Published in final edited form as:

Stat Methodol. 2010 May 1; 7(3): 277–291. doi:10.1016/j.stamet.2009.10.004.

Correlational biases in mean response latency differences[☆]

N. Sriram^{a,*}, Anthony G. Greenwald^b, and Brian A. Nosek^a

^a University of Virginia, United States

^b University of Washington, United States

Abstract

Multifarious psychological constructs are indexed by the mean latency difference (MLD), the within-subject difference between mean response latency on two tasks. Two associations consistently emerge in mean latency data. Firstly, across subjects, mean latencies on distinct tasks are positively correlated. This correlation arises from individual differences in general rates of information processing that are a shared influence on response latency in diverse tasks. Secondly, across tasks, the mean and variance of mean latency are positively correlated. Compared to a simple task, a complex task has both a larger average mean latency and a larger variance of mean latency, across subjects. Taken together, these associations make the interpretation of the MLD problematic by biasing correlations between the MLD and (a) task mean latencies, (b) the average of the mean latencies, (c) external criteria, and (d) other MLDs. A variety of mean latency transformations were evaluated and, while they differed in their effectiveness, they did not satisfactorily rectify MLD biases. An alternative approach, focusing on scale invariant contrasts of within-subject response latency distributions, is introduced in the conclusion.

Keywords

Latency; Difference; Correlation; Bias; Transformation

With a history of over 100 years, the measure of choice reaction time maintains an important role in the modeling of mental processes. In mental chronometry, the focus of interest is often the within-individual contrast between mean response latencies on two distinct tasks. The mean latency difference (MLD) is assumed to reflect the additional processing demands in one condition, relative to a contrasted condition [20]. MLD scores operationalize measures in many paradigms including the Stroop task [16], semantic priming [21], task switching [17], evaluative priming [9], lexical decisions [1], and attentional networks [8].

Despite the advantageous within-individual comparison, problematic aspects of the MLD were recognized in the aging literature [24,26] where older and younger subjects are frequently compared. While older subjects exhibit larger MLDs than younger subjects, they also tend to have larger mean latencies. MLD differences between younger and older subjects may reflect actual group differences in the processing demands of the two conditions, or be an artifact of the overall slower and more variable responding of older adults, or an admixture of the two.

[☆]This research was supported by Project Implicit, by NIH Grant R01 MH68447 and NSF Grant REC-0634041 to University of Virginia, and by the Implicit Cognition Research Fund at University of Washington.

* Corresponding address: Department of Psychology, University of Virginia, Charlottesville, VA 22904, United States. nsriram@virginia.edu (N. Sriram).

From a general perspective, mean response latencies are influenced by (i) subject characteristics, (ii) task demands, and (iii) the interaction of subject characteristics with task demands. In the rate amount framework [10], tasks vary in their information processing amount, individuals differ in their information processing rate, and mean response latency is the product of information amount and the reciprocal of information processing rate. This framework provides a general, task independent, account of variability in mean response latencies, with minimal theoretical commitments. It implies a positive correlation between the mean and standard deviation of mean latencies in that tasks associated with longer mean latencies also have a larger variance of mean latency across subjects. It also implies that individual differences in information processing rates will cause mean latencies to be positively correlated with each other.

This paper illustrates the consequences of these associations for the MLD. We develop a method of analysis to show that these associations bias correlations involving the MLD and make its interpretation problematic. Further, the analysis also shows that a variety of transformations fail to ameliorate these biases. Finally, we briefly introduce an alternative approach that is based on scale invariant contrasts of within-subject response latency distributions.

1. Method of analysis

In this section we develop the methodology for evaluating the properties of correlations involving the MLD. We obtain a general expression for the correlation between two linear combinations. This expression is a function of several parameters, including standard deviation ratios and correlations. Expressions for specific MLD correlations are obtained by fixing certain parameters to appropriate constants. The domain of latency data is defined in terms of the remaining free parameters. MLD correlations are analyzed, within the latency data domain, via visual function plots and linear models that estimate the marginal effect of each free parameter.

