Published in final edited form as:

Comput Methods Biomech Biomed Engin. 2010 ; 13(2): 247–256. doi:10.1080/10255840903170686.

A method for assessing the fit of a constitutive material model to experimental stress–strain data

Duane A. Morrow^a, Tammy Haut Donahue^b, Gregory M. Odegard^b, and Kenton R. Kaufman^{a,*}

^aMayo Clinic, 200 First Street SW, Rochester, MN 55905, USA

^bMichigan Technological University, 1400 Townsend Drive, Houghton, MI 49931-1295, USA

Abstract

Higher-order polynomial functions can be used as a constitutive model to represent the mechanical behaviour of biological materials. The goal of this study was to present a method for assessing the fit of a given constitutive three-dimensional material model. Goodness of fit was assessed using multiple parameters including the root mean square error and Hotelling's T^2 -test. Specifically, a polynomial model was used to characterise the stress–strain data, varying the number of model terms used (45 combinations of between 3 and 11 terms) and the manner of optimisation used to establish model coefficients (i.e. determining coefficients either by parameterisation of all data simultaneously or averaging coefficients obtained by parameterising individual data trials). This framework for model fitting helps to ensure that a given constitutive formulation provides the best characterisation of biological material mechanics.

Keywords

material properties; constitutive model; curve fitting

Introduction

The accuracy of a constitutive model is tied directly to the ability of that model to be well characterised with experimental data. When characterising a model for mechanical response, consideration must be given to the method for handling several data sets. This is especially important for biological materials in which the constitutive equations can be relatively complex and the amount of observed variability between samples can be substantial.

Several transversely isotropic, hyperelastic, three-dimensional formulations have been developed recently for modelling biological soft tissues such as tendon and skeletal muscle (Weiss et al. 1996; Johansson et al. 2000; Criscione et al. 2001; Jenkyn et al. 2002; Van Looke et al. 2006; Odegard et al. 2008). Although many functional forms of constitutive equations are generally used for these materials, a polynomial strain energy density function

© 2010 Taylor & Francis

*Corresponding author. kaufman.kenton@mayo.edu.

Publisher's Disclaimer: The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

The authors affirm that they have no financial affiliation or involvement with any commercial organisation that has direct financial interest in any matter included in this manuscript.

could be used with varying polynomial orders to represent the toe regions typical at low strains. Increases in polynomial order does not always result in a more accurate representation of the data being modelled, as oscillations not observed in the experimental data can appear in curves of models with unnecessarily high order.

Characterisation of constitutive models is often achieved using stress–strain curves from multiple uniaxial load–deformation tests for all manner of biological soft tissues including, but not limited to tendon (Yin and Elliott 2004; Ciarletta et al. 2008), ligament (Weiss et al. 2002; Cheng and Gan 2008), meniscus (Sweigart et al. 2004; Villegas et al. 2007) and muscle tissue (Van Looke et al. 2006, 2008). Constitutive model parameters can be determined through optimisation-based curve-fitting techniques, either simultaneously fitting all available stress–strain data from multiple samples or fitting data curves from individual samples and averaging resultant model parameters. It remains unclear, however, which method, if any, is preferable for this class of materials.

The goal of this study was to examine the differences in methods that can be used to determine the coefficients of a material model and provide a framework for a practical method to determine if a proposed model adequately characterises the data it is to represent. This was explored through the analysis of a model of a transversely isotropic and hyperelastic material using the two techniques described above. The characterised data were previously reported stress–strain data from skeletal muscle tissue under longitudinal extension (LE), transverse extension (TE) and longitudinal shear (LS; Morrow et al. 2008). Characterisation methods have been evaluated using the root mean square error (RMSE) between the resultant model and each dataset. The determined constitutive functions have been assessed through an analysis of the residuals and variances between the models and experimental data.

Materials and methods

Experimental data

Twelve fresh-frozen extensor digitorum longus muscles from skeletally mature New Zealand white rabbits underwent uniaxial material tests, with the institutional approval, experiencing deformations under either LE ($n = 4$), TE ($n = 4$) or LS ($n = 4$). Full details on the experimental set-up are available in Morrow et al. (2009). In brief, the tissue was strained under position control at $0.05\% \text{ s}^{-1}$, to minimise viscoelastic effects, and sampled at 20 Hz. The second Piola–Kirchhoff stress was calculated as the measured force divided by the initial cross-sectional area. Lagrange strain was calculated from the applied displacement and initial specimen length (for LE and TE) or thickness (for LS). Results of these tests are shown in the stress–strain plots of Figure 1.

