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The Design and Test of a New Volume Coil for High Field Imaging

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Abstract

A major problem in the development of high field (>100 MHz) large volume (>6000 cm³) MR coils is the interaction of the coil with the subject as well as the radiation loss to the environment. To reduce subject perturbation of the coil resonance modes, a volume coil that uses an array of freely rotating resonant elements radially mounted between two concentric cylinders was designed for operation at 170 MHz. Substantial electromagnetic energy is stored in the resonant elements outside the sample region without compromising the efficiency of the overall coil. This stored energy reduces the effect of the subject on the circuit and maintains a high Q , facilitating the tuning and matching of the coil. The unloaded Q of the coil is 680; when loaded with a head, it was 129. The ratio of 5.3 of the unloaded to loaded Q supports the notion that the efficiency of the coil was maintained in comparison with previous designs. The power requirement and signal-to-noise performance are significantly improved. The coil is tuned by a mechanism that imparts the same degree of rotation on all of the elements simultaneously, varying their degree of mutual coupling and preserving the overall coil symmetry. A thin radiofrequency shield is an integral part of the coil to reduce the radiation effect, which is a significant loss mechanism at high fields. MR images were collected at 4T using this coil design with high sensitivity and B_1 homogeneity.

Keywords

4T; head; imaging; efficiency

INTRODUCTION

Magnetic resonance studies at high magnetic fields are fundamentally advantageous because of the inherent high sensitivity (1,2), greater spectroscopic resolution (3–5), increased susceptibility effects for contrast with both external and internal agents (deoxyhemoglobin) (6–8), and “black blood” flow quantification schemes that improve with the extended T_1 at high fields (9). To realize the signal to noise advantage of high magnetic fields, the front end of the system (including the radiofrequency coil and receiver) must be optimized. The challenging issue for high field (>100 MHz) large volume coils (>6000 cm³) is the variable coil-subject interaction that significantly changes the field distribution and resonance frequency of the coil. In addition, the radiation loss from the coil to the environment increases drastically with field strength (10,11). Due to the short wavelength at high Larmor frequencies (20 cm for protons at 4T), far field effects cannot be ignored. A useful theoretical tool for coil optimization is the reciprocity relation between coil efficiency and signal-to-noise (SNR) ratio (1,12). This relation must be generalized beyond quasistatic approximation and simple coil-sample geometry to apply to higher field strength, as discussed in the Appendix.

The conventional “birdcage” design that has proven to be successful at lower field strength (13,14) encounters problems at higher fields. At 4 Tesla, the radiation loss of a birdcage head coil without a RF shield degrades the Q value to ~ 20 . After an RF shield is added, the Q value is increased to ~ 100 . However, when loaded with a human head, Q again drops to ~ 35 . This is accompanied by significant perturbations of the resonance modes of the coil indicated by the frequency shift (about 1 MHz for our conventional birdcage coil with a shield). This is probably due to the fact that the human head is placed at the region of the coil where most of the field energy is stored. These specific problems of high field large volume coils call for a coil design that reduces the subject perturbation effects and the radiation loss while maintaining coil efficiency.

METHODS

To overcome the loss factors, subject perturbation effects, and loaded Q degradation, a quadrature-driven volume head coil was designed and constructed with the structure of an array of individually tuned, inductively coupled resonant elements in a relatively enclosed cage to form a low-loss resonator (Fig. 1). The electromagnetic energy stored in the elements outside the loading region of the coil reduces the perturbation of the subject on the resonance modes of the coil while maintaining the efficiency. This design is named “the free element design” (FE design).

The design enables convenient coil tuning with respect to each patient to achieve optimal performance, without disturbing its symmetry. The initial tuning of the coil is greatly simplified to the tuning of individual elements. The design also enables asymmetric adjustments to compensate for the specific part of the body being imaged.

The coil designed and tested has two coaxial cylinders (Figs. 1, 2). Both cylinders are made of 0.635-cm-thick fiberglass. The outer diameter of the exterior cylinder is 40.64 cm, the clear bore of the inner cylinder (the imaging region of the coil) is 27.94 cm, and the length of the coil is 36.83 cm. The inner surface of the outer cylinder is lined with a contiguous piece of thin conductor (in our case, it is 0.0025-cm-thick copper sheet) as the RF shield. Terminal end rings are placed in the interspace between the two concentric cylinders at both ends of the coil.

