Published in final edited form as:

Phys Med Biol. 2009 May 21; 54(10): 2971–2991. doi:10.1088/0031-9155/54/10/001.

Exact image reconstruction with triple-source saddle-curve cone-beam scanning

Yang Lu^{1,2}, Jun Zhao^{1,2,3}, and Ge Wang²

¹Department of Biomedical Engineering, Shanghai Jiao Tong University, Shanghai, 200240, People's Republic of China

²Biomedical Imaging Division, VT-WFU School of Biomedical Engineering, Virginia Tech, Blacksburg, VA 24061, USA

Abstract

In this paper, we propose an exact shift-invariant filtered backprojection (FBP) algorithm for triple-source saddle-curve cone-beam CT. In this imaging geometry, the x-ray sources are symmetrically positioned along a circle, and the trajectory of each source is a saddle curve. Then, we extend Yang's formula from the single-source case to the triple-source case. The saddle curves can be divided into four parts to yield four datasets. Each of them contains three data segments associated with different saddle curves, respectively. Images can be reconstructed on the planes orthogonal to the z-axis. Each plane intersects the trajectories at six points (or three points at the two ends) which can be used to define the filtering directions. Then, we discuss the properties of these curves and study the case of $2N + 1$ sources ($N \geq 2$). A necessary condition and a sufficient condition are given to find efficient curves. Finally, we perform numerical simulations to demonstrate the feasibility of our triple-source saddle-curve approach. The results show that the triple-source geometry is advantageous for high temporal resolution imaging, especially important for cardiac imaging and small animal imaging.

1. Introduction

Since its introduction in 1973, x-ray CT has revolutionized clinical imaging and become a cornerstone of radiology departments. Closely correlated to the development of x-ray CT, the research for better image quality has been pursued for biomedical applications. The first dynamic CT system was the Dynamic Spatial Reconstructor (DSR) built at the Mayo Clinic in 1979 (Robb *et al* 1983). In a 1991 SPIE conference, we presented a spiral cone-beam scanning mode and methods to solve the long object problem (Wang *et al* 1991). In 1990s, single-slice spiral CT became the standard scanning mode of clinical CT (Kalender 1995). In 1998, multi-slice spiral CT entered the market (Taguchi and Aradate 1998). With the fast evolution of the source, detector and reconstruction technology, cone-beam CT is recognized as a central issue for development of the next generation clinical CT systems (Wang *et al* 2000). Moreover, just as there have been strong needs for clinical CT, there are strong demands and exciting results for pre-clinical CT, cone-beam micro-CT of mice and rats in particular.

Despite the impressive advancement of the CT technology, there are critical and immediate needs for better image quality in many biomedical investigations, of which we single out cardiac CT as a primary example. Earlier CT applications are in relatively static body parts such as the head, chest, abdomen and extremities, but the dynamic body parts such as the heart and specifically the coronary arteries remain challenging. Multi-row CT has made imaging of

³Author to whom any correspondence should be addressed.

the lumen of the coronary arteries possible with excellent diagnostic accuracy. Nevertheless, imaging the lumen of the coronary arteries provides only a partial understanding of coronary atherosclerosis. In the early 1990s, it became clear that a majority of myocardial infarctions did not occur at stenotic locations. Clinical trials of thrombolytic agents in acute myocardial infarction showed that the lesion underlying the clot was often not stenotic. More recent results from the 'Courage' trial indicated that fixing the stenosis did not reduce death and other major cardiac events. The current research emphasis is to improve our understanding of the pathobiology and genetics of coronary artery diseases. Preliminary results demonstrated that the CT technology has good correlation with intracoronary ultrasound of plaques and can classify them into lipid containing fibrotic and calcified but with significant overlaps. Hence, cardiac CT has been identified as an important tool.

However, even with the state-of-the-art Siemens dual-source CT scanner (Flohr *et al* 2006), temporal resolution of 83 ms is only achievable with the aid of ECG gating around stable phases (30% or 70% of the R–R period). Major problems still exist, including (1) the incapability of offering similar image quality at an arbitrary cardiac phase for truly dynamic cardiac imaging; (2) prerequisite for breath-holding during the entire cardiac CT scan of 7–13 s, which is not feasible in the cases of pediatric and emergent scans; (3) reconstruction difficulties in the cases of high/irregular heart rates; (4) limitations with circular and helical scanning—while with a circular scan there are more image artifacts further away from the mid-plane, with a helical scan the cardiac motion is only scanned once targeting a single cardiac phase and yet a substantial number of x-ray photons emitted near the two ends of a helical trajectory segment are under-utilized; (5) failure to reach often preferred higher temporal resolution—since the mean velocity of the right coronary artery is 69.5 mm s^{-1} , a temporal resolution of $<20\text{--}80 \text{ ms}$ is needed. In fact, most demanding cardiac imaging used to be done by electron-beam CT (EBCT) with a temporal resolution of 50–100 ms, which is expensive and rarely available, while CT of animals as human cardiac disease models may require much higher temporal resolution; for example, the heart rates of mice/rats are twice as fast or more than that of humans.

The multi-source CT architecture is an effective strategy for the improvement of temporal resolution. To date, most commercial CT scanners are based on the so-called third-generation geometry, in which a single-source detector assembly is rotated around a patient. For the dual-source scanner, the minimum rotation interval is $90^\circ + \alpha$, compared to $180^\circ + \alpha$ in a third-generation geometry. Since the fan angle α is relatively small, there is a $>40\%$ reduction in acquisition time. As a result, the dual-source scanner reduces the data acquisition time to 0.35 s, which leads to 83 ms temporal resolution with cardiac gating. Interestingly, for exact helical cone-beam reconstruction, the trinity is better than the duality because the triple-source arrangement allows a perfect mosaic of longitudinally truncated cone-beam data to satisfy the Orlov condition (Orlov 1975) and yield better noise performance than the dual-source counterpart. In 2006, we presented the first paper on exact image reconstruction with a triple-source helical cone-beam scan (Zhao *et al* 2006, 2009). In the 2008 SPIE conference, we reported our work on exact image reconstruction with a triple-source saddle-curve cone-beam scan (Lu *et al* 2008). In the long run, we believe that the triple-source cone-beam CT scanner will play an important role for medical imaging due to its unique merits relative to the dual-source counterpart despite increased hardware cost.

This paper is organized as follows. In section 2, we overview the key formulas obtained by Pack and Noo (2005) and Yang *et al* (2006a, 2006b). In section 3, we propose the concept of efficient curves that leads to shift-invariant FBP reconstruction. In section 4, we study the $(2N + 1)$ -source geometry and associated temporal resolution. In section 5, we present numerical results. Finally, in section 6, we discuss relevant issues and conclude the paper.

2. Preliminaries

In 2002, Katsevich (2002) proved the first exact and efficient reconstruction formula for helical cone-beam CT. Since then, exact reconstruction algorithms for cone-beam CT have been actively studied (Wang *et al* 2008). In 2005, Pack and Noo derived a general formula (Pack and Noo 2005). It can be summarized as follows.

