjacent points D_{i-1} and D_i is denoted as d . d should be satisfied with the conditions that $d \ll L_x$ and $d \ll L_y$ to minimize the error. A study of the amount of rays N_r and sampling interval d is performed below in the Discussion. A given point (x, y) within the hexagonal hole should satisfy:

$$\begin{cases} |\tilde{y}| < \frac{L_y}{2} \\ |\tilde{y} - \sqrt{3}\tilde{x}| < L_y \\ |\tilde{y} + \sqrt{3}\tilde{x}| < L_y \end{cases} \quad (5)$$

where, $\tilde{x} = x - C_x$, $\tilde{y} = y - C_y$. These three inequality expressions correspond to the functions of the 6 edges of the hexagonal hole. Note this equation varies with different hole geometries.

When point D is moved along a ray, it is difficult to determine in which hexagonal hole the point D is contained. With so many holes in one collimator, it is exceedingly time consuming to determine all the hexagons by (5). Rather, suppose the coordinates of point O'' is (0,0), then the center distance between two adjacent vertical holes along a single column is:

$$S_y = L_x + L_y \quad (6)$$

and the center distance between 2 adjacent horizontal holes along a single row is:

$$S_x = \sqrt{3} \cdot S_y \quad (7)$$

Equation (6) and (7) should be modified slightly for foil collimators. Assuming now that the current hole is located on the 0^{th} row and 0^{th} column, (i.e., location (0,0)), then the coordinates for the hole centered on the n^{th} row and m^{th} column are:

$$\begin{cases} C_x = \frac{s_x}{2} \cdot m \\ C_y = \frac{s_y}{2} \cdot n \end{cases} \quad (8)$$

where n and m can be any integer number.

Observing the hole pattern and septal thickness, when a ray exits out of a hole and proceeds to the next hole, the second hole is always adjacent to the first. For a practical (low/medium/high energy, general purpose/high resolution) collimator, it is usually adequate to consider only the adjacent 18 holes, as depicted by the shaded holes in Fig. 2(a). The number of adjacent holes should be increased only when $2L_y < L_s$, which makes the next point D always traveling in the septa until it gets into a new hole other than the shaded hole. Initially, we will call the current hole that D resides, the center hole, and its 18 adjacent holes as side holes.

The RT process starts with point D moving from O'' along one of the rays, with center hole (C_x, C_y) as (0,0) and initial cumulative septal thickness (CST_0) as 0 cm, with the subscript 0 denoting the starting D . The previous point D is recorded as D_{i-1} , and the current one being D_i . When D is inside the center hole (one of the side holes), it is denoted as $D \in H_c$ ($D \in H_s$), with D within the septa being denoted as $D \in S$. During the movement along ray $O''D$, four possibilities may occur:

- $D_{i-1} \in H_c$ and $D_i \in H_c$. This means point D_i is still within the same hole and thus $CST_i = CST_{i-1}$. The current hole is still the center hole.
- $D_{i-1} \in H_c$ and $D_i \in S$. This means D_i is moving from air to septa. $CST_i = CST_{i-1} + d$. The current hole changes from the center hole to a side hole.
- $D_{i-1} \in S$, the coordinates of D_i are required to be calculated with respect to each side hole using (5) to check whether it is inside one of the side holes. If $D_i \in S$, $CST_i = CST_{i-1} + d$, and the current hole is still a side hole.
- $D_{i-1} \in S$, but $D_i \in H_s$. This means D_i is moving from the septa to a collimator hole, and $CST_i = CST_{i-1}$. The center hole is renewed as the one which holds the current D_i . The current hole changes back to the center hole, and the side holes are updated.

This process is repeated until D_i goes to the end of all the rays being simulated with the CST of each position on the ray being stored in the lookup table LUT_{PL} .