Constitutive model

The skeletal muscle tissue was modelled as an incompressible (Blix 1894), transversely isotropic, hyperelastic material. Because of the incompressible transverse-isotropy, only the terms associated with the Green deformation tensor invariants I_1 , I_2 and I_4 were included (Malvern 1969). A polynomial expansion of these invariants was used for the strain energy function (Holzapfel 2000).

$$W = \sum_{i=1}^3 a_i (I_1 - 3)^i + \sum_{j=1}^3 b_j (I_2 - 3)^j + \sum_{k=2}^6 c_k (I_4 - 3)^k. \quad (1)$$

In order to fit this model to the experimentally derived stress–strain data, the second Piola–Kirchoff stress function was calculated as

$$S = 2 \left(\frac{\partial W}{\partial I_1} \frac{\partial I_1}{\partial \mathbf{C}} + \frac{\partial W}{\partial I_2} \frac{\partial I_2}{\partial \mathbf{C}} + \frac{\partial W}{\partial I_4} \frac{\partial I_4}{\partial \mathbf{C}} \right), \quad (2)$$

where $\mathbf{C}$ is the Green deformation tensor. Substitution of Equation (2) into Equation (1) yields

$$S = 2 \left[\left(\sum_{i=1}^3 i a_i (I_1 - 3)^{i-1} \right) \mathbf{I} + \left(\sum_{j=1}^3 j b_j (I_2 - 3)^{j-1} \right) (I_1 (\mathbf{I} - \mathbf{C})) + \left(\sum_{k=2}^6 k c_k (I_4 - 3)^{k-1} \right) \mathbf{M} \right], \quad (3)$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{M}$ is the structural tensor which defines material symmetry, itself defined as

$$\mathbf{M} = m_0 \otimes m_0. \quad (4)$$

For this case, the axis of transverse isotropic symmetry was defined as $m_0 = [1 \ 0 \ 0]$. The stress function can then be solved for each of the three applied deformations (i.e. LE, TE and LS) through appropriate selection of the Green deformation matrix $\mathbf{C}$.

Curve fitting

Unconstrained nonlinear optimisation was performed to obtain the model parameter coefficients using the FMINUNC function of the MATLAB Optimization Toolbox (Mathworks Inc., Natick, MA, USA). In order to obtain the model parameters for this three-dimensional material, stress–strain data from each test direction needed to be considered simultaneously. Best-fit model parameters were found using two different methods of curve fitting: data was curve-fitted using all data as a single data set (ALL method); obtaining model parameters iteratively using one trial from each of the test directions (LE, TE and LS) and averaging the four sets of model coefficients (EACH method).

In order to evaluate the effect of level of the polynomial order, these methods were further assessed by fitting the model using the EACH and ALL methods for all permutations of the polynomial function. Best-fit parameters were determined for the 45 combinations of the constitutive model, varying between one and three I_1 terms, one and three I_2 terms and one and five I_4 terms. Where model formulations are specified, a three-digit name is used, where the digits denote the included number of I_1 , I_2 and I_4 terms, respectively (i.e. models are referred to as a - b - c , where a is the number of I_1 terms, b is the number of I_2 terms and c is the number of I_4 terms).

Data analysis

Curve-fits were assessed using the RMSE to compare the derived models against each of their experimental stress–strain curves using

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (\sigma_{e,i} - \hat{\sigma}_{m,i})^2}{n - (q+1)}}, \quad (5)$$

where n is the number of points in each data set, $\sigma_{m,i}$ and $\sigma_{e,i}$ are the model-predicted and experimentally measured stresses for the i th strain, respectively and q is the number of terms in the model. The RMSE was determined for each modelling method and polynomial formulation combination. A two-tailed, pairwise Student's t -test was used to determine if there is any difference in the RMSE between the ALL and EACH methods for each material test direction.