Each resonant element is a rectangle of 0.635-cm-diameter copper tube spaced with series capacitors. One of the capacitors is a variable capacitor that tunes the resonant element (Fig. 2). The dimension of the copper tube rectangle is 4.44 cm by 20 cm. Each element is suspended in the space between the two concentric cylinders by attaching its ends to axles mounted on the terminal end rings filling the interspace. The axles are 1.27-cm-diameter lexan rods. One of the end rings contains a mechanism that permits uniform rotation of all of the elements around their axes (Fig. 2). This rotation uniformly changes the mutual inductances between the resonant elements and thereby changes the resonance frequency of the coil.

Alternatively, the coil can be tuned asymmetrically by turning each element individually around its axis or by adjusting the variable capacitor on the element. This asymmetric adjustment may be used to generally compensate for the asymmetric perturbation introduced by the part of the body being imaged.

Denoting the self inductance and capacitance of each resonant element as L and C and the mutual inductance between two elements n units apart as M_n , the resonance modes of the coil are solved from the following equation,

$$i\omega LI_k + \frac{I_k}{i\omega C} + \sum_{j \neq k} i\omega M_{|k-j|} I_j = 0, \quad [1]$$

where I_k is the current in the k th element. The resonance modes of interest are the two degenerate modes with the current distribution

$$I_k = I_0 \cos\left(\frac{2\pi k}{N} + \varphi\right), \quad [2]$$

where N is the total number of elements (in our case, it is 16) and the phase shift φ is 0 and $\pi/2$ for the two modes, respectively. These two modes have a uniform magnetic field distribution in the loading region. The resonance frequency of these two modes is solved from Eq. [1] as

$$\begin{aligned} \omega &= \left\{ CL \left[1 + \sum_{j \neq 0} \frac{M_j}{L} \cos(2\pi j/N) \right] \right\}^{-1/2} \\ &= \omega_0 \left[1 + \sum_{j \neq 0} \frac{M_j}{L} \cos(2\pi j/N) \right]^{-1/2}, \end{aligned} \quad [3]$$

where ω_0 is the resonance frequency of each individual element.

To determine the initial resonance frequency of each individual element such that the desired frequency ω_L falls into the tuning range of the coil, the following procedure is used:

Two resonant elements are tuned to an identical frequency ω_t not far away from the desired frequency ω_L . The two elements are placed in the coil at n units apart. Due to the mutual inductance of the two elements, two resonance modes exist, with the following frequencies:

$$\omega_{\pm} = \omega_t [1 \pm M_n/L]^{-1/2}. \quad [4]$$

These two frequencies are measured for each n , and the ratio M_n/L is then calculated from Eq. [4]. Knowing the ratios M_n/L and assuming they do not change dramatically with frequency, they are used in Eq. [3] to predict the resonance frequency ω_0 . This method has been found to be accurate within 2%.

The coil is driven in quadrature at two elements 90° apart. The actual driving circuit is shown in Fig. 3. For each driven element, a coaxial cable is introduced into the coil with its outer conductor grounded to the shield and its inner conductor connected to a tap point on the element via a flexible copper band. This enables the driven element to pivot freely just like all of the other elements. A series capacitor is used for matching the high impedance of the tap point to 50 ohms.

RESULTS

The prototype head coil operates at 170.74 MHz (4T protons). The tuning range is ± 6 MHz for symmetric tuning with the uniform change of mutual inductances between elements, as

mentioned above. The Q values in both loaded and unloaded conditions were measured with two 2-cm-diameter shielded loops. The transmission coefficient between the two loops was measured with an HP4195A network analyzer (Fig. 4). The Q values from Fig. 4 are 680 without loading and 129 when loaded with a human head. Also indicated by Fig. 4 is that the resonance frequency increased by ~ 120 kHz when loaded, indicating inductive loading of the coil. The high loaded Q signifies a significant reduction in the subject perturbation of the whole coil, whereas the ratio of 5.2 between unloaded Q and loaded Q shows that the efficiency of the coil is well maintained (15).

