Published in final edited form as:

J Biomech. 2009 September 18; 42(13): 2165–2170. doi:10.1016/j.jbiomech.2009.05.023.

Heterogeneity of yield strain in low-density versus high-density human trabecular bone

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Abstract

Understanding the off-axis behavior of trabecular yield strains may lend unique insight into the etiology of fractures since yield strains provide measures of failure independent of elastic behavior. We sought to address anisotropy of trabecular yield strains while accounting for variations in both density and anatomic site and to determine the mechanisms governing this behavior. Cylindrical specimens were cored from vertebral bodies ($n=22$, $BV/TV=0.11\pm0.02$) and femoral necks ($n=28$, $BV/TV=0.22\pm0.06$) with the principal trabecular orientation either aligned along the cylinder axis (on-axis, $n=22$) or at an oblique angle of 15° or 45° (off-axis, $n=28$). Each specimen was scanned with micro-CT, mechanically compressed to failure, and analyzed with nonlinear micro-CT-based finite element analysis. Yield strains depended on anatomic site ($p=0.03$, ANOVA), and the effect of off-axis loading was different for the two sites ($p=0.04$) – yield strains increased for off-axis loading of the vertebral bone ($p=0.04$), but were isotropic for the femoral bone ($p=0.66$). With sites pooled together, yield strains were positively correlated with BV/TV for on-axis loading ($R^2=58\%$, $p<0.0001$), but no such correlation existed for off-axis loading ($p=0.79$). Analysis of the modulus- BV/TV and strength- BV/TV relationships indicated that, for the femoral bone, the reduction in strength associated with off-axis loading was greater than that for modulus, while the opposite trend occurred for the vertebral bone. The micro-FE analyses indicated that these trends were due to different failure mechanisms for the two types of bone and the different loading modes. Taken together, these results provide unique insight into the failure behavior of human trabecular bone and highlight the need for a multiaxial failure criterion that accounts for anatomic site and bone volume fraction.

Keywords

cancellous bone; finite element modeling; off-axis loading; multiaxial loading; yield surface

Introduction

“Off-axis” mechanical behavior of anisotropic materials such as trabecular bone refers to the response to loads that are oblique to the principal material coordinate system of the material.

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Knowledge of the off-axis mechanical behavior of trabecular bone is fundamental to understanding fracture etiology associated with the multiaxial loading that can occur *in vivo* during falls (Cowin, 1986; Lotz et al., 1991; Troy et al., 2007) and at bone-implant interfaces (Cheal et al., 1985; Cheal et al., 1987; Vasu et al., 1981). A unique feature of trabecular bone mechanical behavior is that apparent failure strains may be isotropic despite substantial anisotropy of strength, elastic modulus, and microarchitecture (Chang et al., 1999). Understanding the behavior of failure strains is particularly informative because this quantity provides a measure of failure independent of elastic behavior (Kopperdahl et al., 1998). Furthermore, establishing the dependence of trabecular failure strains on density and orientation is essential to the development of a robust multiaxial failure criterion (Bayraktar et al., 2004a; Cowin et al., 2005; Fenech et al., 1999).

Despite its relevance, the off-axis behavior of trabecular bone has received relatively little attention. Early work performed with high-density human (Brown et al., 1980) and canine (Vahey et al., 1987) femoral bone indicated that the ratio of strength to elastic modulus — a measure of strain at failure — did not depend on orientation of the specimen, suggesting isotropy of failure strains. More explicit tests have since demonstrated that yield strains are indeed isotropic (or nearly so) for compressive, tensile, and torsional loading of on-axis, transverse, and off-axis specimens (Bayraktar et al., 2004a; Chang et al., 1999; Ford et al., 1996; Turner, 1989). However, these experiments only involved high-density bone and some evidence exists to suggest that anisotropy of failure strains can exist in low-density human vertebral bone (Mosekilde et al., 1987), although this finding has yet to be confirmed using refined experimental techniques that control for end-artifacts. Related, while the failure mechanisms of both low- and high-density bone have been well described for on-axis loading (Bayraktar et al., 2004b; Bevill et al., 2006; Fyhrie et al., 1994; Gibson, 1985; Keaveny, 1997; Nazarian et al., 2004), failure mechanisms for off-axis loading are not well understood and may be very different from on-axis behavior since the bone is not adapted to such non-habitual loading.