The goodness-of-fit of the optimised models was further evaluated using the standardised variance (s_{var}). The models for the computed EACH and ALL models for each polynomial formulation were individually compared against experimental stress-strain data using

$$\bar{S}_{\text{res}} = \frac{(\sigma_{e,i} - \hat{\sigma}_{m,i})}{\text{RMSE}}. \quad (6)$$

For a good curve-fit, the variance between a model and experimental for each direction should have an overall mean value of zero. Since LE, TE and LS data are modelled simultaneously, the multivariate Hotelling's T^2 -test was used to evaluate the null hypothesis

$$H_0: \bar{S}_{\text{res}} = 0. \quad (7)$$

For all statistical tests, significance was set at $p < 0.05$.

Results

Model parameters for each of the 45 model combinations are given in Appendix 1; the mean and standard deviations for each curve-fitting method in each test direction are reported in Table 1. The mean RMSEs for models characterised using the ALL method were found to be significantly lower than those characterised using the EACH method ($p < 0.001$). Additionally, the coefficients of variance indicate that the variability in the RMSE for the ALL LE and TE is much smaller (by a factor of two) than the corresponding EACH values.

Hotelling's T^2 -test revealed that the standardised variance was not significantly different from zero for any polynomial formulations derived by the ALL method, and for most of those resulting from use of the EACH method. The only combinations whose variance was significantly different from 0 were 1-1-1, 1-1-2, 1-1-4, 1-1-5 and 2-2-1.

Discussion

While results indicate that the ALL method provides a better fit, the effect of polynomial terms is unknown. Since the ultimate goal is to use this material model in a finite element formulation, having fewer terms, thereby reducing the computational cost of the model, would be beneficial. Figure 2 shows plots of the RMSE for the LE, TE and LS against the total number of terms in the various ALL models. These plots reveal the presence of two or three striations, or mean levels, of RMSE for the available combination of terms. Observing the patterns of the LE and TS RMSE (Figure 2(a) and (c)) indicate that the lower RMSEs for those directions can be attained using four variables. Examination of the TE RMSE (Figure 2(b)), however, reveals that five variables need to be used to minimise RMSE.

Even though the RMSE for curve fits are the largest for testing in the LE direction (Figure 3), it should be noted that the higher magnitudes of the stresses in that direction result in a lower coefficient of variance than either TE or LS (Table 1). This is encouraging since extension in the longitudinal direction is considered the most clinically relevant loading

condition. Further, the magnitudes of the changes in RMSE indicate a substantially better fit when all data are considered in fitting a model; the mean RMSE for EACH method curve fits is greater than that of ALL method fits by 45 and 17% for LE and TE, respectively.

To find which combination of five terms gave the closest fit to the experimental data, the RMSE for each direction was plotted for each five-term model (Figure 3). It is clear from the figure that the 1–2–2 and 2–1–2 models have the smallest overall RMSEs when considering all the three directions of testing. The characteristic variables found for these models are shown in Table 2. Further, with p -values from the Hotelling's T^2 -test of 0.771 and 0.562, respectively, the standardised residuals of these curve-fits are both indistinguishable from zero. In order to further determine whether one model is superior, plots of the model against the experimental data are examined (Figure 4). While the 1–2–2 and 2–1–2 models both appear to fit the general trend of the experimental data well, it is clear by visual inspection that the 1–2–2 LS model (Figure 4(a)) is not a good fit. At the same time, the fit of LE, TE and LS data for the 2–1–2 model all follow the general tenor of their respective experimental data, making it a good fit from RMSE, variance and visual inspection standpoints.

While the RMSE values for ALL method curve fits are significantly lower for EACH models, this does not automatically translate into differences that are biologically and/or clinically significant. As a measure of relative goodness-of-fit, RMSE can be used to discriminate between given models. This discrimination does not, however, give an indication of how the differences in these fits translate into the material properties described by a given model. Also, given the nature of the complex polynomial selected for this constitutive model, a direct comparison of the individual coefficients cannot be used to discern downstream differences in the tissue description.

In order to establish a sense of the magnitude of the biological relevance of the differences in the models found using the EACH and ALL methods, we have chosen to examine the linear elastic portions of the stress–strain curves that the models describe. This was done on the model that was ultimately deemed to best represent the example data, the 2–1–2 model (i.e. two I_1 terms, one I_2 term and two I_4 terms). The linear moduli for the EACH and ALL 2–1–2 models were found for all the three material directions, using the method described by Haut Donahue et al. (2001). These moduli, along with the RMSE values for these two models have now been included in the manuscript as Table 3.