Let $\vec{x}$ be an arbitrary point in an object, $\vec{e}$ be the filtering direction, $\vec{a}(\lambda)$ be a point on a scanning trajectory, $\vec{\omega}$ be a unit vector in S^2 , λ^- and λ^+ be the angular parameters for $\vec{x}$, and $\vec{a}(\lambda^-)$ and $\vec{a}(\lambda^+)$ be collinear. Then,

$$\kappa(\vec{x}, \vec{e}, \lambda^-, \lambda^+) = -\frac{1}{2\pi^2} \int_{\lambda^-}^{\lambda^+} d\lambda \frac{1}{\|\vec{x} - \vec{a}(\lambda)\|} g_F(\lambda, \vec{x}, \vec{e}), \quad (1)$$

$$g_F(\lambda, \vec{x}, \vec{e}) = \int_{-\pi}^{\pi} d\gamma \frac{1}{\sin \gamma} \frac{\partial g(\lambda, \widehat{\theta}(\lambda, \vec{x}, \vec{e}, \gamma))}{\partial \lambda}, \quad (2)$$

where

$$\widehat{\theta}(\lambda, \vec{x}, \vec{e}, \gamma) = \cos \lambda \widehat{a}(\lambda, \vec{x}) + \sin \lambda \widehat{\beta}(\lambda, \vec{x}, \vec{e}), \quad (3)$$

$$\widehat{a}(\lambda, \vec{x}) = \frac{\vec{x} - \vec{a}(\lambda)}{\|\vec{x} - \vec{a}(\lambda)\|}, \quad (4)$$

$$\widehat{\beta}(\lambda, \vec{x}, \vec{e}) = \frac{\vec{e} - (\vec{e} \cdot \widehat{a}(\lambda, \vec{x})) \widehat{a}(\lambda, \vec{x})}{\|\vec{e} - (\vec{e} \cdot \widehat{a}(\lambda, \vec{x})) \widehat{a}(\lambda, \vec{x})\|}. \quad (5)$$

Pack and Noo (2005) proved that

$$\kappa(\vec{x}, \vec{e}, \lambda^-, \lambda^+) = -\frac{1}{8\pi^2} \int_{S^2} d\vec{\omega} (R'' f)(\vec{\omega}, \vec{x} \cdot \vec{\omega}) \sigma(\vec{x}, \vec{\omega}, \vec{e}, \lambda^-, \lambda^+), \quad (6)$$

where

$$\sigma(\vec{x}, \vec{\omega}, \vec{e}, \lambda^-, \lambda^+) = \frac{1}{2} \text{sgn}(\vec{\omega} \cdot \vec{e}) [\text{sgn}(\vec{\omega} \cdot \widehat{a}(\lambda^-, \vec{x})) - \text{sgn}(\vec{\omega} \cdot \widehat{a}(\lambda^+, \vec{x}))]. \quad (7)$$

This formula is attractive because $R'' f$ is the second-order derivative of the 3D Radon transform. This means that if $\sigma(\vec{x}, \vec{\omega}, \vec{e}, \lambda^-, \lambda^+) = 1$, we have $f(\vec{x}) = \kappa(\vec{x}, \vec{e}, \lambda^-, \lambda^+)$. If we can find trajectories with appropriate filtering directions to make $\sigma(\vec{x}, \vec{\omega}, \vec{e}, \lambda^-, \lambda^+)$ a constant, the object function can be exactly reconstructed. Note that this finding is closely linked to Katsevich's general scheme (Katsevich 2003) and the finding by Ye and Wang (2005) as a special case of Katsevich's general scheme (Katsevich 2003).

In 2006, Yang *et al* (2006b) proved that an object function f can be reconstructed along a rectangle on a plane perpendicular to the rotation axis. Then, they extended the formula to a more general case (Yang *et al* 2006a):

Theorem 1

An object function can be reconstructed by

$$f(\vec{x}) = \frac{1}{2} \kappa(\vec{x}, \vec{e}_{N,1}, \lambda_N, \lambda_1) + \frac{1}{2} \sum_{i=1}^{N-1} \kappa(\vec{x}, \vec{e}_{i,i+1}, \lambda_i, \lambda_{i+1}), N=3, 4, 5, \dots, \quad (8)$$

where $\vec{e}_{i,j} = \frac{\vec{a}(\lambda_j) - \vec{a}(\lambda_i)}{\|\vec{a}(\lambda_j) - \vec{a}(\lambda_i)\|}$ are the filtering directions and λ_i are points where the scanning trajectory intersects with the reconstruction plane, and the reconstructed region formed by λ_i is a convex polygon.

3. Efficient curves

Using equation (8), we can immediately reconstruct the image on any plane which has at least three non-collinear intersection points with the trajectory.

This method is convenient but not efficient. For different planes, we have to recalculate the intersection points, determine the filtering directions and then perform the filtration. As shown in figure 1(a), if we directly use equation (8) to reconstruct the plane Π_0 , although the lines l_a and l_b on Π_0 are parallel, at the vertex point $\vec{a}(\lambda)$, the filtration needs to be performed twice, respectively, along the lines l_{ha} and l_{hb} on the detector plane. On the other hand, as shown in figure 1(b) if we directly use equation (8) to reconstruct the planes Π_1 and Π_2 , at the vertex point $\vec{a}(\lambda)$, the filtration will only be performed along the line l_h on the detector plane, where l_h is the projection of both l_1 and l_2 (l_1 and l_2 are on Π_1 and Π_2 , respectively). Thus, unless the trajectory and the reconstructed plane are appropriately selected, the algorithm will not be efficient.

In this section, we first introduce a family of curves which is called efficient, which means that shift-invariant filtration can be performed as in the classic Feldkamp-type reconstruction or Katsevich-type reconstruction. With such an efficient scanning curve, an efficient FBP algorithm can be obtained.

Our question is whether this algorithm can be efficiently applied along a general closed curve. The key is to use a shift-invariant FBP algorithm. The conditions need to be found so that the filtration can be done in a shift-invariant fashion.

Let us first introduce necessary notation. Let C be a continuous closed curve in the space and S be a family of parallel planes each of which intersects C at three points or more. S can be written as a function $S(z) = \{z = z' \in [z_s, z_e]\}$, $z_s, z_e \in R$. When z varies from z_i to z_{i+1} , $z_s \leq z_i < z_{i+1} \leq z_e$, if the polygons formed by the intersection points are similar to each other and the corresponding edges of the polygons are parallel to each other, we consider the curve C between z_i and z_{i+1} efficient. The volume bounded by an efficient part of C or the corresponding portion of $S(z)$ is denoted as $V(z_i, z_{i+1})$ that can be exactly and efficiently reconstructed in a shift-

invariant fashion. If an ROI of the object is in the space $V = \sum_{i=1}^n V(z_i, z_{i+1})$, it can be reconstructed in the shift-invariant FBP format.

Lemma 1

Let $\vec{a}(\lambda_i)$ ($\lambda_i < \lambda_{i+1}$, $i = 1, 2, 3 \dots$) be a series of intersection points at which $S(z_0)$ intersects C . If the curve C is efficient, we must have

$$\vec{a}'(\lambda_i) \times \vec{a}'(\lambda_{i+1}) \cdot [\vec{a}(\lambda_{i+1}) - \vec{a}(\lambda_i)] = 0. \quad (9)$$

Proof. Consider another plane $S(z'_0)$, $z'_0 \neq z_0$, which intersects C at points $\vec{a}'(\lambda'_i)$, $\lambda'_i < \lambda'_{i+1}$, $i = 1, 2, 3 \dots$. If C is efficient, from the definition of an efficient curve, the line connecting $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$ is parallel to the line connecting $\vec{a}'(\lambda'_i)$ and $\vec{a}'(\lambda'_{i+1})$. Moreover, points $\vec{a}(\lambda_i)$, $\vec{a}(\lambda_{i+1})$, $\vec{a}'(\lambda'_i)$ and $\vec{a}'(\lambda'_{i+1})$ are coplanar. When z'_0 approaches z_0 , $\vec{a}'(\lambda'_i)$ moves to $\vec{a}(\lambda_i)$ and $\vec{a}'(\lambda'_{i+1})$ to $\vec{a}(\lambda_{i+1})$. These four points are still coplanar. Therefore, the line connecting $\vec{a}'(\lambda'_i)$ and $\vec{a}(\lambda_i)$, the line connecting $\vec{a}'(\lambda'_{i+1})$ and $\vec{a}(\lambda_{i+1})$, and the line connecting $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$ are coplanar. Taking the limits of z'_0 , $\vec{a}'(\lambda'_i)$ and $\vec{a}'(\lambda'_{i+1})$, we have that the line connecting $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$ and the tangent lines at $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$ are coplanar, i.e., equation (9) is satisfied.