The overall goal of this study was to address anisotropy of trabecular yield strains while accounting for variations in both density and anatomic site and to determine the mechanisms governing this behavior. Focusing on trabecular bone from the femoral neck (high-density) and vertebral body (low-density), and using a combined experimental-computational approach, our specific objectives were to: 1) perform experiments to determine the dependence of yield strains on loading orientation for femoral and vertebral trabecular bone; and 2) use a series of specimen-specific, micro-CT-based finite element models to investigate the micromechanics and explain the observed trends. This study is novel in that it is the first to explicitly examine isotropy of yield strains in human trabecular bone accounting for variations in both density and anatomic site.

Methods

Fifty cylindrical specimens (~8.1 mm diameter, 25 mm length) of trabecular bone were taken from 39 human cadavers, n=22 from L2-L5 vertebral bodies (age=70.1±9.3, 48–87 years; n=8 male, n=5 female; BV/TV=0.11±0.02) and n=28 from femoral necks (age=72.7±11.2, 58–92 years; n=13 male, n=15 female; BV/TV=0.22±0.06). For the femoral specimens, protocols were used to obtain 10 specimens with the principal trabecular orientation aligned along the cylinder axis (on-axis) (Kopperdahl et al., 1998; Morgan et al., 2001). Eighteen specimens were harvested using a similar procedure, but were cored such that the direction of principal trabecular orientation was oblique (15° or 45°) to the longitudinal axis of the cylinder in the coronal plane (off-axis, Fig. 1). The vertebral trabecular cores were machined at two orientations on-axis and 45° off-axis. On-axis cores (n=12) were obtained by coring the vertebral bodies along the inferior-superior direction (Kopperdahl et al., 1998). Off-axis cores

were obtained in the coronal plane by removing a small section of the vertebral body at an angle of 45° relative to the sagittal and transverse planes. A parallel cut was then made opposite to this new surface such that approximately 25 mm separated the two surfaces, and a core was removed orthogonal to the planes. Trabecular orientation was confirmed via contact radiographs taken from two orthogonal viewpoints for all specimens.

Prior to mechanical testing, specimens were cleaned of marrow and BV/TV was measured using Archimedes' Principle. Uniaxial compression tests were conducted using a servohydraulic load frame (858 Mini-Bionix, MTS, Eden Prairie, MN). To minimize the end-artifacts, the vertebral cores were affixed in brass endcaps (Keaveny et al., 1997) and strain was measured using a 25 mm gage length extensometer (632.11F-20, MTS, Eden Prairie, MN) attached to the endcaps. Because the strength of the on-axis femoral bone exceeded the shear strength of the cyanoacrylate glue, all femoral specimens were tested between unlubricated platens and end-artifacts were accounted for using the techniques described by Morgan et al. (2001). The testing protocol included 10 preconditioning cycles to 0.3% strain followed by a final ramp at 0.5% strain per second to 2.0% apparent strain. Modulus was determined as the slope at 0% strain of a quadratic curve fit to the portion of the stress-strain curve from 0 to 0.2% strain (Morgan et al., 2001), and yield was determined using the 0.2% offset technique.

High-resolution finite element models were created using a voxel-based conversion technique with an element size of 66 µm for the femoral images and 20 µm for the vertebral images. These resolutions were chosen based on recommended numerical convergence criteria (Niebur et al., 1999). Images were thresholded to match the experimental BV/TV measurement. Materially and geometrically nonlinear finite element analysis was conducted for each model to 1% compressive strain using roller-type boundary conditions. The trabecular tissue was modeled using a validated finite plasticity model (Bevill et al., 2006; Papadopoulos et al., 1998; 2001). However, different tissue properties were assigned based on anatomic site (Bevill, 2008): femoral bone was assigned an elastic modulus of 18.5 GPa, Poisson's ratio of 0.3, and tissue-level yield strains of 0.81% and 0.33% (compression and tension, respectively) (Bevill et al., 2006); the respective values for the vertebral bone were 10.0 GPa, 0.3, 0.69%, and 0.33% (Bevill, 2008). Apparent-level yield was determined using the 0.2% offset method. Additional outcome variables included the number of gauss points failed at each step and the loading mode (tensile or compressive) by which they failed. The distribution, type, and amount of tissue failure from the finite element analyses was used to elucidate any relationship between tissue-level failure mechanisms and apparent-level yield strain behavior.