While these results show that there is roughly a 10% change in the moduli across all directions between the ALL and EACH methods, this would not be enough to put the change into context. As such, we have gone to the literature to look for what would constitute significant changes in linear modulus of passive skeletal muscle. Examination of the data in Table 3 shows that there are differences in the linear moduli of 60, 1 and 0.1 kPa for the LE, TE and LS directions, respectively. While there has been little in the way of examining passive skeletal muscle tissues (in comparison to the prolific volume of work done on active muscle properties), there has been even less to show differences in transverse material properties in particular. However, in the data on passive longitudinal properties, Bensamoun et al. (2007) showed that a difference of 3.41 kPa in stiffness could be attributed to improvement in muscle function in patients undergoing treatment to hyperthyroidism, and Van Ee et al. (2000) found that stiffness would increase by over 40 kPa as a skeletal muscle progressed from immediate post-mortem to post-mortem hour 10. Given these supporting data, we feel that the differences that are found between modelling methods are biologically and clinically significant.

Additionally, it may be noted that, while assessments concerning inter-specimen variability could be addressed through testing a single specimen under each test condition, the destructive nature of these experiments (e.g. straining samples beyond 15%) precluded the use of specimens for more than one trial.

The fitting of multi-dimensional data to a multivariate model is complex, and often requires a multifaceted approach to find the solution that best realises the goal of the modeller. The RMSE is very useful as one gauge of the goodness-of-fit between a model and the data it represents. By the nature of its mean normalisation, the coefficient of variance can help to further elucidate relative differences in samples. While the Hotelling's T^2 -test did not eliminate many of the models under consideration from contention, it is still necessary to make sure that the overall variance between data and representative model is zero overall. In general terms, this indicates that the model effectively splits the data, with data equally spread above and below the model. Lastly, as was obvious in the final criteria for making a distinction between the 1–2–2 and 2–1–2 models, a computational model should never be accepted entirely based on overall statistical measures; qualitative inputs, such as visual inspection, can provide invaluable assistance in the selection between the otherwise indistinguishable models.

With a lower RMSE between experimental data and characterised models than that of the EACH method for every evaluated case, the use of the ALL method fits data better for every combination of terms in this polynomial formulation. In turn, this formulation, having up to three I_1 terms, three I_2 terms and five I_4 terms, is highly flexible, making it capable of replicating other isotropic or transversely isotropic constitutive functions used in the mechanical analysis of biological tissues. While a particular case may exist for which the EACH method would be suitable, the RMSE findings and general flexibility of the model suggest that the ALL method will be the best at characterising 3D stress-strain data.

Conclusions

When fitting a material model to experimental data from multiple trials, a better fit is obtained by fitting all data simultaneously. While RMSE can be used to find which combination of terms in a polynomial constitutive formulation most closely characterises experimental data, visual inspection should still be performed to ensure that the resulting model has an appropriate general shape and inflection points.

Acknowledgments

Funding for this study was gratefully provided by the NIH Grant No. HD31476 from the National Institute of Child Health and Human Development.