On the other hand, if one line does not satisfy equation (9), we can have the following four points on the curve: $\vec{a}(\lambda_i + \beta)$, $\vec{a}(\lambda_i - \beta)$, $\vec{a}(\lambda_{i+1} + \beta)$ and $\vec{a}(\lambda_{i+1} - \beta)$, where β is some positive number. When β is sufficiently small, these four points are non-coplanar. Thus, for any δ_1 and $\delta_2 \in [-\beta, \beta]$, the line through $\vec{a}(\lambda_i + \delta_1)$ and $\vec{a}(\lambda_{i+1} + \delta_2)$ is not parallel to the line through $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$. For any plane which intersects the trajectory at two points in a neighborhood of $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$, its filtering direction (which is determined by $\vec{a}(\lambda_i + \delta_1)$ and $\vec{a}(\lambda_{i+1} + \delta_2)$) is always different from that of the plane intersecting the trajectory at $\vec{a}(\lambda_i)$ and $\vec{a}(\lambda_{i+1})$. Therefore, these planes will not share the same filtering line on the detector. Since the set of these planes is of nonzero Lebesgue measure, it ruins the desirable structure for shift-invariant reconstruction. Thus, only efficient or partially efficient curves can be used in the aforementioned FBP algorithm.

Since it is not convenient to use equation (9) directly, in the following we provide two theorems to help us find the efficient curves. In doing so, we need the concept of Gaussian curvature, which is an intrinsic measure of curvature. It can be explained in terms of normal sectional curvature. Given a point on a surface and a direction in its tangent plane, the normal sectional curvature is computed by intersecting the surface with the plane determined by the point, the normal vector to the surface at that point and the direction. The normal sectional curvature is the signed curvature of this curve at that point. In this way, we can find the maximum and minimum normal sectional curvature values. Gaussian curvature is the product of these maximum and minimum values. A surface with zero Gaussian curvature is called a developable surface. It can be flattened onto a plane without any distortion.

Theorem 2

Any efficient curve must be on a series of developable surfaces.

Proof. For any continuous efficient curve, we can divide it into different parts according to the filtering directions. Suppose the curve has a filtering direction $\vec{e}_i(\lambda)$, $\lambda \in [\lambda_i, \lambda_{i+1}]$. Let $\vec{a}_0$, $\vec{a}_1$ be a pair of intersection points defining the filtering direction $\vec{e}_i(\lambda)$, and η be the distance from $\vec{a}_0$ to $\vec{a}_1$. As shown in figure 2, we have

$$\vec{a}_1 = \vec{a}_0 + \vec{e}_i \eta, \quad (10)$$

where λ varies from λ_i to λ_{i+1} , and all the filtering lines form a surface. Any point $\vec{q}$ on the surface can be denoted as

$$\vec{q} = \vec{a}_0 + \vec{e}_i \mu, \mu \in R. \quad (11)$$

Then,

$$\begin{aligned} \vec{q}_\lambda = \frac{\partial \vec{q}}{\partial \lambda} = \vec{a}'_0, \quad \vec{q}_\mu = \frac{\partial \vec{q}(\mu)}{\partial \mu} = \vec{e}_i, \quad \vec{n} = \frac{\vec{a}'_0 \times \vec{e}_i}{\|\vec{a}'_0 \times \vec{e}_i\|}, \\ \vec{q}_{\lambda\lambda} = \vec{a}''_0, \quad \vec{q}_{\mu\mu} = 0, \quad \vec{q}_{\lambda\mu} = 0. \end{aligned} \quad (12)$$

Note that we write $\vec{e}_i(\lambda)$ as $\vec{e}_i$ since all filtering lines are parallel. Consequently, the Gaussian curvature equals zero at any point on this surface:

$$K = \frac{(\vec{q}_{\lambda\lambda} \cdot \vec{n})(\vec{q}_{\mu\mu} \cdot \vec{n}) - (\vec{q}_{\lambda\mu} \cdot \vec{n})^2}{(\vec{q}_\lambda \cdot \vec{q}_\lambda)(\vec{q}_\mu \cdot \vec{q}_\mu) - (\vec{q}_\lambda \cdot \vec{q}_\mu)^2} = 0. \quad (13)$$

Therefore, this part of the scanning trajectory must be on the developable surface (Toponogov 2006). Similarly, all parts of the curve should be on the developable surfaces. This finishes the proof.

In other words, theorem 2 gives a necessary condition for efficient curves. From the above discussion, we know that both the curve C and the set S are important. Usually, the choice of S may determine whether C is efficient or not. For the same C , various choices of S may lead to different computational efficiencies. Hence, how to choose an appropriate S becomes a major issue after the curve is known. However, we do not have a general procedure yet and have to find the optimal solution case by case.

An important family of curves with medical CT relevance can be described as

$$C(\lambda) = (R \cos \lambda, R \sin \lambda, H(\lambda)), \quad (14)$$

where $\lambda \in [0, 2\pi]$, $H(s)$ is continuous and $H(0) = H(2\pi)$. In this case, no matter how $H(\lambda)$ changes, $C(\lambda)$ is on the developable surface $P(\lambda, z) = (R \cos \lambda, R \sin \lambda, z)$, $z \in R$. Then, S can be specified to contain involved planes perpendicular to the z -axis.

Theorem 3

The curve $C(\lambda)$ is efficient if for any minimum or maximum value of $H(\lambda_0)$ at λ_0 , $H(\lambda_0 - \lambda) = H(\lambda_0 + \lambda)$, $\lambda \in [0, \Delta\lambda]$, where $\Delta\lambda$ is the smaller length of the two angular intervals reaching the adjacent extreme values of $H(\lambda_0)$ and the range between $H(\lambda_0 - \Delta\lambda)$ and $H(\lambda_0)$ covers an ROI in an object in the z -direction.

Proof. Consider two points $\vec{a}(\lambda_0 - \lambda)$ and $\vec{a}(\lambda_0 + \lambda)$,

$$\vec{a}(\lambda_0 - \lambda) = (R \cos(\lambda_0 - \lambda), R \sin(\lambda_0 - \lambda), H(\lambda_0 - \lambda)), \quad (15)$$

$$\vec{a}(\lambda_0 + \lambda) = (R \cos(\lambda_0 + \lambda), R \sin(\lambda_0 + \lambda), H(\lambda_0 + \lambda)). \quad (16)$$

The derivatives of $\vec{a}(\lambda_0 - \lambda)$ and $\vec{a}(\lambda_0 + \lambda)$ are

$$\vec{a}'(\lambda_0 - \lambda) = \left. \frac{d\vec{a}(s)}{ds} \right|_{s=\lambda_0-\lambda} = (-R \sin(\lambda_0 - \lambda), R \cos(\lambda_0 - \lambda), H'(\lambda_0 - \lambda)), \quad (17)$$

$$\vec{a}'(\lambda_0 + \lambda) = \left. \frac{d\vec{a}(s)}{ds} \right|_{s=\lambda_0+\lambda} = (-R \sin(\lambda_0 + \lambda), R \cos(\lambda_0 + \lambda), H'(\lambda_0 + \lambda)). \quad (18)$$