To further elucidate the mechanisms governing yield behavior, additional finite element analyses were conducted in which parametric variations were made to the original models. These variations included: 1) performing small deformation (geometrically linear) analysis; 2) removing the tension-compression asymmetry from the tissue material model (using the average of the tensile and compressive values given above); and 3) combining the small deformation analysis with tension-compression symmetry. Analysis of variance (ANOVA) was used to determine the dependence of yield strain on trabecular orientation and anatomic site. Differences in the modulus- and strength-BV/TV relationships were examined using general linear regressions. Paired t-tests with Bonferroni adjustment were used to compare finite element versus experimental measurements of yield strain.

Results

The magnitude of yield strains primarily depended on anatomic site ($p=0.03$, ANOVA), and the effect of off-axis loading was different between anatomic sites ($p=0.04$). Off-axis loading of the femoral trabecular bone resulted in a statistically insignificant decrease in yield strains ($p=0.66$, Fig. 2), but yield strains increased for off-axis loading of the vertebral trabecular bone

($p=0.04$, Fig. 2), indicating that the bone from these two sites respond in fundamentally different ways to off-axis loading. While the magnitude of the change in yield strain due to off-axis loading was similar between the anatomic sites, the vertebral yield strains were statistically anisotropic since they displayed a much smaller standard deviation (approximately one-half) than the femoral bone. Similar trends were observed in the modulus-BV/TV and strength BV/TV relationships (Table 1). For the femoral bone, the reduction in strength due to off-axis loading was greater than that for modulus, while the opposite trend was seen for the vertebral bone. For both sites pooled, there was a strong positive correlation between yield strain and BV/TV, but no such correlation existed for off-axis loading (Fig. 3).

The finite element predictions of yield strain were statistically indistinguishable from experiment for all the femoral groups ($p=0.52$, Figs. 2, 4A) and the on-axis vertebral bone ($p=0.44$, Figs. 2, 4B), although the models slightly over-predicted yield strain for the 45° vertebral group ($p=0.05$). Vertebral yield strains were relatively unaffected by tension-compression symmetry, but substantially increased for geometrically linear analysis (more so for the off- than on-axis group, Fig. 4B). As such, anisotropy of yield strains was retained for all variations to the vertebral models ($p<0.0001$). However, in the femoral bone, the parametric variations affected on- versus off-axis yield strain behavior in a substantially different way (Fig. 4A). For example, the geometrically linear models with tension-compression symmetry predicted increasing yield strains for off-axis loading in the femoral trabecular bone — an opposite trend to that observed in experiment, although statistical isotropy was retained ($p>0.64$) in all cases.

The percent of tissue failed at the apparent yield point decreased with off-axis loading and decreasing BV/TV (Fig. 5A). The largest percentage of tissue failure ($16.0\pm6.0\%$) was observed in the on-axis femoral bone, and the least ($2.9\pm1.1\%$) was in the off-axis vertebral bone. The ratio of tissue predicted to fail in tension versus compression increased with off-axis loading and increasing BV/TV (Fig. 5B). This ratio was significantly greater for the 45° off-axis groups than all other groups ($p<0.03$), and was greatest for the 45° vertebral group despite the fact that the tissue-level tension-compression asymmetry was less for the vertebral bone ($0.69/0.34 = 2.0$) than the femoral bone ($0.81/0.34 = 2.4$). Bending failure was evident in off-axis specimens by regions of opposed tensile and compressive failure (Fig. 6), as well as increased failure in horizontal trabeculae.