C. Collimator Response Lookup Table

Once generated, the collimator response can be calculated using the table, LUT_{PL} , by reprojecting the cumulative 2D septal thickness back to the 3D septal thickness based upon (2). By reading $P_{O''B}$ in Fig. 1 from LUT_{PL} , the polar coordinates ($L_{O''B}, \gamma$) of B and the cumulative septal thickness $P_{O''B}$ are known. The coordinates ($L_{O''A'}, \gamma$) of A' can be derived from (3) and (4), and the subsequent value of $P_{O''A'}$ can be determined by searching LUT_{PL} . Furthermore, P_{OB} can be determined based on (2). Therefore, the intensity of the photon flux striking detector location B is determined by (1). As the detector surface is typically divided into a number of detector bins, this intensity gets added to the value of the detector bin corresponding to this location and the number of photons striking at this bin is subsequently incremented.

$$\begin{cases} V_{i,j}=V_{i,j}+I_B \\ CNT_{i,j}=CNT_{i,j}+1 \end{cases} \quad (9)$$

where, i, j denote the i^{th} row, j^{th} collimator bin. $V_{i,j}$ is the value of this bin, I_B is the intensity of certain photon flux on point B , as calculated from (1). $CNT_{i,j}$ is the total amount of the photons striking on this bin.

This procedure is repeated for all the locations on a given ray and for all possible emission angles over 360° . Eventually, the value of each bin can be given by:

$$V_{i,j} = \frac{V_{i,j}}{CNT_{i,j}}. \quad (10)$$

In order to generate the LUT_{CR} , the point source is moved along the axis OO'' at a step of d_p and the value of μ at a step of d_μ in (1) is altered according to different photon energies. The sum of each resultant projection image is normalized to 1 before the image is stored in LUT_{CR} for each point source location and different μ . The matrix size of RT resultant images should be twice of the projection images to ensure the corners of the projection images get effective data when the photons are traveling above the opposite corners on the diagonals.

The sampling intervals for the d_p and d_μ are analyzed in the Discussion.

D. Implementation of Lookup Table, LUT_{CR} , Into SIMIND

Following the generation of LUT_{CR} , it is then implemented into a CFD version of the SIMIND MC program (RT-MC) [1], [2]. In CFD, when a photon of energy E , undergoes scatter within the object, it is directed to the path perpendicular to the detector. The photon probability P is determined by (11), which is the product of photon history weight (PHW) beginning with 1 at the initial emission location, the probability of Compton scattering, as calculated by the Klein-Nishina (KN) function, and the probability $(1 - P_{pe}(E))$ that the photon is not absorbed by photoelectric interaction.

$$P = PHW_i \times KN(\varphi) \times (1 - P_{pe}(E)) \exp(-\mu(E) \cdot l_{FP}) \quad (11)$$

where, i is the photon interaction number, φ is the scattered angle, l_{FP} is the photon forced path, and $\exp(-\mu(E) \cdot l_{FP})$ is the attenuation along the forced path.

LUT_{CP} is then consulted to determine the projection images that most likely matches the photon energy and its scatter location. If an exact match is not found, then the projection image is obtained via linear interpolation.

III. Results

In order to evaluate RT-MC generated models, a GE Millennium VG camera is simulated and compared with experimental data for a block water phantom. A high energy collimator ($L_y = 0.4$ cm, $L_s = 0.18$ cm) and I-131 point sources are used with 10,000 rays simulated over 360° , and with d is set to 0.004 cm. A 10% energy window centered at 364.5 keV is used to create the projection images.

A. Point Source Response

Fig. 3 shows the results of RT-MC along with experimental data for I-131 when the point source is placed at 30 cm above the collimator in the air. The RT-MC generated image matches the experimental data very well. Additionally, the RT image for the HEGP collimator has been rotated counterclockwise by 15° to match experimental results. The center dark circle region corresponds to the geometric response with the long tails of each image corresponding to septal penetration, which have been individually illustrated in Fig. 3(c) and (d). Septal penetration appears as very well-known 6-sided star pattern with a second, less obvious star pattern situated 30° away from the first one. This can be explained in Fig. 1(d), we can see that in the vertical direction (black lines), the rays at every 60° travel through the thinnest septal thickness. The second, less intense, star pattern appears offset 30° (gray lines) from the first pattern due to another thin septal thickness compared to its surrounding direction. Fig. 3(e) and (f) respectively give the linear-plot and log-plot of one tail profile to show the difference between RT-MC and experimental data. RT-MC simulation shows more greatly fluctuated ripples is because our model is lack of the photon interactions such as collimator scatter, X-ray emission, and else interactions after crystal.