The coil can be conveniently tuned and matched with the symmetric tuning mechanism and the matching capacitors. Figure 5 shows the reflection coefficient from one of the two driving ports in the three states: tuned and matched to empty load, loaded with a head, and retuned and matched with the head. The 3dB width of the empty load condition was 345 KHz. When retuned and matched to the head, the 3dB width broadened to 1.44 MHz. The relatively high value of loaded Q made the resonance width narrow enough to accurately tune the coil. When tuned and matched with a head, the isolation between the two quadrature channels was between 25 and 30 dB.

As a test of the B_1 homogeneity of the coil, a spin echo image was acquired on a phantom made of a matrix of test tubes containing saline water (Fig. 6). An array of test tubes was used to avoid any long current paths, which give rise to sample resonance effects at 170 MHz (4). This segmented sample more realistically reflected the field distribution of the coil when compared to a solid sample. The signal intensity of the test tube array across the diameter and along the axis of the coil is plotted in Fig. 7. In the transaxial plane, the variation over 20 cm distance is $\sim 10\%$. Along the axial direction, the signal stayed within 10% over 15 cm, which is $3/4$ of the resonant element's length.

To evaluate the B_1 homogeneity and SNR of the coil in a human subject, a single acquisition proton density spin echo image was collected on a normal volunteer (Fig. 8). The voxel size was $1 \times 1 \times 4$ mm, $TE = 26$ ms, $TR = 3$ s. The SNR averaged 80:1 in a single white matter pixel. The improved efficiency of the coil was demonstrated by the modest 229 watts peak power required for a 90° 3-ms sinc pulse ($\text{sinc} = 2$) used in this study. As a comparison, a conventional birdcage coil with an RF shield similar to the FE coil was used for a spin echo image with the same imaging parameters. The RF shield, not usually used on birdcage coils at low field strengths, was necessary at 170 MHz to bring the loaded Q to a reasonable level for tuning purpose. The unloaded Q of the coil was 96; it was 35 when loaded with a head. The peak power required for a 90° 3-ms sinc pulse ($\text{sinc} = 2$) was 620 watts.

The homogeneity of the coil was perturbed slightly by the subject, requiring a small amount of asymmetric tuning of the individual elements located near the back of the head. The actual asymmetric tuning was done empirically by adjusting the resonance frequencies of the four elements at the back of the head. Their resonance frequencies were tuned down by $\sim 0.6\%$ to compensate for the decreased B_1 there.

DISCUSSION

The cylindrical arrangement of resonant elements is an effective solution to the problems encountered with large volume NMR coils at high fields. This arrangement permitted accurate planning of the coil's tuning and matching characteristics, reduced the net perturbation of the subject on the coil, enabled precise tuning and matching with each patient, as well as reduction in dielectric and radiation losses of the coil. These factors resulted in a very efficient MR volume coil, which produced improved MR images at 4T. A simple mathematical model was shown to be adequate for planning and construction of the coil within reasonable range of the

target values. This is an important aspect because it reduces the empirical iterations usually required. For high field coil optimization problems, a generalized form of the reciprocity relation between coil efficiency and sensitivity was derived. It is applicable to all frequencies and coil-subject geometries.

The net perturbation of the subject on the coil was reduced significantly, as illustrated by the high loaded Q value. However, the efficiency and sensitivity of the coil is well maintained. The B_1 distribution of the coil without sample dielectric resonance was demonstrated to be quite uniform with a segmented phantom. The B_1 homogeneity with a head was slightly perturbed and could be easily corrected with a small amount of asymmetric tuning of a few resonant elements located near the back of the head.

The radiation loss of the coil is significantly reduced by the RF shield as shown in the high unloaded Q . The self-shielding nature of each resonant element may also contribute to the decrease in radiation loss (16).

Because the design consists of individual tuned elements, multiple tuning can be easily realized by making each element multiple tuned. Because each resonant element is relatively small, many multiple tuning schemes can be applied to it without losing much of its efficiency. This, in turn, also guarantees the efficiency of the whole coil in the multiple tuned configuration.

Whereas the prototype head coil has cylindrical symmetry, the general design is not confined to this form. In the case of thoracic or abdominal imaging, for example, a coil of elliptical cross section with the elements asymmetrically arranged is conceivably preferential.