Discussion

In this study we sought to determine the dependence of yield strains on loading orientation in trabecular bone from both low- and high-density anatomic sites, and to determine the failure mechanisms governing the observed behavior. The results demonstrated a fundamental difference in the yield strain response of these anatomic sites to off-axis loading — yield strains increased for off-axis loading of the vertebral bone, but decreased (non-significant) for the femoral bone. Yield predictions from the fully nonlinear finite element models with tissue-level tension-compression asymmetry agreed well with experiment. However, when the tension-compression asymmetry and large deformations were artificially removed from the models, both the magnitude and trends for yield strains could be substantially altered, demonstrating the important role of these parameters in trabecular failure behavior — which depended on the anatomic origin of the specimens. Particularly complex interactions were observed for the femoral bone, presumably due to the fact that both material and geometric failure mechanisms can be important in the failure of higher-density bone. As such, the results of this study provide unique insight into the apparent-level failure behavior and the micromechanical failure mechanisms for on- and off-axis loading of trabecular bone.

The main strength of our study was the combined use of experimental and computational methods to estimate yield strains for the same specimens, which enabled us to probe the mechanisms associated with off-axis failure in much greater detail than would be possible using purely experimental techniques. Both techniques indicated the same trends regarding isotropy of yield strains, providing a strong level of validation for the models (which was further supported by the agreement with the experimental data on a specimen-specific basis). Lastly, our finite element analyses included a number of controlled parameter studies — removal of tissue-level strength asymmetry and large deformation effects — that provided additional insight into the failure mechanisms associated with off-axis loading.

Despite these strengths, a potential source of uncertainty in our finite element modeling was the use of different tissue material properties for each anatomic site. These material properties represent our best estimate for vertebral and femoral site-specific properties (Bevill et al., 2006; Bevill, 2008), and are supported by recent evidence that suggests trabecular tissue from low- and high-density bone can have different mechanical properties (Bevill, 2008). Regardless, our apparent-level failure results should be robust with respect to these assumptions — for example, the failure behavior in the vertebral bone was dominated by geometric mechanisms, and the anisotropy of failure strains in this type of bone persisted even when we applied symmetric tensile and compressive tissue failure strains. Although the tissue-level failure results (Figs. 5 and 6) may be more sensitive to the use of site-specific tissue-failure properties, the differences between the anatomic sites would actually be accentuated if the same tissue failure properties had been applied to all of the models, thus strengthening the conclusions derived from these analyses. Finally, apparent yield strain does not depend on tissue modulus and thus our results are completely independent of the use of site-specific tissue moduli.

Cellular solid models can lend insight into the mechanisms governing off-axis failure behavior, but must be taken in the context of all mechanical data available and interpreted cautiously due to the idealized nature of such analyses. For example, low-density bone can be represented as an out-of-plane rod-like structure (Gibson et al., 1997). Yield strains are predicted to increase for off-axis loading in this type of model (and elastic modulus and failure stress also follow the correct trends, see Appendix). This suggests that the anisotropy of yield strains in the vertebral bone is likely due to the substantial bending deformations that occur prior to failure. However, while isotropic failure strains are predicted (calculations not shown) for in-plane loading of the regular hexagonal structures that have been used to represent high-density bone (Bayraktar et al., 2004b), these models also predict isotropy for elastic modulus (Gibson et al., 1997) and failure stress, which is inconsistent with our experimental observations. Rather, we suggest the yield strain behavior of high-density bone appears to be a result of competing effects: a small amount of bending can occur in high-density femoral neck trabecular bone (Bayraktar et al., 2004b), but the increase in bending for off-axis specimens is offset by the increasing amount of tensile tissue failure that occurs with off-axis loading.

The findings from this study have direct implications to the development of strain-based multiaxial failure criteria for trabecular bone (Bayraktar et al., 2004a; Chang et al., 1999; Cowin et al., 2005). For example, one such recent criterion that assumed isotropy of yield strains for trabecular bone of all densities (Cowin et al., 2005) may benefit from refinements that account for the variations in yield strain that can occur across orientation and density as found in this study. Another criterion, which was developed for high-density bone and was found to be nearly isotropic (Bayraktar et al., 2004a), may require adjustment in shape, size, and the number of parameters when extended to low-density sites. Specifically, the development of a comprehensive multiaxial failure criterion for trabecular bone of any density will likely require more complex mathematical surfaces than have previously been employed

(Bayraktar et al., 2004a; Cowin et al., 2005) such that the failure surface can vary in strain-space depending on both bone density and architecture.