Then, we have the triple product of $\vec{a}'(\lambda_0 - \lambda)$, $\vec{a}'(\lambda_0 + \lambda)$ and $\vec{a}(\lambda_0 + \lambda) - \vec{a}(\lambda_0 - \lambda)$ as follows:

$$\begin{aligned} & \vec{a}'(\lambda_0 - \lambda) \times \vec{a}'(\lambda_0 + \lambda) \cdot [\vec{a}(\lambda_0 + \lambda) - \vec{a}(\lambda_0 - \lambda)] \\ &= \begin{vmatrix} -R \sin(\lambda_0 - \lambda) & R \cos(\lambda_0 - \lambda) & H'(\lambda_0 - \lambda) \\ -R \sin(\lambda_0 + \lambda) & R \cos(\lambda_0 + \lambda) & H'(\lambda_0 + \lambda) \\ R \cos(\lambda_0 + \lambda) - R \cos(\lambda_0 - \lambda) & R \sin(\lambda_0 + \lambda) - R \sin(\lambda_0 - \lambda) & 0 \end{vmatrix} \\ &= R^2 [\cos(2\lambda) - 1] (H'(\lambda_0 + \lambda) + H'(\lambda_0 - \lambda)). \end{aligned} \quad (19)$$

Note that $H(\lambda) = H(2\lambda_0 - \lambda)$ and $H'(\lambda) + H'(2\lambda_0 - \lambda) = 0$. Therefore, we obtain that $H'(\lambda_0 - \lambda) + H'(\lambda_0 + \lambda) = 0$, and the triple product always equals zero when $\lambda \in [0, \Delta\lambda]$. That is, the curve $C(\lambda)$ is efficient. This finishes the proof.

Theorem 3 provides a sufficient condition for the efficient curves. Using theorems 2 and 3, it can be shown that the saddle curve is efficient. Since the filtering directions are independent of the reconstruction point $\vec{x}$, we can immediately obtain an efficient FBP algorithm characterized by the shift-invariant filtration of cone-beam projection data.

4. Analyses on FOV and temporal resolution

4.1. Field of view analysis

In the case of $2N + 1$ sources, when the sources and the detector arrays are equally spaced, the radius of the field of view (FOV) is

$$r = R \sin\left(\frac{\pi}{4N+2}\right), \quad (20)$$

which is the best choice to maximize the FOV (see figure 3). According to equation (20), the radius of FOV is decreased if the number of sources is increased. Hence, if we do not want to reduce FOV, we must enlarge the distance between the source and the object. This will decrease the effective x-ray flux and increase the gantry size, which is highly undesirable. With the Siemens SOMATOM volume zoom scanner as an example, the radii of the scanning circle and the FOV are 570 mm and 403 mm, respectively. Then, we can calculate the corresponding radii in a few representative cases by equation (20), as summarized in table 1. It is observed

that in the triple-source geometry, the FOV (radius of 285 mm) is sufficiently large to cover a normal patient's chest without any size increment of the scanning trajectory.

4.2. Temporal resolution analysis in the case of triple sources

In this case, the three x-ray sources are symmetrically positioned along a circle, and the detectors are opposite the corresponding sources. Note that the detector area should be sufficiently large to cover all the projection lines through an object to be reconstructed. Cone-beam projections are collected when the sources and detectors are, respectively, moved along the three saddles defined by

$$\begin{aligned} C_1(\lambda) &= (R \cos \lambda, R \sin \lambda, h \cos (2\lambda - 4\pi/3)), \\ C_2(\lambda) &= (R \cos \lambda, R \sin \lambda, h \cos (2\lambda - 2\pi/3)), \\ C_3(\lambda) &= (R \cos \lambda, R \sin \lambda, h \cos (2\lambda)), \end{aligned} \quad (21)$$

where λ is the angular parameter, and R and h are the radius and the pitch of each saddle, respectively.

In one period of the saddles, the curves intersect at 12 points $P_1, P_2, \dots, P_{12}$, as shown in figures 4 and 5. Let

$$\begin{aligned} \Lambda_1 &= \{C_1(8, 10)\} \cup \{C_2(10, 12)\} \cup \{C_3(12, 8)\}, \\ \Lambda_2 &= \{C_1(3, 5)\} \cup \{C_2(5, 1)\} \cup \{C_3(1, 3)\}, \\ \Lambda_3 &= \{C_1(11, 7)\} \cup \{C_2(7, 9)\} \cup \{C_3(9, 11)\}, \\ \Lambda_4 &= \{C_1(6, 2)\} \cup \{C_2(2, 4)\} \cup \{C_3(4, 6)\}, \end{aligned} \quad (22)$$

where $\Lambda_1, \Lambda_2, \Lambda_3$ and Λ_4 are the rebinned new trajectories, respectively, and for simplicity $C_1(8, 10)$ denotes $\{C_1(\lambda) | \lambda_{P8} < \lambda < \lambda_{P10}\}$, etc. All the rebinned curves are continuous and closed. Note that the scanning trajectories are periodic, and $\Lambda_1, \Lambda_2, \Lambda_3$ and Λ_4 will appear in turn.

The intersection points can be grouped into two sets, $\{P_1, P_2, P_3, P_4, P_5, P_6\}$ and $\{P_7, P_8, P_9, P_{10}, P_{11}, P_{12}\}$, each of which is characterized by an identical z -coordinate. The planes spanned by these sets divide the whole reconstructible volume into three parts, i.e. V_1, V_2 and V_3 , as illustrated in figure 4. Each rebinned curve covers two parts, as listed in table 2. In one period, V_1 and V_3 can be reconstructed twice while V_2 can be reconstructed four times. Also, V_2 appears in all four cases, and in one period the interval of the four phases of the object motion is the same, as shown in figure 6. In figure 6, suppose that the period is 1 s, i.e. the interval between two dashed lines is 1/12 s; then for V_1 and V_3 , the time interval is 1/3 s with a break time of 1/6 s, and for V_2 , the time interval is 1/3 s without any break time. This means that with triple-source saddle CT, we can continuously perform dynamic reconstruction for V_2 with three times better temporal resolution than with single-source saddle CT. Furthermore, with triple-source saddle CT, we can perform dynamic reconstruction in some specific time windows for V_1 and V_3 with three times better temporal resolution than with single-source saddle CT. More rebinned trajectories can be defined but the above conclusion on temporal resolution improvement remains the same.

Consider the plane $\Pi_{z_0} = \{(x, y, z) : z = z_0, z_0 \in [z_{\min}, z_{\max}]\}$, which intersects any $\Lambda \in \{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4\}$ at six points, where $z_{\min}, z_{\max}$ indicate the minimum and maximum values of the saddle in the z -direction. We denote these intersection points as $F_n, n = 1, 2, \dots, 6$, and let Ω_{z_0} be the convex polygon formed by them. By theorem 1, we have the reconstruction formula:

$$\begin{aligned}
 f(\vec{x}) = & \frac{1}{2} [\kappa(\vec{x}) \\
 & , \vec{e}_{F1,F2}, \lambda_{F1}, \lambda_{F2}) \\
 & + \kappa(\vec{x}) \\
 & , \vec{e}_{F2,F3}, \lambda_{F2}, \lambda_{F3}) \\
 & + \kappa(\vec{x}, \vec{e}_{F3,F4}, \lambda_{F3}, \lambda_{F4}) \\
 & + \kappa(\vec{x}) \\
 & , \vec{e}_{F4,F5}, \lambda_{F4}, \lambda_{F5}) \\
 & + \kappa(\vec{x}) \\
 & , \vec{e}_{F5,F6}, \lambda_{F5}, \lambda_{F6}) \\
 & + \kappa(\vec{x}) \\
 & , \vec{e}_{F6,F1}, \lambda_{F6}, \lambda_{F1} + 2\pi)].
 \end{aligned} \tag{23}$$

4.3. Temporal resolution analysis in the case of (2N + 1) sources

The main purpose of the multiple-source geometry is to achieve superior temporal resolution for dynamic imaging. Thus, it is important to analyze how the temporal resolution changes when the number of the sources increases. Recalling what we have done in the triple-source case, the original saddles are broken into many segments and then rebinned to new curves. The same strategies can be used in the case of (2N + 1) sources for $N \geq 2$.