Appendix 1

ALL fit method				I_1 term coefficients			I_2 term coefficients			I_4 term coefficients					
# I_1 terms	# I_2 terms	# I_4 terms		a_1	a_2	a_3	b_1	b_2	b_3	c_2	c_3	c_4	c_5	c_6	
1	1	1	1	5.15	-	-	-2.72	-	-	46.94	-	-	-	-	-
1	1	2	2	4.26	-	-	-1.69	-	-	22.28	31.58	-	-	-	-
1	1	3	3	4.23	-	-	-1.65	-	-	19.67	39.58	-6.16	-	-	-
1	1	4	4	4.24	-	-	-1.66	-	-	22.00	26.90	16.56	-13.12	-	-
1	1	5	5	4.17	-	-	-1.59	-	-	-4.58	251.18	-670.86	881.68	-418.43	-
1	2	1	1	3.53	-	-	-2.80	2.98	-	47.94	-	-	-	-	-
1	2	2	2	3.46	-	-	-1.82	1.62	-	25.00	28.79	-	-	-	-
1	2	3	3	3.47	-	-	-1.83	1.62	-	25.34	27.76	0.78	-	-	-
1	2	4	4	3.48	-	-	-1.87	1.66	-	31.53	-5.42	59.94	-34.07	-	-
1	2	5	5	3.46	-	-	-1.80	1.59	-	8.69	185.17	-521.84	721.68	-352.99	-
1	3	1	1	3.59	-	-	-2.95	4.10	-1.39	47.87	-	-	-	-	-
1	3	2	2	3.37	-	-	-1.56	-0.10	2.08	24.30	29.81	-	-	-	-
1	3	3	3	3.36	-	-	-1.53	-0.18	2.15	23.06	33.49	-2.78	-	-	-
1	3	4	4	3.37	-	-	-1.57	-0.07	2.05	27.72	8.83	40.87	-25.05	-	-
1	3	5	5	3.33	-	-	-1.46	-0.36	2.30	2.39	218.10	-594.94	798.98	-384.36	-
2	1	1	1	0.69	3.34	-	-1.07	-	-	48.06	-	-	-	-	-
2	1	2	2	1.65	2.01	-	-0.77	-	-	24.84	29.16	-	-	-	-
2	1	3	3	1.65	2.01	-	-0.77	-	-	24.36	30.62	-1.12	-	-	-
2	1	4	4	1.64	2.03	-	-0.78	-	-	29.41	3.35	47.66	-28.14	-	-
2	1	5	5	1.66	1.98	-	-0.75	-	-	5.68	202.44	-561.38	763.90	-370.18	-
2	2	1	1	3.37	0.17	-	-2.71	2.84	-	47.96	-	-	-	-	-
2	2	2	2	1.33	2.38	-	-0.58	-0.34	-	24.75	29.30	-	-	-	-
2	2	3	3	1.30	2.42	-	-0.55	-0.38	-	24.00	31.55	-1.71	-	-	-
2	2	4	4	1.40	2.32	-	-0.63	-0.26	-	28.95	5.10	45.25	-26.99	-	-
2	2	5	5	1.15	2.56	-	-0.43	-0.54	-	4.03	211.88	-583.85	788.75	-380.57	-
2	3	1	1	1.14	2.83	-	-1.68	3.13	-3.04	47.96	-	-	-	-	-
2	3	2	2	2.29	1.24	-	-1.00	-0.51	1.35	24.42	29.71	-	-	-	-
2	3	3	3	2.28	1.24	-	-0.97	-0.59	1.42	23.15	33.49	-2.85	-	-	-
2	3	4	4	2.32	1.21	-	-1.03	-0.47	1.34	27.70	9.36	39.85	-24.49	-	-

Appendix 2

ALL fit method				I_1 term coefficients			I_2 term coefficients			I_4 term coefficients					
# I_1 terms	# I_2 terms	# I_4 terms	# I_4 terms	a_1	a_2	a_3	b_1	b_2	b_3	c_2	c_3	c_4	c_5	c_6	
2	3	5		2.21	1.30	-	-0.88	-0.79	1.54	2.19	220.18	-600.36	805.12	-386.93	
3	1	1		-0.19	6.08	-2.08	-0.85	-	-	48.09	-	-	-	-	
3	1	2		2.03	0.86	0.85	-0.86	-	-	24.30	29.83	-	-	-	
3	1	3		2.06	0.78	0.90	-0.85	-	-	23.05	33.57	-2.84	-	-	
3	1	4		2.03	0.88	0.84	-0.86	-	-	27.42	10.20	38.73	-23.92	-	
3	1	5		2.11	0.61	1.00	-0.84	-	-	2.29	219.28	-598.68	803.67	-386.43	
3	2	1		1.43	3.61	-1.39	-1.73	1.41	-	48.03	-	-	-	-	
3	2	2		3.15	-0.85	1.33	-1.47	0.98	-	24.28	29.80	-	-	-	
3	2	3		3.12	-0.83	1.34	-1.44	0.93	-	23.29	32.75	-2.24	-	-	
3	2	4		3.21	-0.91	1.33	-1.51	1.03	-	28.12	6.99	43.48	-26.27	-	
3	2	5		3.00	-0.77	1.36	-1.33	0.78	-	3.00	215.30	-590.18	795.34	-383.30	
3	3	1		2.92	-2.49	3.99	-2.82	7.73	-8.21	47.75	-	-	-	-	
3	3	2		3.59	-2.64	2.91	-1.76	2.85	-2.43	24.32	29.64	-	-	-	
3	3	3		3.55	-2.55	2.84	-1.73	2.75	-2.32	23.89	30.94	-0.98	-	-	
3	3	4		3.76	-3.04	3.18	-1.89	3.30	-2.87	29.69	0.32	52.89	-30.77	-	
3	3	5		3.36	-2.10	2.54	-1.58	2.23	-1.83	4.59	206.13	-569.18	772.99	-374.20	