From a biomechanics perspective, our results demonstrate that the off-axis yield strain behavior is very different for bone from low- versus high-density anatomic sites, and the relationship between yield strength and modulus is also altered by off-axis loading since yield strain is described by the ratio of these two parameters. Since off-axis loading often occurs *in vivo* as a result of falls (Lotz et al., 1991; Troy et al., 2007), utilization of these altered off-axis material property relationships may improve the fidelity of whole-bone finite element models and enable such analyses to provide additional insight into fall-related fracture etiology in the elderly and osteoporotic population.

Acknowledgments

Funding for this work was provided by the National Institutes of Health (AR43784). Cadaveric material was obtained from NDRI and UCSF Willd Body Program. Super-computing resources were obtained from the National Partnership for Advanced Computational Infrastructure (NPACI UCB266). Dr. Keaveny has a financial interest in O.N. Diagnostics and both he and the company may benefit from the results of this research.

References

- Bayraktar HH, Gupta A, Kwon RY, Papadopoulos P, Keaveny TM. The modified super-ellipsoid yield criterion for human trabecular bone. *Journal of Biomechanical Engineering* 2004a;126(6):677–684. [PubMed: 15796326]
- Bayraktar HH, Keaveny TM. Mechanisms of uniformity of yield strains for trabecular bone. *Journal of Biomechanics* 2004b;37(11):1671–1678. [PubMed: 15388309]
- Bevill G, Eswaran SK, Gupta A, Papadopoulos P, Keaveny TM. Influence of bone volume fraction and architecture on computed large-deformation failure mechanisms in human trabecular bone. *Bone* 2006;39(6):1218–1225. [PubMed: 16904959]
- Bevill, G. PhD Thesis. University of California; Berkeley: 2008. Micromechanical Modeling of Failure in Trabecular Bone.
- Brown TD, Ferguson AB. Mechanical property distributions in the cancellous bone of the human proximal femur. *Acta Orthop Scand* 1980;51(3):429–437. [PubMed: 7446022]
- Chang WCW, Christensen TM, Pinilla TP, Keaveny TM. Isotropy of uniaxial yield strains for bovine trabecular bone. *Journal of Orthopaedic Research* 1999;17:582–585. [PubMed: 10459766]
- Cheal EJ, Hayes WC, Lee CH, Snyder BD, Miller J. Stress analysis of a condylar knee tibial component: influence of metaphyseal shell properties and cement injection depth. *Journal of Orthopaedic Research* 1985;3(4):424–434. [PubMed: 4067701]
- Cheal EJ, Snyder BD, Nunamaker DM, Hayes WC. Trabecular bone remodeling around smooth and porous implants in an equine patellar model. *Journal of Biomechanics* 1987;20(11–12):1121–1134. [PubMed: 3429458]
- Cowin SC. Wolff's Law of trabecular architecture at remodeling equilibrium. *Journal of Biomechanical Engineering* 1986;108:83–88. [PubMed: 3959556]
- Cowin SC, He QC. Tensile and compressive stress yield criteria for cancellous bone. *Journal of Biomechanics* 2005;38(1):141–144. [PubMed: 15519349]
- Fenech CM, Keaveny TM. A cellular solid criterion for predicting the axial-shear failure properties of trabecular bone. *Journal of Biomechanical Engineering* 1999;121:414–422. [PubMed: 10464696]
- Ford CM, Keaveny TM. The dependence of shear failure properties of bovine tibial trabecular bone on apparent density and trabecular orientation. *Journal of Biomechanics* 1996;29:1309–1317. [PubMed: 8884476]
- Fyhrie DP, Schaffler MB. Failure mechanisms in human vertebral cancellous bone. *Bone* 1994;15(1):105–109. [PubMed: 8024844]
- Gibson LJ. The mechanical behavior of cancellous bone. *Journal of Biomechanics* 1985;18(5):317–328. [PubMed: 4008502]