1.1. The correlation between linear combinations

Correlations involving the MLD are special cases of the correlation between L_1 and L_2 , linear combinations of random variable pairs (X_1, Y_1) and (X_2, Y_2) that are weighted by coefficients (α_1, β_1) and (α_2, β_2) .

$$L_1 = \alpha_1 X_1 + \beta_1 Y_1, \quad L_2 = \alpha_2 X_2 + \beta_2 Y_2 \quad (1)$$

$$\text{Define } \lambda_1 = \frac{\sigma_{Y_1}}{\sigma_{X_1}}, \quad \lambda_2 = \frac{\sigma_{Y_2}}{\sigma_{X_2}}, \quad \lambda_X = \frac{\sigma_{X_2}}{\sigma_{X_1}} \quad (2)$$

$$\sigma_{Y_1} = \lambda_1 \sigma_{X_1}, \quad \sigma_{X_2} = \lambda_X \sigma_{X_1}, \quad \sigma_{Y_2} = \lambda_2 \lambda_X \sigma_{X_1} \quad (3)$$

$r_{X_1 Y_1}, r_{X_2 Y_2}, r_{X_1 X_2}, r_{Y_1 Y_2}, r_{X_1 Y_2}, r_{X_2 Y_1}$ are the correlations among X_1, Y_1, X_2, Y_2 .

$$\sigma_{L_1 + L_2}^2 = \sigma_{L_1}^2 + \sigma_{L_2}^2 + 2\sigma_{L_1} \sigma_{L_2} r_{L_1 L_2} \quad (4)$$

$$\sigma_{L_1}^2 = \sigma_{a_1 X_1 + \beta_1 Y_1}^2 = \alpha_1^2 \sigma_{X_1}^2 + \beta_1^2 \sigma_{Y_1}^2 + 2\alpha_1 \beta_1 \sigma_{X_1} \sigma_{Y_1} r_{X_1 Y_1} = \sigma_{X_1}^2 (\alpha_1^2 + \beta_1^2 \lambda_1^2 + 2\alpha_1 \beta_1 \lambda_1 r_{X_1 Y_1}) \quad (5)$$

$$\begin{aligned} \sigma_{L_2}^2 &= \sigma_{a_2 X_2 + \beta_2 Y_2}^2 = \alpha_2^2 \sigma_{X_2}^2 + \beta_2^2 \sigma_{Y_2}^2 + 2\alpha_2 \beta_2 \sigma_{X_2} \sigma_{Y_2} r_{X_2 Y_2} \\ &= \sigma_{X_2}^2 \lambda_X^2 (\alpha_2^2 + \beta_2^2 \lambda_2^2 + 2\alpha_2 \beta_2 \lambda_2 r_{X_2 Y_2}). \end{aligned} \quad (6)$$

Substituting (5) and (6) in (4),

$$\sigma_{L_1+L_2}^2 = \frac{\sigma_{X_1}^2 (\alpha_1^2 + \beta_1^2 \lambda_1^2 + 2\alpha_1 \beta_1 \lambda_1 r_{X_1 Y_1}) + \sigma_{X_2}^2 \lambda_X^2 (\alpha_2^2 + \beta_2^2 \lambda_2^2 + 2\alpha_2 \beta_2 \lambda_2 r_{X_2 Y_2})}{+ 2\sigma_{X_1}^2 \lambda_X \sqrt{(\alpha_1^2 + \beta_1^2 \lambda_1^2 + 2\alpha_1 \beta_1 \lambda_1 r_{X_1 Y_1}) (\alpha_2^2 + \beta_2^2 \lambda_2^2 + 2\alpha_2 \beta_2 \lambda_2 r_{X_2 Y_2})} r_{L_1 L_2}} \quad (7)$$

$$\begin{aligned} \sigma_{L_1+L_2}^2 &= \sigma_{a_1 X_1 + a_2 X_2 + \beta_1 Y_1 + \beta_2 Y_2}^2 \\ &= \alpha_1^2 \sigma_{X_1}^2 + \alpha_2^2 \sigma_{X_2}^2 + \beta_1^2 \sigma_{Y_1}^2 + \beta_2^2 \sigma_{Y_2}^2 + 2\alpha_1 \alpha_2 \sigma_{X_1} \sigma_{X_2} r_{X_1 X_2} + 2\alpha_1 \beta_1 \sigma_{X_1} \sigma_{Y_1} r_{X_1 Y_1} \\ &\quad + 2\alpha_1 \beta_2 \sigma_{X_1} \sigma_{Y_2} r_{X_1 Y_2} + 2\alpha_2 \beta_1 \sigma_{X_2} \sigma_{Y_1} r_{X_2 Y_1} + 2\alpha_2 \beta_2 \sigma_{X_2} \sigma_{Y_2} r_{X_2 Y_2} + 2\beta_1 \beta_2 \sigma_{Y_1} \sigma_{Y_2} r_{Y_1 Y_2}. \end{aligned} \quad (8)$$