For $N = 2$, we have five original saddles C_1, C_2, C_3, C_4, C_5 and eight rebinned continuous and closed curves $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, \Lambda_7, \Lambda_8$:

- $\Lambda_1: \{C_1\{(9, 30), (22, 3)\}; C_2\{(1, 22), (24, 5)\}; C_3\{(3, 24), (26, 7)\}; C_4\{(5, 26), (28, 9)\}; C_5\{(7, 28), (30, 1)\}\}$,
- $\Lambda_2: \{C_1\{(30, 22)\}; C_2\{(22, 24)\}; C_3\{(24, 26)\}; C_4\{(26, 28)\}; C_5\{(28, 30)\}\}$,
- $\Lambda_3: \{C_1\{(32, 13), (15, 36)\}; C_2\{(34, 15), (17, 38)\}; C_3\{(36, 17), (19, 40)\}; C_4\{(38, 19), (11, 32)\}; C_5\{(40, 11), (13, 34)\}\}$,
- $\Lambda_4: \{C_1\{(13, 15)\}; C_2\{(15, 17)\}; C_3\{(17, 19)\}; C_4\{(19, 11)\}; C_5\{(11, 13)\}\}$,
- $\Lambda_5: \{C_1\{(4, 25), (27, 8)\}; C_2\{(6, 27), (29, 10)\}; C_3\{(8, 29), (21, 2)\}; C_4\{(10, 21), (33, 14)\}; C_5\{(2, 23), (25, 6)\}\}$,
- $\Lambda_6: \{C_1\{(25, 27)\}; C_2\{(27, 29)\}; C_3\{(29, 21)\}; C_4\{(21, 23)\}; C_5\{(23, 25)\}\}$,
- $\Lambda_7: \{C_1\{(20, 31), (37, 18)\}; C_2\{(12, 33), (39, 20)\}; C_3\{(14, 35), (31, 12)\}; C_4\{(16, 37), (33, 14)\}; C_5\{(18, 39), (35, 16)\}\}$,
- $\Lambda_8: \{C_1\{(18, 20)\}; C_2\{(20, 12)\}; C_3\{(12, 14)\}; C_4\{(14, 16)\}; C_5\{(16, 18)\}\}$,

where $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, \Lambda_7$ and Λ_8 are the rebinned new trajectories, respectively, and for simplicity, $C_1(9, 30)$ denotes $\{C_1(\lambda) | \lambda_{P9} < \lambda < \lambda_{P30}\}$, etc. Note that the scanning trajectories are periodic, and $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, \Lambda_7$ and Λ_8 will appear in turn.

The intersection points can be grouped into four sets, $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}\}$, $\{P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}\}$, $\{P_{21}, P_{22}, P_{23}, P_{24}, P_{25}, P_{26}, P_{27}, P_{28}, P_{29}, P_{30}\}$ and $\{P_{31}, P_{32}, P_{33}, P_{34}, P_{35}, P_{36}, P_{37}, P_{38}, P_{39}, P_{40}\}$, each of which is characterized by an identical z -coordinate. The planes spanned by these sets divide the whole reconstructible volume into five parts, i.e., V_1, V_2, V_3, V_4 and V_5 , as illustrated in figure 7.

Similar to what we have done in the triple-source case, we can obtain the relationship between the rebinned curves and the reconstructible volume parts as listed in table 3.

According to table 3, V_1 and V_5 can be reconstructed only twice while V_2 , V_3 and V_4 can be reconstructed four times. The result is quite similar to that in the triple-source case. The only difference is that V_1 here is smaller than that in the triple-source case, and V_5 here is also smaller than V_4 there. In fact, if we use seven sources or more, the conclusion remains the same. Thus, the increment of the source number does not improve the temporal resolution except for a small fraction of the volume.

Also, V_2 , V_3 and V_4 appear over the whole time axis, as shown in figure 8. Although in one period, all of these three parts have four phases, the time intervals for them are not the same. In figure 8, suppose that the interval between two dashed lines is 0.05 s; then for V_1 and V_5 , the time interval is 0.2 s with break time 0.3 s, and for V_2 and V_4 , the time intervals are 0.4 s and 0.2 s without any break time, and for V_3 , the time interval is 0.4 s without any break time. Remember that the shorter the time interval, the higher the temporal resolution will be. Interestingly, for V_2 and V_4 , the time intervals are shorter in one phase but longer in the next phase than the counterparts in the triple-source case; for V_3 , the time intervals are longer than that in the triple-source case. In general, the temporal resolution does not become better in the quintuple-source case than in the triple-source case. In fact, V_2 and V_4 will become smaller and smaller when the number of source increases and V_3 will be the major part of the object. Unlike in the triple-source case, the segment we select in a single curve is no longer continuous, i.e., $C_1\{(9, 30), (22, 3)\}$ in Λ_1 . We do this because the segment $C_1\{(22, 24)\}$ is redundant data for Λ_1 . However, when we calculate the time interval for a rebinned curve, such redundant segments must be taken into consideration since the x-ray source is still on for this segment.

Thus, the time interval is $\frac{N}{2N+1}$, which means if a single x-ray source finishes one saddle in 1 s (for a single source, one complete saddle is required to perform a fast FBP reconstruction), then it will take $\frac{N}{2N+1}$ s for $2N+1$ sources to acquire sufficient projections for a fast FBP reconstruction. As $\frac{N}{2N+1}$ increases monotonically (with a limit 0.5), the minimum 1/3 is reached at $N=1$.

Recall that in the $(2N+1)$ -source helical CT case (Zhao *et al* 2006, 2009), the more sources, the higher the temporal resolution. Very interestingly, in the $(2N+1)$ -source saddle CT case, the best temporal resolution for continuous dynamic reconstruction of the central volume is achieved by triple-source saddle CT. As the number of sources is increased beyond three, the temporal resolution for continuous dynamic reconstruction of the central volume is decreased, although at the two end-parts of the object the temporal resolution within some time windows is indeed increased, and outside the time windows exact reconstruction cannot be achieved. For example, with quintuple-source saddle CT, we can continuously perform dynamic reconstruction for V_2 , V_3 , V_4 with 2.5 times better temporal resolution than with single-source saddle CT, and for V_1 and V_5 within some time windows with five times better temporal resolution than with single-source saddle CT. Usually, we will not improve temporal resolution for a small portion of an object in an intermittent way. Therefore, the triple-source geometry is the best in our context.