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APPENDIX

The General Reciprocity of an NMR Transceiver

Relations between transceiver sensitivity and efficiency have been derived for simple coil geometries, often under quasistatic approximations (1,12). At 4T, the Larmor frequency of protons is 170 MHz, high enough so that the wavelength of 20 cm in water approaches the scale of the subject in human imaging studies. Theoretical description beyond quasistatic approximation and simple coil-sample geometry is needed. It is shown here theoretically that a general reciprocal relation exists between the sensitivity and the efficiency of a transceiver (CGS units):

$$SNR = \frac{1}{8} \sin(\alpha) e^{-\tau} n V_{ox} N^{1/2} \hbar^2 \omega^3 B_0^{-1} P_{1G}^{-1/2} (kT)^{-3/2} (\Delta f)^{-1/2}, \quad [A1]$$

where SNR is the signal to noise level of a voxel in the image, α is the flip angle of the experiment, $e^{-\tau}$ represents the spin relaxation factor of the nuclear magnetization, n is the spin density, V_{ox} is the volume of the voxel, N is the number of scans in the image acquisition, P_{1G} is the power needed to generate a circularly polarized RF magnetic field of 1 Gauss magnitude at the voxel, Δf is the frequency bandwidth of each voxel, ω is the Larmor frequency, B_0 is the main static magnetic field, T is the temperature, and $\hbar$ is the Planck's constant.

In the following discussion, all electromagnetic fields and moments are oscillating at frequency ω , and the complex notation $\mathbf{a}(t) = \mathbf{a} e_{i\omega t}$ is used, $\mathbf{a}$ being a constant complex vector. The factor $e_{i\omega t}$ will be omitted unless specified with “(t)”, and the following notation is also used:

$$\frac{1}{T} \int_{-T}^T \mathbf{a}(t) \cdot \mathbf{b}(t) dt = \frac{\mathbf{a} \cdot \mathbf{b}^* + \mathbf{b} \cdot \mathbf{a}^*}{2} = \langle \mathbf{a} | \mathbf{b} \rangle. \quad [A2]$$

Apparently

$$\langle \mathbf{a} | \mathbf{b} \rangle = \langle \mathbf{a}^* | \mathbf{b}^* \rangle. \quad [A3]$$

Consider the simple case of two general electromagnetic “current” elements (electric dipole, magnetic dipole, etc.) $\mathbf{I}_1$ and $\mathbf{I}_2$ at coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ in free space, the conjugate field of $\mathbf{I}_2$ at $\mathbf{r}_2$ generated by $\mathbf{I}_1$ is represented by the retarded free space. Green function operator G_{21} ,

$$\mathbf{v}_{21}(t, \mathbf{r}_2) = \int G_{21}(t, \mathbf{r}_2, t', \mathbf{r}_1) \mathbf{I}_1(t') dt', \quad [A4]$$

vice versa,

$$\mathbf{v}_{12}(t, \mathbf{r}_1) = \int G_{12}(t, \mathbf{r}_1, t', \mathbf{r}_2) \mathbf{I}_2(t') dt'. \quad [\text{A5}]$$

From the explicit form of the free space retarded Green functions, it is easy to see that

$$\langle \mathbf{I}_1 | G_{12} \mathbf{I}_2 \rangle = \langle \mathbf{I}_2^* | G_{21} \mathbf{I}_1^* \rangle. \quad [\text{A6}]$$

Now consider the general case of two current elements $\mathbf{I}_s$ and $\mathbf{I}_d$ at $\mathbf{r}_s$ and $\mathbf{r}_d$. The two current elements are emersed in a linear dissipative environment, which can be represented by many discrete point “elements,” each having a linear response to the local electromagnetic field:

$$\mathbf{I}_i(t) = \boldsymbol{\varepsilon}_i \mathbf{v}_i(t, \mathbf{r}_i), \quad [\text{A7}]$$

where $\mathbf{I}_i$ is the general current of the element i in response to its conjugate field $\mathbf{V}_i$ at its locale $\mathbf{r}_i$, $\boldsymbol{\varepsilon}_i$ is the response operator. If the linear environment is at or near thermal equilibrium, Onsager's relation means that for any oscillating fields $\mathbf{V}_1(\mathbf{r}_i)$ and $\mathbf{V}_2(\mathbf{r}_i)$, the following is satisfied by the response operator,

$$\langle \boldsymbol{\varepsilon}_i \mathbf{v}_1(\mathbf{r}_i) | \mathbf{v}_2(\mathbf{r}_i) \rangle = \langle \mathbf{v}_1(\mathbf{r}_i)^* | \boldsymbol{\varepsilon}_i \mathbf{v}_2^*(\mathbf{r}_i) \rangle. \quad [\text{A8}]$$