There is an additional structure seen in the PSF images as regions of higher and lower intensity along the septal penetration tails in Fig. 3. This can be explained from the CST 's stored in the LUT_{PL} . Fig. 4(a) plots $P_{O''B}$ in the horizontal direction for a single ray. Fig. 4(b) shows a magnified view of the selected region in (a), and Fig. 4(c) and (d) plot the septal thickness $P_{A'B}$ and P_{AB} for differing B when the point source is located 30cm above the collimator, CST appears as a staircase pattern, with the flat lines corresponding with point B in the collimator hole and the increasing lines associated with the points in the collimator septa. In Fig. 4(c) and (d), we can see some ripples in this pattern whereby several ripples combine to a larger packet. These ripples and bumps come from the numerator term in (2). The increasing portions or flat portions in Fig. 4(a) minus another increasing portion results in periodically appearing ripples. Between every 2 bumps, $P_{O''B}$ and $P_{O''A}$ moves along two flat lines and the minus term results in the slowly increasing line, which denotes the increasing azimuth angle canceling with the increase in the septal thickness. The sin term in the denominator of (2) flattens the septal thickness P_{AB} , but enlarges the first bump more than others because of the small photon emission angle.

Full width at half maximum (FWHM) has been introduced to evaluate the RT-MC geometric compartment. Because the result of RT does not include the intrinsic resolution of the detector, it is necessary to further convolve the result of RT with the camera intrinsic resolution. This has been found to consist of a $\sigma = 0.2$ cm Gaussian function for HEGP collimator using 2 cm thick crystal. The experimental and RT-MC generated images are convolved with an additional $\sigma = 0.2$ cm Gaussian function, to remove the effect of the collimator hole pattern as seen in Fig. 3. Fig. 5 shows the FWHM evaluation of RT and experimental PSF's when the point source is located in the air and water, respectively. The correlation coefficient (r^2) has then been introduced to determine how well the RT models fit the experimental data.

Table I shows the r^2 evaluation of HEGP collimator for sources in both air and water, together with the same evaluations of medium energy general purpose collimator (MEGP collimator: $L_y = 0.3$ cm, $L_s = 0.1$ cm) with In-111 and low energy high resolution collimator (LEHR collimator: $L_y = 0.15$ cm, $L_s = 0.02$ cm) with Tc-99m when the source is placed in the air.

We can see that all the r^2 of the PSF's of these three collimators are greater than 0.995, showing that the RT models are consistent with experimental data. Fig. 5(a) and (b) shows that the simulated detector sensitivity (CPS/MBq) of RT-MC agrees well with the measured data, for both air or water.

Full width at tenth maximum (FWTM) is taken as the measurement criteria to assess the accuracy of the septal penetration model [10]. Excellent agreement between the RT-MC models and the experimental PSF's for these three collimators is also seen in the Table I.

B. Energy Spectrum

RT models do not include the photon interactions in the collimator and gamma camera. However, photon scatter within the collimator and crystal, X-ray emission, and backscatter from the materials behind the crystal can be very important when medium or high energy photons are simulated. Fig. 6 shows the energy spectra of I-131 using HEGP collimator. Photon interactions within the collimator and gamma camera affect the photons with energies lower than 200 keV much more than those with higher energies. In practice, the 10% energy window centered at 364.5 keV is used, which makes the photon interactions not a significant problem. For more accurate simulation, the new SIMIND MC code developed by Ljungberg, *et al.*, is recommended to be combined with the RT models [5].

C. Computation Time

In order to evaluate the speed of the RT method when implemented into the SIMIND MC program, we have simulated a $50 \times 50 \times 50$ I-131 block source contained within a $128 \times 128 \times 128$ voxelized air space using a HEGP collimator. Different numbers of photons have been simulated and the computation time recorded. As the projection image is expected to be relatively uniform, the coefficient of variation (CV) of the center 50×50 dark region in the resulting images is calculated to measure the degree of uniformity.

Fig. 7 shows a plot of CV vs computation time. As seen, the RT method requires approximately 100 seconds in order to achieve a stable level of CV and a projection image with $CV = 0.006$. Forced detection MC is performed as a non-RT Monte Carlo method (non-RT MC) to compare with RT-MC. It takes the non-RT SIMIND program 1494 seconds to achieve an image with $CV = 0.03$. In order to achieve a noise free image with $CV = 0.006$, it takes the non-RT MC longer than 750,000 s (9 days) to run, indicating a speed improvement for the RT-MC of about 7,500 times. Fig. 8(a) and (b) shows the projection images of non-RT MC and RT-MC with $CV = 0.006$. It is easy to see that the result of RT-MC is relatively noise free but even after 9 days simulation, the non-RT MC is not. The center regions both depict the hexagonal pattern, which is due to the geometry of the collimator. The resultant non-RT image looks sharper than RT possibly because the maximum emission angle of septal penetration rays is limited in SIMIND MC for difference source-detector distances. Only one single maximum emission angle can be used for the cubic source which include several different distances. The limitation will result in insufficient septal penetration photons and an overestimate of the geometric photons, which can also be seen in the linear and log center profiles shown in Fig. 8(c) and (d).