- Gibson, LJ.; Ashby, MF. Cellular solids: structures & properties. 2. Pergamon Press; Oxford: 1997. p. 510
- Keaveny TM. Mechanistic approaches to analysis of trabecular bone. *Forma* 1997;12:267–275.
- Keaveny TM, Pinilla TP, Crawford RP, Kopperdahl DL, Lou A. Systematic and random errors in compression testing of trabecular bone. *Journal of Orthopaedic Research* 1997;15:101–110. [PubMed: 9066533]
- Kopperdahl DL, Keaveny TM. Yield strain behavior of trabecular bone. *Journal of Biomechanics* 1998;31(7):601–608. [PubMed: 9796682]
- Lotz JC, Cheal EJ, Hayes WC. Fracture prediction for the proximal femur using finite element models: Part I-Linear analysis. *Journal of Biomechanical Engineering* 1991;113(4):353–360. [PubMed: 1762430]
- Morgan EF, Keaveny TM. Dependence of yield strain of human trabecular bone on anatomic site. *Journal of Biomechanics* 2001;34(5):569–577. [PubMed: 11311697]
- Mosekilde L, Mosekilde L, Danielsen CC. Biomechanical competence of vertebral trabecular bone in relation to ash density and age in normal individuals. *Bone* 1987;8(2):79–85. [PubMed: 3593611]
- Nazarian A, Muller R. Time-lapsed microstructural imaging of bone failure behavior. *Journal of Biomechanics* 2004;37(1):55–65. [PubMed: 14672568]
- Niebur GL, Yuen JC, Hsia AC, Keaveny TM. Convergence behavior of high-resolution finite element models of trabecular bone. *Journal of Biomechanical Engineering* 1999;121(6):629–635. [PubMed: 10633264]
- Papadopoulos P, Lu J. A general framework for the numerical solution of problems in finite elasto-plasticity. *Computer Methods in Applied Mechanics and Engineering* 1998;159(1–2):1–18.
- Papadopoulos P, Lu J. On the formulation and numerical solution of problems in anisotropic finite plasticity. *Computer Methods in Applied Mechanics and Engineering* 2001;190(37–38):4889–4910.
- Troy KL, Grabiner MD. Off-axis loads cause failure of the distal radius at lower magnitudes than axial loads: a finite element analysis. *Journal of Biomechanics* 2007;40(8):1670–1675. [PubMed: 17368466]
- Turner CH. Yield behavior of bovine cancellous bone. *Journal of Biomechanical Engineering* 1989;111(3):256–260. [PubMed: 2779192]
- Vahey JW, Lewis JL, Vanderby RJ. Elastic moduli, yield stress, and ultimate stress of cancellous bone in the canine proximal femur. *Journal of Biomechanics* 1987;20(1):29–33. [PubMed: 3558426]
- Vasu R, Carter DR, Harris WH. Stress distributions in the acetabular region. I. Before and after total joint replacement. *Journal of Biomechanics* 1981;15:155–164. [PubMed: 7096368]

Appendix

We sought to use cellular solid theory (Gibson et al., 1997) to determine the dependence of failure strains on loading orientation for open-cell rod-like structures. The model assumed that the longitudinal axis of the structure could be oriented at varying oblique angles (θ) to the applied apparent compressive load (Fig. 7), and that the deformation of the structure was dominated by bending of the longitudinal struts, thus reducing the analysis to that of an individual oblique beam.

Given an apparent compressive stress of σ^* and a structure with horizontal trabecular length of l and longitudinal length of $2l$, the internal force (P) and moment (M) are:

$$P = \frac{\sqrt{3}l^2\sigma^*}{4}, \quad \text{Eq. 1}$$

and

$$M = \frac{Pl \sin(\theta)}{2}. \quad \text{Eq. 2}$$

The vertical deflection (δ) due to P and M (assuming a square cross section with area of t^2 and a tissue modulus of E_t) is:

$$\delta = \frac{8Pl^3 \sin^2(\theta)}{E_t t^4}. \quad \text{Eq. 3}$$

The apparent strain (ϵ^*) is the ratio of δ to the apparent height of the structure ($H=2l\cos(\theta)$), and the apparent elastic modulus becomes:

$$E^* = \frac{\sigma^*}{\epsilon^*} = \frac{t^4 E_t}{\sqrt{3} l^4 \tan(\theta) \sin(\theta)}. \quad \text{Eq. 4}$$