5. Numerical simulation

5.1. Filtering lines

Let

$$\begin{aligned}
\vec{e}_u(\lambda) &= (-\sin\lambda, \cos\lambda, 0), \\
\vec{e}_v(\lambda) &= (0, 0, 1), \\
\vec{e}_w(\lambda) &= (-\cos\lambda, -\sin\lambda, 0),
\end{aligned} \tag{24}$$

be a local coordinate system with cone-beam projection data measured on the detector plane parallel to $\vec{e}_u(\lambda)$ and $\vec{e}_v(\lambda)$ at a distance D from $\vec{a}(\lambda)$. Any position on the detector plane is specified by Cartesian coordinates (u, v) , which are signed distances along $\vec{e}_u(\lambda)$ and $\vec{e}_v(\lambda)$, respectively, and $(0, 0)$ denotes the orthogonal projection of $\vec{a}(\lambda)$ onto the detector plane. (u, v) can also be expressed as $\vec{x} = (x_x, x_y, x_z)$. Let $\vec{a} = (R \cos \lambda, R \sin \lambda, H(\lambda))$ be any point on the scanning trajectory and $\vec{n}_0 = (0, 0, H(\lambda) + \hat{z})$ a point where the filtering plane $\Pi(\lambda, \hat{z})$ intersects with the z -axis. Let $\vec{n} = (n_x, n_y, n_z)$ be any vector in the space and $\vec{e} = (e_x, e_y, 0)$ be the filtering direction in the x - y plane.

Supposing that $\vec{n} - \vec{n}_0$ is the normal vector of the filtering plane $\Pi(\lambda, \hat{z})$, we have

$$(\vec{n} - \vec{n}_0) \cdot (\vec{x} - \vec{n}_0) = 0 \Rightarrow x_x n_x + x_y n_y + (n_z - H(\lambda) - \hat{z})(x_z - H(\lambda) - \hat{z}) = 0. \tag{25}$$

Recall that the filtering plane $\Pi(\lambda, \hat{z})$ passes through $\vec{a} - \vec{n}_0$ and is parallel to the filtering direction $\vec{e}$. Therefore,

$$(\vec{n} - \vec{n}_0) \cdot \vec{e} = 0 \Rightarrow n_x \cdot e_x + n_y \cdot e_y = 0, \tag{26}$$

$$(\vec{n} - \vec{n}_0) \cdot (\vec{a} - \vec{n}_0) = 0 \Rightarrow R \cos \lambda \cdot n_x + R \sin \lambda \cdot n_y = \hat{z}(n_z - H(\lambda) - \hat{z}). \tag{27}$$

According to our detector setup, any point on the detector plane can be expressed as

$$\begin{cases} x_x = (R - D) \cos \lambda - u \sin \lambda \\ x_y = (R - D) \sin \lambda - u \cos \lambda \\ x_z = H(\lambda) + v. \end{cases} \tag{28}$$

Inserting equations (26)–(28) into equation (25), we have

$$v(R \sin \lambda \cdot e_x - R \cos \lambda \cdot e_y) = \hat{z}[(D \sin \lambda - u \cos \lambda) \cdot e_x - (D \cos \lambda + u \sin \lambda) \cdot e_y], \tag{29}$$

which describes the filtering lines on the detector plane. Figure 9 shows the filtering lines on the detector plane for two source positions.

5.2. Algorithm description

In the triple-source case, the algorithm can be implemented as follows:

Step 1: Rearrange the saddles using equation (22).

Step 2: Choose one dataset and determine the filtering directions.

For example, for the rebinned curve Λ_1 , the filtering directions are $\vec{e}_{1,2}, \vec{e}_{2,3}, \vec{e}_{3,4}, \vec{e}_{4,5}, \vec{e}_{5,6}, \vec{e}_{6,1}$, as shown in figure 10.

Step 3: Compute the derivative data for every projection:

$$g_1(\lambda, u, v) = \left(\frac{\partial}{\partial \lambda} + \frac{D'(\lambda)u + u^2 + D^2(\lambda)}{D(\lambda)} \frac{\partial}{\partial u} + \frac{D'(\lambda)v + uv}{D(\lambda)} \frac{\partial}{\partial v} \right) g_f(\lambda, u, v). \quad (30)$$

Step 4: Perform the filtration:

Step 4.1: Pre-weighting:

$$g_2(\lambda, u, v) = \frac{D(\lambda)}{\sqrt{D^2(\lambda) + u^2 + v^2}} g_1(\lambda, u, v). \quad (31)$$

Step 4.2: Forward height-based rebinning:

$$g_3(\lambda, u, \widehat{z}) = g_2(\lambda, u, v(u, \widehat{z})). \quad (32)$$

In table 4, $\lambda_2, \lambda_4, \lambda_6$ are points where Λ_1 reaches its maximum value along the z -axis while $\lambda_1, \lambda_3, \lambda_5$ are points where Λ_1 takes its minimum value along the z -axis.

Step 4.3: Hilbert transform in u :

$$g_4(\lambda, u, \widehat{z}) = \int_{-\infty}^{\infty} du' h_H(u - u') g_3(\lambda, u, \widehat{z}), \quad (33)$$

where h_H is the kernel of the Hilbert transform.

Step 4.4: Backward height-based rebinning:

$$g_5(\lambda, u, v) = g_4(\lambda, u, \widehat{z}(u - v)), \quad (34)$$

where

$$v(R \sin \lambda \cdot e_x - R \cos \lambda \cdot e_y) = \widehat{z}[(D \sin \lambda - u \cos \lambda) \cdot e_x - (D \cos \lambda + u \sin \lambda) \cdot e_y], \quad (35)$$

e_x and e_y are the projections of the filtering direction $\vec{e}$ on the x -axis and y -axis, respectively. The filtering directions for different positions are listed in table 4.

Step 4.5: Post-weighting:

$$g_f^F(\lambda, u, v) = \frac{\sqrt{D^2(\lambda) + u^2 + v^2}}{D(\lambda)} g_5(\lambda, u, v). \quad (36)$$

Step 5: Perform the backprojection:

$$f(\vec{x}) = - \frac{1}{4\pi^2} \int_{\lambda_s}^{\lambda_s + 2\pi} d\lambda \frac{1}{\|\vec{x} - \vec{d}(\lambda)\|} g_f^F(\lambda, u, v). \quad (37)$$

For the rebinned curves Λ_2, Λ_3 and Λ_4 , the steps are essentially the same. If the object is stationary, one dataset is theoretically sufficient for exact image reconstruction. If the object changes over time, the four datasets should be used in turn for dynamic image reconstruction.

5.3. Numerical results

The performance of the proposed algorithm was numerically tested in the static and dynamic cases. As expected, the artifacts were significant when an object was dynamic. A conventional solution is to perform a motion correction in image and projection domains (Grangeat *et al* 2002, Li *et al* 2006, Yu and Wang 2007). On the other hand, the motion artifacts can be reduced if the scanning time is shortened (Flohr *et al* 2007, Yin and De Man 2007). Our triple-source geometry reduces the scanning time by 1/3 of that with the single-source geometry. In the dynamic case, we compared these two geometries in a numerical study.

The key simulation parameters are listed in table 5. In the static case, the Shepp–Logan phantom was used; the reconstructed slice and its representative profile in figure 11 show that our proposed algorithm performed very well. In the dynamic case, the clock phantom (Turbell and Danielsson 2000) was used. For comparison, two simulated reconstructions were performed using single-source cone-beam scanning along a saddle trajectory (Yang's algorithm) and triple-source cone-beam scanning along helices (Zhao's algorithm, Zhao *et al* 2006), as shown in figures 12(a) and (b). The rotation speed of the x-ray source was 1 turn per second while the phantom was counterclockwise rotated by 10° per second. Figures 12(c), (d), (e) and (f) show the four reconstructed images corresponding to the time intervals 0–0.33 s, 0.25–0.58 s, 0.5–0.83 s and 0.75–1.08 s, respectively. To indicate the rotation direction of the phantom, figure 12(g) presents the difference between figures 12(f) and (c).