Equating (7) with (8) and eliminating common terms,

$$\begin{aligned} &\sqrt{(\alpha_1^2 + \beta_1^2 \lambda_1^2 + 2\alpha_1 \beta_1 \lambda_1 r_{X_1 Y_1}) (\alpha_2^2 + \beta_2^2 \lambda_2^2 + 2\alpha_2 \beta_2 \lambda_2 r_{X_2 Y_2})} r_{L_1 L_2} \\ &= \alpha_1 \alpha_2 r_{X_1 X_2} + \alpha_1 \beta_2 \lambda_2 r_{X_1 Y_2} + \alpha_2 \beta_1 \lambda_1 r_{X_2 Y_1} + \beta_1 \beta_2 \lambda_1 \lambda_2 r_{Y_1 Y_2} \end{aligned} \quad (9)$$

$$r_{L_1 L_2} = \frac{\alpha_1 \alpha_2 r_{X_1 X_2} + \alpha_1 \beta_2 \lambda_2 r_{X_1 Y_2} + \alpha_2 \beta_1 \lambda_1 r_{X_2 Y_1} + \beta_1 \beta_2 \lambda_1 \lambda_2 r_{Y_1 Y_2}}{\sqrt{(\alpha_1^2 + \beta_1^2 \lambda_1^2 + 2\alpha_1 \beta_1 \lambda_1 r_{X_1 Y_1}) (\alpha_2^2 + \beta_2^2 \lambda_2^2 + 2\alpha_2 \beta_2 \lambda_2 r_{X_2 Y_2})}} \quad (10)$$

The correlation between two linear combinations is a function of four coefficients, two standard deviation ratios, and six inter-variable correlations. It does not depend on the variances of the random variables, is independent of λ_X , and applies to all distributional forms of the random variables.

1.2. Empirical parameter ranges

Mean latency in the task with slower responses is identified as Y and mean latency in the task

with faster responses is X , $Y - X \geq 0$, $\lambda = \frac{\sigma_Y}{\sigma_X}$. Faust et al. [10] found that the correlations among mean response latencies were consistently positive. The first unrotated principal component extracted by PCA, identified as the “general information processing rate”, typically accounted for over two thirds of the overall variance. In the present analyses the correlation between mean latencies r_{XY} is placed in $[0, 0.9]$, a range that covers all but the most extremely positive correlations.

In a meta-analysis covering old and young subject samples, Hale et al. [12] reported a correlation of 0.93 between the group mean RT and the SD of the mean RT . This correlation was similar even when the old and young samples were considered separately, suggesting that the increase in variability of mean latency was not specifically related to aging. Using the Hale et al. empirical equation $SD = 0.354RT - 43$ (in millisecond units), $\lambda = 1.84$ for $Y = 1000$ ms and $X = 600$ ms, giving a very large MLD of 400 ms. In the present analyses λ is placed in [1,2], a range that covers most latency data.

1.3. Expressing MLD correlations

Table 1 lists MLD correlations that are of particular interest. The random variables, X and Y , represent distinct mean latencies and $Y - X$ is the MLD. $r_{Y, Y-X}$ and $r_{X, Y-X}$ are the correlations of the MLD with its components. $r_{Y+X, Y-X}$ is the correlation of the sum or average of the mean latencies with the MLD. $r_{C, Y-X}$ is the correlation between the MLD and an external criterion and $r_{Y_1-X_1, Y_2-X_2}$ is the correlation between two MLDs.

The equations in Table 1 express correlations involving the MLD as functions of parameters that include standard deviation ratios and inter-variable correlations. In the case of $r_{X, Y-X}$ it is clear that the parameter r_{XY} has a positive effect on the magnitude of the MLD correlation because its presence in the numerator and its presence in the denominator result in changes in the same direction. However, the roles of λ and r_{XY} in the $r_{Y, Y-X}$ MLD correlation are difficult to discern because their effects in the numerator and denominator operate in opposite directions.