IV Discussion

In the implementation of the RT method, it is possible to alter the number of the rays (N_r), ray step size (d), the distance between two adjacent source/detector planes (d_p) and the linear attenuation coefficient spacing (d_μ) used in the LUT_{PL} corresponding to different photon energies and LUT_{CR} lookup tables. By increasing N_r , and by reducing d , d_p , and d_μ , the resultant septal penetration model should yield higher accuracy, but at the expense of increased computation time and storage requirements for LUT_{PL} and LUT_{CR} . As such, a study of these parameters has been undertaken to determine the optimal setting in order to maintain accuracy, yet reduce computational and storage requirements. The following section discusses the appropriate parameters for HEGP, MEGP, and LEHR collimators.

A. Path Length Lookup Table Parameter Determination

An optimization has been performed to evaluate the minimum N_r necessary, and the optimal d in order to minimize computer requirements while maintaining accuracy. The CST and L_y of different collimators vary, which makes the optimal simulated parameters of each collimator different. The evaluation depends on the hole size and cumulative septal thickness. For reference, we have used projection images generated using a lookup table calculated with $N_r = 10,000$ and $d = 0.002$ cm, which have shown high accuracy compared with experimental data. PSF's modeled using reduced numbers of rays, N_r , and larger step sizes, d , have been compared to these reference images using the normalized mean square error (NMSE):

$$NMSE = \frac{\sum_{i=1}^n (O_i - F_i)^2}{\sum_{i=1}^n F_i^2} \quad (12)$$

where, O_i and F_i are the i^{th} pixel values of the estimated projection images and reference images, respectively.

Fig. 9(a) shows the average value of the NMSE for the same N_r when the point source moves from 10 cm to 60 cm in 10 cm intervals, and d is altered from 0.002 cm to 0.15 cm. Fig. 9(b) shows the average value of the NMSE for the same d when the point source moves from 10 cm to 60 cm at each step of 10 cm, and N_r is altered from 10,000 to 500. Fig. 9(c) and (d) are the log value of Fig. 9(a) and (b). We can see that d affects the result more than N_r . In fact, it can be seen that the LEHR collimator requires more precise parameters due to its thinner septal thickness and smaller hole size. Table II shows the optimal value of N_r and d when the corresponding NMSE value gets to a stable level. For all collimators, it is seen that 1,000 rays are enough for RT simulation when the step size is set to half of the collimator septal thickness.

B. Collimator Response Lookup Table Optimization

As the location of an emitted photon is variable and the resultant collimator attenuation coefficient varies widely as a function of energy (ranging from 1.167 cm^{-1} at 700 keV to 62.13 cm^{-1} at 100 keV), it is impossible to store all possible source location and energy combinations in the lookup table. Rather, we have implemented the LUT_{CR} for set distances from the detector surface, and for set photon energies. Thus, for a given source location and photon energy, we determine the correct collimator response model via linear interpolation between the stored values. Thus, we must determine the optimal spacing for source-detector distance, d_p and linear attenuation coefficient, d_μ .

For the determination of d_p , we have simulated the PSF's for two point sources with a specified d_p . A linear interpolation is performed between these PSF's and compared to the actual PSF at that location using the NMSE. The evaluation for d_μ is performed in an analogous manner. HEGP collimator are used in the following evaluation. As seen in Fig. 10, when the separation between calculated values for d_μ increases, the NMSE increases as the linear interpolation scheme is not accurate. It can generally be seen that reasonable accuracy exists in the simulated PSF even for $d_\mu = 0.5 \text{ cm}^{-1}$. As expected, it is seen that by reducing the plane separation, d_p , more accurate PSF's are obtained. This trend is seen to be essentially linear, thereby suggesting that the optimal plane spacing may be equal to the voxel size as simulated in the subsequent MC simulation.