Evidently, the images reconstructed in the triple-source geometry, whether along saddle curves or helical trajectories, were much better than those in the single-source geometry. Also, the images reconstructed using Zhao's algorithm looked quite similar to those using our algorithm we have proposed here. Interestingly, in Zhao's algorithm there were three backprojection segments for each point in an ROI, which were located on the different helices and determined by three inter-helix PI-lines. In our experiment, it took less than 1/3 s to collect sufficient projections to reconstruct a single point; thus the motion artifacts were greatly reduced. However, in Zhao's algorithm the combination of backprojection segments for each point was unique. To reconstruct the entire volume, we would need all the cone-beam projections collected along the trajectories. That is why we can only reconstruct one image from a full-scan dataset using Zhao's algorithm. In contrast, from a full-scan dataset, four images in the four time intervals can be reconstructed using the algorithm we have proposed here.

6. Discussions and conclusions

While image artifacts are a main problem with circular cone-beam CT, saddle-curve scanning offers an attractive solution. For example, the Toshiba Aquiline One scanner utilizes 320 detector rows (0.5 mm width) to cover up to 16 cm longitudinally, which is sufficient to capture the entire heart or brain and show the physiological and pathological dynamics continuously (Rybicki *et al* 2008). Thus, it can greatly reduce the diagnostic imaging time and replace comprehensive exams and invasive procedures with a single scan. Unfortunately, well-known cone-beam artifacts are significant. The severity of these artifacts increases as a function of the distance from the mid-plane and may introduce substantial reconstruction errors and simulate/hide critical features. In addition to the above-discussed temporal resolution gain, our proposed cone-beam scanning along multi-saddle trajectories seems an ideal solution.

One concern with triple-source CT is that the improvement in temporal resolution may be offset by the increment in scattering artifacts. In fact, there were similar arguments against the dual-source system in comparison with the single-source counterpart prior to the introduction of the Siemens definition dual-source CT scanner. Actually, the Siemens dual-source CT scanner has received a very positive market response, which is encouraging for our proposed triple-source extension. The Siemens dual-source CT scanner has been claimed to have a dose benefit

because the second detector is smaller, and the dose is reduced at the edge of the field of view. The dual-source CT scanner can double the power for cardiac applications without any compromise in image quality. It has been reported that cross scatter is smaller with the decreased collimation z -width or the decreased object size for dual-source CT (Kyriakou and Kalender 2007). Also, a potential strategy is to utilize shutters so that at any time instant the object is only exposed to one or two sources since the shutters can be rapidly opened in an alternative fashion. These guidelines should be valuable for optimization of triple-source CT. Furthermore, the recently developed interior tomography technology can significantly narrow the cone-beam aperture, thus reducing the scatter problem effectively (Ye *et al* 2007, Kudo *et al* 2008).

Another issue to be analyzed is the detector size for triple-source cone-beam CT. Briefly speaking, the necessary height of the detector remains the same as discussed by Yang *et al* (2006b). The width of the flat detector or the fan angle of the cylinder detector cannot be larger than 60° due to the angular limitation of the scanning mode. This may result in transverse data truncations when a patient is too large. Conventionally, the detector must cover all the projections through the patient or he/she cannot be exactly reconstructed. Again, interior tomography is being developed to allow projections to be truncated on one side or both sides (Ye *et al* 2007, Kudo *et al* 2008). Even when an ROI is entirely inside the chest, it can now be exactly reconstructed provided *a priori* knowledge is known on a sub-region in the ROI such as blood density in the aorta. With this new approach, triple-source CT can perform well with quite flexible data truncation to minimize radiation dose and scattering artifacts.

In conclusion, we have derived a necessary condition and a sufficient condition for construction of efficient curves so that the exact and efficient FBP algorithm can be easily designed for clinical applications. Also, we have derived a new exact and efficient algorithm for triple-source saddle-curve cone-beam CT. It reduces the scanning time, thus suppressing the motion artifacts. Finally, we have shown that triple-source saddle-curve cone-beam CT is the optimal solution in terms of FOV, hardware cost and temporal resolution among all multi-source saddle-curve cone-beam scanning modes. Therefore, we believe that triple-source saddle-curve cone-beam CT is particularly promising in reconstructing a localized dynamic volume with a beating heart as a primary example.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China (30570511 and 30770589), National High Technology Research and Development Program of China (863 Program) (2007AA02Z452), and NIH/NIBIB (EB002667 and EB004287).

References

- Flohr TG, McCollough CH, Bruder H, Petersilka M, Gruber K, Süß C, Grasruck M, Stierstorfer K, Krauss B, Raupach R. First performance evaluation of a dual-source CT (DSCT) system. *Eur. Radiol* 2006;16:256–268. [PubMed: 16341833]
- Flohr T, Schoepf U, Ohnesorge B. Chasing the heart: new developments for cardiac CT. *J. Thorac. Imaging* 2007;22:4. [PubMed: 17325571]
- Grangeat P, Koenig A, Rodet T, Bonnet S. Theoretical framework for a dynamic cone-beam reconstruction algorithm based on a dynamic particle model. *Phys. Med. Biol* 2002;47:2611–2625. [PubMed: 12211208]
- Kalender WA. Thin-section three-dimensional spiral CT: is isotropic imaging possible? *Radiology* 1995;197:578–580. [PubMed: 7480719]
- Katsevich A. Theoretically exact filtered backprojection-type inversion algorithm for spiral CT. *SIAM J. Appl. Math* 2002;62:2012.

- Katsevich A. A general scheme for constructing inversion algorithms for cone beam CT. *Int. J. Math. Math. Sci* 2003;21:1305–1321.
- Kudo H, Courdurier M, Noo F, Defrise M. Tiny *a priori* knowledge solves the interior problem in computed tomography. *Phys. Med. Biol* 2008;53:2207–2231. [PubMed: 18401067]
- Kyriakou Y, Kalender W. Intensity distribution and impact of scatter for dual-source CT. *Phys. Med. Biol* 2007;52:6969. [PubMed: 18029988]
- Li T, Schreiber E, Yang Y, Xing L. Motion correction for improved target localization with on-board cone-beam computed tomography. *Phys. Med. Biol* 2006;51:253. [PubMed: 16394337]
- Lu Y, Zhao J, Wang G. Exact image reconstruction for triple-source cone-beam CT along saddle trajectories. *Proc. SPIE*. 2008 70780K.
- Orlov SS. Theory of three-dimensional reconstruction: 1. Conditions of a complete set of projections. *Sov. Phys. Crystallogr* 1975;20:312–314.
- Pack JD, Noo F. Cone-beam reconstruction using 1D filtering along the projection of M-lines. *Inverse Problems* 2005;21:1105.
- Robb RA, Hoffman EA, Sinak LJ, Harris LD, Ritman EL. High-speed three-dimensional x-ray computed tomography: the dynamic spatial reconstructor. *Proc. IEEE* 1983;71:308–319.
- Rybicki FJ, Otero HJ, Steigner ML, Vorobiof G, Nallamshetty L, Mitsouras D, Ersoy H, Mather RT, Judy PF, Cai T. Initial evaluation of coronary images from 320-detector row computed tomography. *Int. J. Cardiovasc. Imaging (formerly Card. Imaging)* 2008;24:535–546.
- Taguchi K, Aradate H. Algorithm for image reconstruction in multi-slice helical CT. *Med. Phys* 1998;25:550. [PubMed: 9571623]
- Toponogov, V. *Differential Geometry of Curves and Surfaces: A Concise Guide*. Boston, MA: Birkhauser; 2006.
- Turbell H, Danielsson PE. Helical cone-beam tomography. *Int. J. Imaging Syst. Technol* 2000;11:91–100.
- Wang G, Crawford CR, Kalender WA. Guest editorial—Multirow detector and cone-beam spiral/helical CT. *IEEE Trans. Med. Imaging* 2000;19:817–821. [PubMed: 11127597]
- Wang G, Lin TH, Cheng PC, Shinozaki DM, Kim HG. Scanning cone-beam reconstruction algorithms for x-ray microtomography. *Proc. SPIE* 1991;1556:99.
- Wang G, Yu H, De Man B. An outlook on x-ray CT research and development. *Med. Phys* 2008;35:1051. [PubMed: 18404940]
- Yang, H.; Li, M.; Koizumi, K.; Kudo, H. Application of pack and Noo's cone-beam inversion formula to a wide class of trajectories; *IEEE Nucl. Sci. Symp. Conf. Record*; 2006a.
- Yang H, Li M, Koizumi K, Kudo H. Exact cone beam reconstruction for a saddle trajectory. *Phys. Med. Biol* 2006b;51:1157. [PubMed: 16481685]
- Yin, Z.; De Man, B. 3D analytic cone-beam reconstruction for less than a full scan using a multi-source CT system; *IEEE Nucl. Sci. Symp. Conf. Record*; 2007.
- Ye Y, Wang G. Filtered backprojection formula for exact image reconstruction from cone-beam data along a general scanning curve. *Med. Phys* 2005;32:42. [PubMed: 15719953]
- Ye Y, Yu H, Wei Y, Wang G. A general local reconstruction approach based on a truncated Hilbert transform. *Int. J. Biomed. Imaging* 2007;2007:63634. [PubMed: 18256734]
- Yu H, Wang G. Data consistency based rigid motion artifact reduction in fan-beam CT. *IEEE Trans. Med. Imaging* 2007;26:249. [PubMed: 17304738]
- Zhao J, Jiang M, Zhuang T, Wang G. An exact reconstruction algorithm for triple-source helical cone-beam CT. *J. X-Ray Sci. Technol* 2006;14:191.
- Zhao J, Jin Y, Lu Y, Wang G. A filtered backprojection algorithm for triple-source helical cone-beam CT. *IEEE Trans. Med. Imaging* 2009;28:384–393. [PubMed: 19244010]