1.4. Interpreting parameter effects

For MLD correlations involving an external criterion and another MLD, there are three or more correlation parameters in the respective expressions. In these cases the domain is discontinuous because many parameter combinations are invalid. For instance, it is not possible for the correlations among three variables to be 0.9, 0.9, and -0.9. The correlation between two MLDs is a function of two standard deviation ratios and six interdependent correlations and difficult to depict. For these reasons linear models are used to index the magnitude and direction of parameter effects on the MLD correlation. These linear models, along with the means and standard deviations of the MLD correlations, are exact *descriptors* of data in the defined parameter domain which is representative of latency data. Imprecision in these descriptive statistics is simply a consequence of the size of the finite sample of parameter combinations used in the computation. A single observation (row) in the data used to estimate the linear model, consisting of an MLD correlation and its parameters, can be mapped to a hypothetical subject sample who provide mean latencies for two contrasted tasks. The analyses examine valid parameter combinations, uniformly sampled from the domain, and are not concerned with measurement error. We do not claim that the parameter domain accurately represents the “actual” distribution of latency data samples. In the absence of such knowledge, we use uniformity in the restricted parameter domain as a convenient prior.

2. Properties of MLD correlations

2.1. Correlation of the MLD with Y

10 000 parameter combinations were selected using uniform random sampling from $1 \leq \lambda \leq 2$, $0 \leq r_{XY} \leq 0.9$. The mean $r_{Y, Y-X}$ was 0.75 with $SD = 0.12$. As can be seen in Fig. 1, $r_{Y, Y-X}$ is always positive, increases with λ and declines with increases in r_{XY} at lower values of λ . Also, λ had the stronger of these two influences on $r_{Y, Y-X}$ and the impact of r_{XY} was negative in general. A linear model predicting $r_{Y, Y-X}$ from the parameters had $R^2 = 0.84$ (see Table 2) and this increased to 0.93 when the product of the parameters was included to account for the interaction. In summary, the slower, larger mean Y shows an increasing dominance of the MLD as its variability gets relatively larger than the faster, smaller mean X . When the variances of

Y and X are relatively similar, a strong correlation between them (partly reflecting a shared influence such as general processing speed) weakens the relationship between Y and the MLD.

2.2. Correlation of the MLD with X

In Fig. 2, the component correlation $r_{X, Y-X}$ spans both negative and positive values. The mean $r_{X, Y-X}$ was -0.21 with $SD = 0.30$. $r_{X, Y-X}$ is positively associated with both r_{XY} and λ , with the former having the bigger impact (see Table 2). It can be deduced from the equation that the effect of r_{xy} should be strongly positive and this is confirmed here. Inclusion of a multiplicative term, the product of the two parameters, increases R^2 from 0.94 to 0.96.

2.3. Correlation of the MLD with the mean latency

Across tasks, general speed or cognitive fluency covaries negatively with mean latency. Cognitively fluent subjects have smaller mean latencies than less fluent subjects. The overall mean latency, the average of Y and X , is correlationally identical to the sum $Y + X$ and can be considered as an indicator of the general speed factor. How is $r_{Y+X, Y-X}$, the correlation between mean latency and the MLD, influenced by λ and r_{XY} ?

Fig. 3 shows that the correlation of the mean latency with the MLD is always positive with a mean of 0.41 and $SD = 0.20$. That is, subjects with larger overall mean latency tend to have larger MLDs. Across samples, λ accounts for much of the variability in $r_{Y+X, Y-X}$. The regression model reinforces our conclusion that λ substantially influences $r_{Y+X, Y-X}$ by allowing the more variable component Y to dominate both the sum and the difference (see Table 2). This finding emphasizes the importance of being mindful of λ when comparing correlations of general processing speed and the MLD across tasks.

2.4. Correlation of the MLD with external criteria: Convergent validity

Latency differences are routinely correlated with external criteria. Table 1 expresses $r_{C, Y-X}$ as a function of λ , r_{XY} , r_{CY} and r_{CX} . If r_{CY} and r_{CX} are opposed in sign, the difference score combines the predictive association of the components with the criterion. Consider contexts in which $r_{C, Y-X} > 0$, $r_{CY} \geq 0$, and $r_{CX} \leq 0$. The conclusions of this analysis also apply to the symmetric scenario wherein $r_{C, Y-X} < 0$, $r_{CY} \leq 0$, and $r_{CX} \geq 0$. These cases exemplify *convergent validity* because the correlation between the difference score and the external criterion is non-zero.