Taking into account the effect of the medium, the conjugate field of $\mathbf{I}_d$ generated by $\mathbf{I}_s$ at $\mathbf{r}_d$ is

$$\begin{aligned} \mathbf{v}_{ds} &= \mathbf{v}_{ds} + \sum_i \mathbf{v}_{di} \\ &= G_{ds} \mathbf{I}_s + \sum_i G_{di} \mathbf{I}_i, \end{aligned} \quad [\text{A9}]$$

where lower case $\mathbf{v}_{ds}$ is the field at $\mathbf{r}_d$ generated by $\mathbf{I}_s$ in the absence of the medium, $\mathbf{v}_{di}$ is the field contribution of the current $\mathbf{I}_i$ of medium “element” i . Because

$$\begin{aligned} \mathbf{I}_i &= \boldsymbol{\varepsilon} \mathbf{v}_i \\ &= \boldsymbol{\varepsilon} (\mathbf{v}_{is} + \sum_{j \neq i} \mathbf{v}_{ij}) \\ &= \boldsymbol{\varepsilon} (\mathbf{v}_{is} + \sum_{j \neq i} G_{ij} \mathbf{I}_j), \end{aligned} \quad [\text{A10}]$$

substituting Eq. [A10] into Eq. [A9] iteratively gives

$$\begin{aligned}
\mathbf{V}_{ds} &= \mathbf{v}_{ds} = \sum_i \mathbf{v}_{di} \\
&= G_{ds} \mathbf{I}_s + \sum_i G_{di} \mathbf{I}_i \\
&= G_{ds} \mathbf{I}_s + \sum_i G_{di} \epsilon_i G_{is} \mathbf{I}_s + \sum_i G_{di} \epsilon_i \sum_{j \neq i} G_{ij} \mathbf{I}_j \\
&= G_{ds} \mathbf{I}_s + \sum_i G_{di} \epsilon_i G_{is} \mathbf{I}_s + \sum_i G_{di} \epsilon_i \sum_{j \neq i} G_{ij} \epsilon_j G_{js} \mathbf{I}_s + \sum_i G_{di} \epsilon_i \sum_{j \neq i} G_{ij} \epsilon_j \sum_{k \neq j} G_{jk} \epsilon_k G_{ks} \mathbf{I}_s + \dots \\
&= F_{ds} \mathbf{I}_s,
\end{aligned} \tag{A11}$$

where F_{ds} denotes the Green function in the presence of the medium. With the Eq. [A3], [A6], and [A8],

$$\begin{aligned}
\langle \mathbf{I}_a | G_{ab} \epsilon_b \mathbf{V}_b \rangle &= \langle G_{ba} \mathbf{I}_a^* | (\epsilon_b \mathbf{V}_b)^* \rangle \\
&= \langle (G_{ba} \mathbf{I}_a^*)^* | \epsilon_b \mathbf{V}_b \rangle \\
&= \langle \epsilon_b G_{ba} \mathbf{I}_a^* | \mathbf{V}_b^* \rangle.
\end{aligned} \tag{A12}$$

Thus

$$\begin{aligned}
&\langle \mathbf{I}_d | G_{di} \epsilon_i G_{ij} \epsilon_j \dots G_{ms} \mathbf{I}_s \rangle \\
&= \langle \epsilon_j G_{id} \mathbf{I}_d^* | (G_{ij} \epsilon_j \dots G_{ms} \mathbf{I}_s)^* \rangle \\
&= \langle (\epsilon_i G_{id} \mathbf{I}_d^*)^* | G_{ij} \epsilon_j \dots G_{ms} \mathbf{I}_s \rangle \\
&= \langle \epsilon_j G_{ji} \epsilon_i G_{id} \mathbf{I}_d^* | (\dots G_{ms} \mathbf{I}_s)^* \rangle \\
&= \langle \dots \epsilon_j G_{ji} \epsilon_i G_{id} \mathbf{I}_d^* | (G_{ms} \mathbf{I}_s)^* \rangle \\
&= \langle (\dots \epsilon_j G_{ji} \epsilon_i G_{id} \mathbf{I}_d^*)^* | (G_{ms} \mathbf{I}_s) \rangle \\
&= \langle G_{sm} \dots \epsilon_j G_{ji} \epsilon_i G_{id} \mathbf{I}_d^* | \mathbf{I}_s^* \rangle \\
&= \langle \mathbf{I}_s^* | G_{sm} \epsilon_m \dots \epsilon_j G_{ji} \epsilon_i G_{id} \mathbf{I}_d^* \rangle.
\end{aligned} \tag{A13}$$