Figure 1.

Analysis on the computational efficiency. The detector coordinates v and u are used with $(u, v) = (0, 0)$ being the orthogonal projection of $\vec{a}(\lambda)$ onto the detector plane. (a) l_a and l_b are parallel lines on the reconstructed plane Π_0 , and they correspond to different filtering lines l_{ha} and l_{hb} on the detector. (b) Π_1 and Π_2 are two planes perpendicular to the z -axis, and lines l_1 and l_2 on different planes have the same filtering line l_h on the detector with respect to a fixed source point $\vec{a}(\lambda)$.

Figure 2.
Filtering lines on an efficient curve.

Figure 3.
FOV determined by the number of the sources.

Figure 4.

Distribution of the 12 intersection points on the unraveled cylinder scanning surface. In the 2D view, a saddle is simply a sinusoid curve, and $\lambda^T = \lambda \bmod (2\pi)$. The solid, dashed and dotted lines represent different saddles, respectively.

Figure 5.

Rebinned curves in 3D views. The solid lines in (a), (b), (c) and (d) highlight the rebinned curves $\Lambda_1, \Lambda_2, \Lambda_3$ and Λ_4 , respectively. The dotted, dash dotted with '+' marks and dashed curves represent saddles C_1, C_2 and C_3 , respectively.

Figure 6. Distributions of rebinned curves and reconstructible partial volumes in the case of triple sources. The horizontal axis is time. Vertical dashed lines divide each cycle into equal intervals. The gray boxes above the time axis indicate the minimum time required to form the rebinned curves, and those below the time axis show the partial volumes that can be reconstructed in these time intervals.

Figure 7.

Intersection points of the quintuple-source saddle trajectories on the unraveled cylinder scanning surface. In the 2D view, a saddle is simply a sinusoid curve, and $\lambda^T = \lambda \bmod (2\pi)$. The lines with different patterns represent different saddles.

Figure 8.

Distributions of rebinned curves and reconstructible partial volumes in the quintuple-source case. The horizontal axis is for time. Vertical dashed lines divide each cycle into equal intervals. The gray boxes above the time axis indicate the minimum time required to form the rebinned curves, and those below the time axis show the partial volumes that can be reconstructed in these time intervals.

Figure 9. Filtering lines on the detector plane for two source positions. (a) $\lambda = 0.83 \pi$ and (b) $\lambda = 1.27 \pi$ in Λ_4 .

Figure 10.
Illustration of the filtering directions.

Figure 11. Reconstructed image of the Shepp-Logan phantom on the plane $z = -15$ mm and its profile along the line $y = 4.9$ mm. The display window is $[1.0, 1.05]$.

Figure 12.

Reconstructed images of the clock phantom on the plane $z = 0$. (a) The image reconstructed using Yang's algorithm (Yang *et al* 2006b); (b) the image reconstructed using Zhao's algorithm (Zhao *et al* 2006); (c), (d), (e) and (f) the images reconstructed using our algorithm over the time intervals $[0, 0.33]$ s, $[0.25, 0.58]$ s, $[0.5, 0.83]$ s and $[0.75, 1.08]$ s, respectively, and (g) the difference between (f) and (c). The display window is $[0.5, 1.5]$ for (b)–(f), and $[-0.5, 0.5]$ for (g).

Table 1Radii of the scanning circle and FOV for $2N + 1$ sources.

	$N = 1$	$N = 2$	$N = 3$	$N = 4$
Radius (mm) of the scanning circle for the current FOV size	806	1304	1811	2321
Enlargement of the scanning trajectory (%)	41.4	128	217	307
Radius of the FOV (mm) for the same scanning trajectory	285	176	127	99.0
Reduction of the FOV (%)	29.3	56.3	68.6	75.5

Table 2

Reconstructible partial volumes according to the different rebinned curves in the triple-source case.

Rebinned curve	Reconstructible volume
Λ_1	V_1, V_2
Λ_2	V_2, V_3
Λ_3	V_1, V_2
Λ_4	V_2, V_3

Table 3
Reconstructible partial volumes according to the rebinned curves in the quintuple-source case.

Rebinned curve	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8
Reconstructible partial volume	V_2, V_3	V_4, V_5	V_3, V_4	V_1, V_2	V_2, V_3	V_4, V_5	V_3, V_4	V_1, V_2

Table 4

Selection of filtering directions for efficient FBP reconstruction.

	$\lambda \in (\lambda_1, \lambda_2]$	$\lambda \in (\lambda_2, \lambda_3]$	$\lambda \in (\lambda_3, \lambda_4]$	$\lambda \in (\lambda_4, \lambda_5]$	$\lambda \in (\lambda_5, \lambda_6]$	$\lambda \in (\lambda_6, \lambda_1 + 2\pi]$
$\hat{z} < 0$	$\vec{e}_{1,2}$	$\vec{e}_{3,4}$	$\vec{e}_{3,4}$	$\vec{e}_{3,4}$	$\vec{e}_{5,6}$	$\vec{e}_{5,6}$
$\hat{z} \geq 0$	$\vec{e}_{6,1}$	$\vec{e}_{2,3}$	$\vec{e}_{2,3}$	$\vec{e}_{4,5}$	$\vec{e}_{4,5}$	$\vec{e}_{6,1}$

Table 5

Parameters used in the numerical simulation.

Scanning radius	200 mm
Spherical support radius	60 mm
Source to detector distance	200 mm
Detector size	125 mm $\times$ 125 mm
Number of detector rows	500
Number of pixels per row	500
Pixel size	0.25 mm $\times$ 0.25 mm
Number of projections per turn	1200
Saddle pitch	30 mm
Image matrix	513 $\times$ 513 $\times$ 513