EACH fit method				I_1 term coefficients			I_2 term coefficients			I_4 term coefficients					
# I_1 terms	# I_2 terms	# I_4 terms		a_1	a_2	a_3	b_1	b_2	b_3	c_2	c_3	c_4	c_5	c_6	
1	1	1	1	4.69	-	-	-2.31	-	-	42.81	-	-	-	-	-
1	1	2	2	4.28	-	-	-1.77	-	-	22.89	35.17	-	-	-	-
1	1	3	3	4.20	-	-	-1.67	-	-	11.10	87.14	-66.48	-	-	-
1	1	4	4	4.20	-	-	-1.67	-	-	8.75	114.76	-176.76	142.38	-	-
1	1	5	5	4.20	-	-	-1.66	-	-	3.85	165.39	-362.00	419.46	-137.86	-
1	2	1	1	3.80	-	-	-2.64	2.86	-	44.08	-	-	-	-	-
1	2	2	2	3.53	-	-	-1.92	1.81	-	27.13	28.80	-	-	-	-
1	2	3	3	3.46	-	-	-1.77	1.61	-	18.92	61.53	-40.25	-	-	-
1	2	4	4	3.45	-	-	-1.77	1.60	-	21.47	44.54	-11.49	-2.79	-	-
1	2	5	5	3.45	-	-	-1.77	1.60	-	21.03	38.90	60.96	-230.02	218.93	-
1	3	1	1	3.26	-	-	-2.52	5.96	-6.25	43.78	-	-	-	-	-
1	3	2	2	3.33	-	-	-1.80	2.31	-1.47	26.81	29.09	-	-	-	-
1	3	3	3	3.37	-	-	-1.65	1.32	-0.11	17.86	64.53	-42.51	-	-	-
1	3	4	4	3.38	-	-	-1.66	1.30	-0.04	19.72	52.82	-26.18	5.87	-	-
1	3	5	5	3.38	-	-	-1.65	1.26	0.00	17.97	63.37	-29.60	-67.21	104.57	-
2	1	1	1	0.92	3.89	-	-1.00	-	-	44.22	-	-	-	-	-
2	1	2	2	1.76	2.46	-	-0.91	-	-	26.67	29.86	-	-	-	-
2	1	3	3	1.89	2.21	-	-0.88	-	-	17.60	67.04	-46.18	-	-	-
2	1	4	4	1.90	2.20	-	-0.88	-	-	18.97	60.37	-48.74	29.09	-	-
2	1	5	5	1.90	2.18	-	-0.88	-	-	14.28	109.78	-236.85	330.18	-169.39	-
2	2	1	1	3.11	0.87	-	-2.21	2.05	-	44.07	-	-	-	-	-
2	2	2	2	2.31	1.71	-	-1.19	0.42	-	26.48	30.04	-	-	-	-
2	2	3	3	1.86	2.20	-	-0.84	-0.08	-	17.21	68.15	-47.05	-	-	-
2	2	4	4	1.86	2.21	-	-0.84	-0.08	-	18.43	63.12	-54.00	32.50	-	-
2	2	5	5	1.85	2.22	-	-0.83	-0.09	-	16.76	72.93	-53.24	-51.94	115.70	-
2	3	1	1	2.21	1.49	-	-1.97	5.25	-6.80	43.86	-	-	-	-	-
2	3	2	2	2.26	1.62	-	-1.24	1.52	-2.04	26.82	29.23	-	-	-	-
2	3	3	3	2.01	1.98	-	-0.93	0.31	-0.70	17.53	66.42	-44.79	-	-	-
2	3	4	4	2.01	1.99	-	-0.93	0.26	-0.61	18.88	59.04	-41.67	19.61	-	-