Equations [A11] and [A13] together lead to the general reciprocal relation of the Green function in a linear environment at or near thermal equilibrium:

$$\langle \mathbf{I}_d | F_{ds} \mathbf{I}_s \rangle = \langle \mathbf{I}_s^* | F_{ad} \mathbf{I}_d^* \rangle. \tag{A14}$$

To apply the general reciprocity to the discussion of sensitivity versus efficiency, the two specific current elements are chosen to represent the nuclear magnetic moment at $\mathbf{r}_d$ in the subject, and the RF power input/signal output port of the transceiver. The element at $\mathbf{r}_d$ is thus the circularly polarized nuclear magnetic dipole moment

$$\mathbf{I}_d = \mathbf{m} = m_0 (\mathbf{e}_x + i\mathbf{e}_y), \tag{A15}$$

where $\mathbf{m}_0$ is the amplitude of the magnetic moment, assuming the main static magnetic field is in the z direction. The other element is simply a current source $\mathbf{I}_s$ at the I/O port of the transceiver. The conjugate field of $\mathbf{I}_d$ generated by $\mathbf{I}_s$ is

$$\mathbf{V}_{ds} = \nabla \times \mathbf{E}_1(t) = - \frac{d\mathbf{B}_1(t)}{dt} = -i\omega \mathbf{B}_1 \tag{A16}$$

The conjugate field of $\mathbf{I}_s$ generated by $\mathbf{I}_d$ is simply the electric voltage across the current source. If V_s is the voltage across $\mathbf{I}_s$ from the nuclear magnetic moment $\mathbf{m}^* = m_0 (\mathbf{e}_x - i\mathbf{e}_y)$, then Eq. [A14] results in

$$\langle I_s^* | V_s \rangle = \langle -i\omega B_1 | m \rangle. \quad [\text{A17}]$$

Note $\mathbf{e}_x + i\mathbf{e}_y$ as the clockwise polarization, $\mathbf{I}_s$ the magnitude of $\mathbf{I}_s$, $V_{s,cc}$ the amplitude of the voltage generated by the counter-clockwise polarized $\mathbf{m}^*$, and $B_{1,c}$ the amplitude of the clockwise component of $\mathbf{B}_1$. It is easy to see from Eq. [A17] that

$$I_s V_{s,cc} = 2\omega B_{1,cc} m_0. \quad [\text{A18}]$$

Because $B_{1,c}/I_s$ reflects the efficiency of the transceiver and $V_{s,cc}/m_0$ reflects the sensitivity of the transceiver, Eq. [A18] is the basis for the eventual relation [A1].

Notice that Eq. [A18] relates the coil's efficiency to generate a circularly polarized magnetic field to its sensitivity toward the nuclear magnetic moment of the other circular polarization. This change of polarization makes it necessary to distinguish linear transceivers from quadrature transceivers, as discussed below.

A linear transceiver generates linearly polarized $\mathbf{B}_1$. Because a linear field is the sum of the two circular polarizations of equal magnitude, Eq. [A18] can be written as

$$I_s V_{s,cc} = 2\omega B_{1,cc} m_0, \quad [\text{A19}]$$

where $B_{1,cc}$ is the counter-clockwise component of $\mathbf{B}_1$. If the impedance into the output port of the transceiver (where the element $\mathbf{I}_s$ lies) is Z_m , its resistive part R_m , then the RF power needed to generate $B_{1,cc}$ is

$$P_t = \frac{I_s^2 R_m}{2}. \quad [\text{A20}]$$

With Eq. [A20], Eq. [A19] becomes

$$\begin{aligned} V_{s,cc} &= \frac{2\omega B_{1,cc} m_0}{\sqrt{2P_t/R_m}} \\ &= \frac{\sqrt{2}\omega \sqrt{R_m} m_0}{\sqrt{P_{1G}}}, \end{aligned} \quad [\text{A21}]$$