The tissue was assumed to behave as an elastic-perfectly plastic material with tension-compression asymmetry in the tissue-level failure stresses (σ_y^T in tension and σ_y^C in compression). The apparent stress for complete plastification of the beam cross section is:

$$\sigma_{pl}^* = \frac{4t^3 c \sigma_y^C}{\sqrt{3} l^3 (1+c) \sin(\theta)}, \quad \text{Eq. 5}$$

where c is the ratio of σ_y^T to σ_y^C . Taking the ratio of σ_{pl}^* to E^* , it is found that failure strain is proportional to the tangent of θ :

$$\epsilon_{pl}^* = \frac{4lc \sigma_y^C \tan(\theta)}{t(1+c) E_t}. \quad \text{Eq. 6}$$

The relation found in Eq. 6 is linear in $\tan(\theta)$ and thus corresponds to an increasing magnitude of failure strain with increasing off-axis loading.

Figure 1. Renderings of 1 mm thick longitudinal cross-sections from representative cores of femoral neck (FN) and vertebral body (VB) trabecular bone for each of the on- and off-axis angles considered.

Figure 2.

Experimental yield strains for on-axis and 45° off-axis femoral neck and vertebral trabecular bone. Yield strains were isotropic for the femoral bone ($p>0.39$), but slightly anisotropic for the vertebral bone ($p=0.037$). Data for the 15° off-axis femoral bone was omitted for clarity (yield strains were $0.76\pm0.10\%$).

Figure 3.

Yield strain versus BV/TV for on-axis and 45° off-axis loading (anatomic sites are pooled together). A strong positive correlation existed for on-axis loading ($p < 0.0001$), but not for off-axis loading ($p = 0.79$).

Figure 4.

Yield strains for (A) femoral neck and (B) vertebral trabecular bone from the parametrically altered finite element models (G.L.=geometrically linear, T/C Sym.=tissue failure strains were assumed to be tension-compression symmetric). Data for the 15° off-axis femoral bone was omitted for clarity (yield strains were $0.75 \pm 0.08\%$, $0.81 \pm 0.08\%$, $0.69 \pm 0.06\%$, and $0.73 \pm 0.08\%$ for the original, G.L., T/C sym., and G.L.+T/C sym. models, respectively). Statistical isotropy was retained for all variations of the femoral bone FE analyses ($p > 0.64$), while anisotropy was retained for all variations of the vertebral bone FE analyses (< 0.0001).

Figure 5.
 (A) The percent of total tissue failed and (B) the ratio of tissue failed in tension to compression. Data for the 15° off-axis femoral bone was omitted for clarity (total failure was $11.1 \pm 7.5\%$, and ratio of tensile to compressive failure was 1.21 ± 0.61).

Figure 6.

Tissue failure distributions at the apparent yield point for typical on-axis and 45° off-axis femoral neck (FN) and vertebral body (VB) specimens. Red elements correspond to tissue failure in tension and blue correspond to failure in compression. Off-axis specimens tended to have less total tissue failure, increased failure in horizontal trabeculae, and a higher ratio of tensile to compressive tissue failure.

Figure 7.

(Left) An off-axis, open-celled cellular solid structure shown with out-of-plane compressive loading. (Right) Free body diagram of an individual oblique beam with compressive force (P) and internal moment (M).

Table 1

Modulus- and yield stress-BV/TV relationships for each anatomic site and orientation. Data is given as σ_y , $E = m * BV/TV + b$, where yield stress is in units of MPa and elastic modulus is in units of GPa. All reported regressions are statistically significant ($p < 0.003$).

		0°			15°			45°		
		m	b	R ²	m	b	R ²	m	b	R ²
FN	E	11.19	-0.37	0.79	9.79	-0.78	0.82	6.83	-0.79	0.95
	σ_y	85.5	-6.0	0.83	55.6	-4.6	0.65	44.1	-5.7	0.82
VB	E	4.81	-0.16	0.69				1.89	-0.08	0.68
	σ_y	21.8	-0.6	0.66				12.3	-0.6	0.73