EACH fit method				I_1 term coefficients			I_2 term coefficients			I_4 term coefficients					
# I_1 terms	# I_2 terms	# I_4 terms		a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	c_4	c_5	c_6
2	3	5		1.99	2.02	-	-0.90	0.19	-0.55	13.80	111.80	-242.12	342.23	-184.03	
3	1	1		0.21	9.38	-8.18	-0.88	-	-	43.97	-	-	-	-	-
3	1	2		1.66	3.73	-2.17	-0.90	-	-	26.86	29.29	-	-	-	-
3	1	3		1.95	2.41	-0.72	-0.89	-	-	17.51	66.69	-45.29	-	-	-
3	1	4		1.95	2.34	-0.61	-0.90	-	-	18.82	59.83	-44.53	22.98	-	-
3	1	5		1.97	2.29	-0.56	-0.90	-	-	17.02	70.67	-47.14	-56.37	112.65	-
3	2	1		0.08	9.47	-8.10	-0.80	-0.01	-	43.97	-	-	-	-	-
3	2	2		1.79	3.51	-2.00	-0.96	0.04	-	26.79	29.41	-	-	-	-
3	2	3		1.89	2.55	-0.77	-0.86	-0.09	-	17.42	67.08	-45.74	-	-	-
3	2	4		1.94	2.44	-0.67	-0.89	-0.05	-	18.74	60.49	-46.39	24.78	-	-
3	2	5		1.94	2.40	-0.63	-0.88	-0.06	-	16.94	71.49	-50.14	-51.94	110.66	-
3	3	1		1.58	2.25	1.05	-1.60	4.44	-6.99	43.83	-	-	-	-	-
3	3	2		3.44	-3.02	5.04	-1.84	4.16	-5.36	26.55	29.54	-	-	-	-
3	3	3		2.53	-0.19	2.49	-1.19	1.52	-2.30	17.44	66.36	-44.39	-	-	-
3	3	4		2.50	-0.02	2.28	-1.18	1.44	-2.16	18.99	57.75	-38.01	16.16	-	-
3	3	5		2.43	0.15	2.17	-1.13	1.27	-2.01	13.92	110.28	-237.50	338.22	-185.26	-

References

- Bensamoun SF, Ringleb SI, Chen QS, Ehman RL, An KN, Brennan M. Thigh muscle stiffness assessed with magnetic resonance elastography in hyperthyroid patients before and after medical treatment. *J Magn Reson Imaging*. 2007; 26(3):708–713. [PubMed: 17729336]
- Blix M. Die langrund die spannung des muskels. *Scand Arch Physiol*. 1894; 5:149–206.
- Cheng T, Gan RZ. Mechanical properties of anterior malleolar ligament from experimental measurement and material modeling analysis. *Biomech Model Mechanobiol*. 2008; 7(5):387–394. [PubMed: 17710457]
- Ciarletta P, Dario P, Micera S. Pseudo-hyperelastic model of tendon hysteresis from adaptive recruitment of collagen type i fibrils. *Biomaterials*. 2008; 29(6):764–770. [PubMed: 17997481]

- Criscione JC, Douglas AS, Hunter WC. Physically based strain invariant set for materials exhibiting transversely isotropic behavior. *J Mech Phys Solids*. 2001; 49(4):871–897.
- Haut Donahue TL, Gregersen C, Hull ML, Howell SM. Comparison of viscoelastic, structural, and material properties of double-looped anterior cruciate ligament grafts made from bovine digital extensor and human hamstring tendons. *J Biomech Eng*. 2001; 123(5):162–169. [PubMed: 11340877]
- Holzappel, GA. *Nonlinear solid mechanics: A continuum approach for engineering*. Wiley; West Sussex, England: 2000.
- Jenkyn T, Koopman B, Huijing PA, Lieber RL, Kaufman KR. Finite element model of intramuscular pressure during isometric contraction of skeletal muscle. *Phys Med Biol*. 2002; 47:4043–4061. [PubMed: 12476981]
- Johansson T, Meier P, Blickman R. A finite element model for the mechanical analysis of skeletal muscles. *J Theo Biol*. 2000; 206:131–149.
- Malvern, LE. *Introduction to the mechanics of the continuous medium*. Prentice-Hall; Upper Saddle River, NJ: 1969.
- Morrow DA, Haut Donahue TL, Odegard GM, Kaufman KR. Transversely isotropic tensile material properties of skeletal muscle tissue. *J Mech Behav Biomed Mater*. In press.
- Odegard GM, Donahue TL, Morrow DA, Kaufman KR. Constitutive modeling of skeletal muscle tissue with an explicit strain-energy function. *J Biomech Eng*. 2008; 130(6):061017. [PubMed: 19045546]
- Sweigart MA, Zhu CF, Burt DM, DeHoll PD, Agrawal CM, Clanton TO, Athanasiou KA. Intraspecies and interspecies comparison of the compressive properties of the medial meniscus. *Ann Biomed Eng*. 2004; 32(11):1569–1579. [PubMed: 15636116]
- Van Ee CA, Chasse AL, Myers BS. Quantifying skeletal muscle properties in cadaveric test specimens: Effects of mechanical loading, postmortem time, and freezer storage. *J Biomech Eng*. 2000; 122(1):9–14. [PubMed: 10790824]
- Van Loocke M, Lyons CG, Simms CK. A validated model of passive muscle in compression. *J Biomech*. 2006; 39(16):2999–3009. [PubMed: 16313914]
- Van Loocke M, Lyons CG, Simms CK. Viscoelastic properties of passive skeletal muscle in compression: Stress-relaxation behaviour and constitutive modelling. *J Biomech*. 2008; 41(7):1555–1566. [PubMed: 18396290]
- Villegas DF, Maes JA, Magee SD, Donahue TL. Failure properties and strain distribution analysis of meniscal attachments. *J Biomech*. 2007; 40(12):2655–2662. [PubMed: 17359982]
- Weiss JA, Gardiner JC, Bonifasi-Lista C. Ligament material behavior is nonlinear, viscoelastic and rate-independent under shear loading. *J Biomech*. 2002; 35(7):943–950. [PubMed: 12052396]
- Weiss JA, Maker BN, Govindjee S. Finite element implementation of incompressible, transversely isotropic hyperelasticity. *Comput Methods Appl Mech Eng*. 1996; 135(1–2):107–128.
- Yin L, Elliott DM. A biphasic and transversely isotropic mechanical model for tendon: Application to mouse tail fascicles in uniaxial tension. *J Biomech*. 2004; 37(6):907–916. [PubMed: 15111078]