where P_{1g} is the power needed to generate a circularly polarized $\mathbf{B}_1$ of 1 Gauss magnitude at the voxel. If the preamplifier's input impedance is Z_p the actual voltage on the preamplifier is

$$V_p(t) = V_{s,cc}(t) \frac{Z_p}{Z_p + Z_m}. \quad [\text{A22}]$$

The noise voltage V_n from the transceiver is also divided between Z_p and Z_m ,

$$V_{np}(t) = V_n(t) \frac{Z_p}{Z_p + Z_m}. \quad [\text{A23}]$$

The SNR is thus

$$\begin{aligned} SNR^2 &= \frac{\langle V_p^2(t) \rangle}{\langle V_{np}^2(t) \rangle} \\ &= \frac{\langle V_{sec}^2(t) \rangle}{\langle V_n^2(t) \rangle} \\ &= \frac{V_{sec}^2}{2\langle V_n^2(t) \rangle}. \end{aligned} \quad [\text{A24}]$$

If the frequency bandwidth of each voxel in the data acquisition is Δf , then

$$\langle V_n^2(t) \rangle = 4\Delta f R_m kT, \quad [\text{A25}]$$

where k is the Boltzman's constant. Substitution of Eq. [A21] and Eq. [A25] into Eq. [A24] gives

$$SNR^2 = \frac{\omega^2 n_0^2}{4\Delta f k T_{1G}}. \quad [\text{A26}]$$

For a voxel of volume V_{ox} and spin density n , the magnitude of the transverse magnetic moment can be expressed as

$$m_0 = e^{-\tau} \sin(\alpha) M_0 n V_{ox}, \quad [\text{A27}]$$

where M_0 is the average magnetization of each spin, and the other parameters are defined under Eq. [A1]. It has been shown (17) that for magnetic dipoles,

$$M_0 = \frac{\hbar^2 \omega^2}{4B_0 kT}. \quad [\text{A28}]$$

Substituting Eq. [A28] and Eq. [A27] into Eq. [A26], taking into account the factor $N^{1/2}$ if N scans are performed to acquire the image, the general relation Eq. [A1] is obtained.

A quadrature transceiver is essentially two linear coils combined at a quadrature hybrid (see, e.g., ref. 15). The hybrid produces 90° phase difference between the two linear coils and a 180° phase flip on one of the linear coils between the transmission and the reception. Thus, the transceiver transmits and receives in the same circular polarization, and Eq. [A1] still holds. The difference is that a quadrature transceiver generates only the correct circular polarization when transmitting, whereas the linear transceiver generates an equal amount of both circular

polarizations. Thus, everything being equal, a quadrature coil needs only half of the power to produce the same magnitude of $B_{1,cc}$, which is reflected in the value of P_{1G} in Eq. [A1].

An important noise source that is not included in Eq. [A1] is the coherent noise in the environment. Depending on the shielding of the magnetic room and the general geometry of the coil and the subject, etc., the coherent noise can be picked up by the coil to various degrees. The general reciprocity relation [A1] therefore predicts an SNR that is usually higher than the measured value.

FIG. 1.
A picture of the prototype head coil.

FIG. 2.

A schematic of a section of the coil and the resonant element. For symmetric tuning, all elements are rotated simultaneously by the angle.

FIG. 3.
A sketch of the driving circuit to one driven element.

FIG. 4. Transmission coefficient between two shielded probe loops in the coil. The Q of unloaded and loaded conditions were calculated from this measurement.

FIG. 5.
Reflection coefficient at one drive port in loaded and unloaded conditions.

FIG. 6.

A spin echo transaxial phantom image taken as a measure of the B_1 homogeneity of the coil.
 $TE = 21$ ms; $TR = 1$ s, voxel = $1 \times 1 \times 4$ -mm.

a

FIG. 7. Signal intensity in the transaxial and axial directions of the coil; (a) is of the transaxial plane, (b) is of the axial direction.

FIG. 8. Spin echo proton density image of a head at 4T. NEX = 1, $TR = 3$ s, $TE = 26$ ms, voxel = $1 \times 1 \times 4$ mm.