V. Conclusion

In this work, we have introduced an effective ray tracing code into the SIMIND Monte Carlo program to account for septal penetration within the collimator. In order to efficiently implement this method, a lookup table storing the cumulative septal path lengths for a point source must first be generated. Once evaluated and stored, specific collimator response functions can then be calculated based on the path length lookup table. The collimator response functions are then stored in a subsequent lookup table for different source–detector distances and various photon energies. In order to accelerate Monte Carlo modeling, the collimator response lookup table is referenced for each photon that is emitted from the object simulated. The point spread function referenced from the lookup table is then convolved with the detection probability on the camera surface, thereby resulting in convolution-based forced detection incorporating septal penetration modeling.

An optimization of various parameters stored in the lookup tables has been performed for different types of collimators. A validation has been performed assessing the correlation coefficient for the point spread functions, and has shown that the ray tracing method is very accurate. When incorporated into the SI-MIND Monte Carlo program, it has been shown that the speed of the ray tracing method has increased the simulation speed by a factor of 7,500 compared to the conventional Monte Carlo program. Subsequent work will focus on using this RT-MC for image reconstruction in SPECT imaging.

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Fig. 1.

Representation of a point source above a parallel hole collimator. (a) Rays emitted from point source to the collimator. (b) Simplified illustration of the ray tracing method. (c) Collimator side view. (d) Collimator top view.

Fig. 2. Septal thickness calculation on the collimator back plane. (a) Rays on collimator back plane. (b) Schematic for checking whether point D is inside or outside a hexagonal hole.

Fig. 3.

Comparison between RT-MC model and experimental result using HEGP collimator when point source located at 30 cm above the collimator. (a) and (b) compare the experimental measured data and RT-MC resultant image, which has been separated into geometric (c) and septal penetration (d) compartments. The linear and log center profiles of one tail are plotted in (e) and (f) showing the great agreement between the measured data and RT simulation.

Fig. 4.

The cumulative septal thicknesses from O'' and O to point B along a ray. The x -axis denotes the location of point B away from the center O'' with the unit of cm. (a) Cumulative septal thickness $P_{O''B}$. (b) The magnified view of the block region in (a). (c) Septal thickness $P_{A''B}$ on the collimator plane. (d) The 3D septal thickness P_{AB}

Fig. 5. RT method sensitivity evaluation for PSF with point source in air (left) and in water (right). It presented the sensitivity of the detection based upon the detected photon counts per second for 1MBq activity.

Fig. 6.
The difference of energy spectra with and without photon interactions when photons hit on the collimator and gamma camera.

Fig. 7.

cv vs time. The solid line is the computation time of rt, and the dashed line is computation time of non-RT mc. The computation time of rt-mc is much less than non-RT mc, and the stable noise level of it is lower than non-RT mc.

Fig. 8.

Noise-free projection images of non-RT MC and RT-MC. The projection image of non-RT MC in (a) is still not noise free even after 9 days running, comparing with the noise free projection image of RT-MC in (b). The non-RT MC resultant image looks sharper than RT-MC image might because of the rays maximum emission angle limitation of the SIMIND MC code, which results in the insufficient septal penetration photon estimation shown by the linear and log center profiles in (c) and (d).

Fig. 9.

NMSE comparison of the model for the 3 collimators with the reference projection images over various N_r and d . (a) shows the effect of N_r is not very obvious, 1,000 rays is enough for simulation. (b) shows the effect of d is significant. The optimal value is about half of the septal thickness and results in accurate modeling. (c) Log value of (a). (d) Log value of (b).

Fig. 10.

The estimation of d_p and d_μ . (a) NMSE comparison of the interpolated images with the actual PSF at different d_μ . (b) NMSE comparison of the interpolated images with the actual PSF at different d_p . (c) Log NMSE of (a). (d) Log NMSE of (b).

TABLE I

r^2 Value for FWTM Measurement of Different Collimators Subscript h Denotes Horizontal Direction, With v Being Vertical Direction

collimator	$FWHM_h$	$FWHM_v$	$FWTM_h$	$FWTM_v$
HEGP (air)	0.9976	0.9966	0.9985	0.9932
HEGP (h2o)	0.9988	0.9931	0.9931	0.9921
MEGP (air)	0.9954	0.9954	0.9960	0.9933
LEHR (air)	0.9956	0.9964	0.9708	0.9872

TABLE II

Optimal Parameters for Different Collimators

Collimator	hole size L_y (cm)	septal thickness L_s (cm)	ray number N_r	stepsize d (cm)
HEGP	0.4	0.18	500	0.1
MEGP	0.3	0.1	500	0.05
LEHR	0.15	0.02	1000	0.01