Figure 1.
Experimental data to be modelled, including (a) LE, (b) TE and (c) LS.

Figure 2. Plots of the RMSE versus total number of modelled terms for (a) LE, (b) TE and (c) LS data. As indicated by the LE and LS, four or more terms must be used to minimise RMSE. Consideration of TE urges the use of five or more terms to minimise RMSE for this model.

Figure 3.

Plots of LE, TE and LS RMSE for all models with five total terms. The two model formulations on the far left (1-2-2 and 2-1-2) clearly have the lowest RMSEs for all the three directions.

Figure 4.

Models 1–2–2 (a, c and e, left) and 2–1–2 (b, d and f, right) plotted against experimental data for (a–b) LE, (c–d) TE and (e–f) LS. While both models report similar RMSEs for all directions, the 1–2–2 LS plot's shape clearly departs from the shape of the typical experimental data plot.

Table 1

Summary RMSE data for ALL and EACH methods by material testing direction.

	ALL			EACH		
	Mean (kPa)	SD (kPa)	CV	Mean (kPa)	SD (kPa)	CV
LE	8.57 ^a	1.66	0.19	12.50 ^a	5.01	0.40
TE	2.22 ^a	0.46	0.21	2.60 ^a	1.11	0.43
LS	0.48	0.24	0.50	0.49	0.21	0.42

^aSignificant difference ($p < 0.05$) between mean RMSE for ALL and EACH.

Table 2
Parameter coefficients for 1-2-2 and 2-1-2 models as determined by using ALL and EACH curve-fit methods.

Method	Model terms numbers			I ₁ term coefficients		I ₂ term coefficients		I ₄ term coefficients		
	# I ₁ terms	# I ₂ terms	# I ₄ terms	a ₁	a ₂	b ₁	b ₂	c ₂	c ₃	
ALL	1	2	2	3.46	-	-1.82	1.62	25.00	28.79	
	2	1	2	1.65	2.01	-0.77	-	24.84	29.16	
EACH	1	2	2	3.53	-	-1.92	1.81	27.13	28.80	
	2	1	2	1.76	2.46	-0.91	-	26.67	29.86	

Note: kPa values.

Table 3

RMSE and linear slope for 2-1-2 models defined using EACH and ALL methods.

	RMSE			Linear slope		
	EACH (kPa)	ALL (kPa)	Difference (%)	EACH (kPa)	ALL (kPa)	Difference (%)
LE	8.5E+0	7.8E+0	9	4.7E+2	5.3E+2	12
TE	2.1E+0	2.0E+0	6	1.3E+1	1.2E+1	9
LS	3.6E-1	3.5E-1	3	1.5E+0	1.4E+0	14