The domain of the criterion correlation function is discontinuous because the magnitude and sign of two correlations constrain the values that can be assumed by the third correlation. The valid domain is a subset of 10 000 cases that were randomly sampled from $1 \leq \lambda \leq 2$, $0 \leq r_{XY} \leq 0.9$, $0 \leq r_{CY} \leq 0.5$, $-0.5 \leq r_{CX} \leq 0$. Every case was verified to be legitimate by ensuring that the determinant of the 3×3 correlation matrix was positive. This check eliminated 791 invalid cases from consideration.

The regression model explained most of the variance in the criterion correlation $r_{C, Y-X}$, $R^2 = 0.94$, mean = 0.44, $SD = 0.20$ (see Table 2). For $\lambda = 1$, r_{CY} and r_{CX} are symmetric and contribute equally to $r_{C, Y-X}$. As λ is increased, the magnitude of the criterion correlation is lowered due to the underweighting of r_{CX} and overweighting of r_{CY} . Consistent with the role of r_{XY} in the denominator of the criterion correlation in Table 1, increasing the correlation between X and Y , on average, raises the criterion correlation $r_{C, Y-X}$. Summing up, the correlation associated with the more variable latency, r_{CY} , plays a larger role in determining the value of the criterion correlation and, in the cases examined here, larger values of λ reduce the criterion correlation.

2.5. Correlation of the MLD with external criteria: Divergent validity

This section considers contexts wherein $r_{C, Y-X} \approx 0$, $r_{CY} \geq 0$, $r_{CX} \geq 0$. The positive validities of the two MLD components were constrained to be similar to each other in magnitude and result in $r_{C, Y-X} \approx 0$. The term *divergent validity* is used to indicate that the magnitude of the correlation of the difference score with the criterion is close to zero. An example in which this type of pattern is expected is when the criterion is a measure of general cognitive fluency. In such cases, the MLD should ideally not be influenced by cognitive fluency unlike the mean response latencies X and Y .

To construct this domain, parameter combinations were randomly generated by uniform sampling from $1 \leq \lambda \leq 2$, $0 \leq r_{XY} \leq 0.9$, $0.05 \leq r_{CY} \leq 0.85$, $r_{CY} - 0.5 \leq r_{CX} \leq r_{CY} + 0.5$. This ensured that r_{CX} and r_{CY} were similar in magnitude. There were 467 invalid parameter combinations and the regression analysis was based on 9533 cases (see Table 2). If λ was constrained to be 1, then the mean $r_{C, Y-X}$ was 0 (unbiased), with $SD = 0.03$. λ had a substantial impact on the magnitude of the criterion correlation while the effect of r_{XY} was relatively modest. The mean $r_{C, Y-X}$ was 0.16, with $SD = 0.13$. In the domain of divergent validity, λ created a positive bias in the criterion correlation, r_{CY} , the correlation associated with the more variable latency, had a larger influence on the criterion correlation than r_{CX} . Thus, when Y and X have discrepant variances, Y dominates the MLD and the criterion correlation mostly reflects the relationship between Y and the criterion. This finding implies that observed correlations between measures of general cognitive fluency (e.g., IQ) and MLD constructs are likely biased upwards.

2.6. The correlation between MLDs

$r_{Y_1-X_1, Y_2-X_2}$ is a function of six correlations and two standard deviation ratios. Given the complexity, it is difficult to legibly display graphical plots without imposing excessive constancy constraints on parameters. Consequently the linear approximation aided interpretation; $1 \leq \lambda_1 \leq \lambda_2 \leq 2$, with the larger value in every randomly drawn pair being identified as λ_2 . This led to $\bar{\lambda}_1 = 1.33$ and $\bar{\lambda}_2 = 1.67$. The parameters λ_1 and λ_2 were positively correlated at 0.498 because smaller values of λ_2 , on average, were paired with smaller values of λ_1 as a side-effect of the non-random assignment. Consequently, λ_1 and λ_2 were replaced

by $\frac{\lambda_1 + \lambda_2}{2} = \lambda_{avg}$ and $\lambda_2 - \lambda_1 = \lambda_{diff}$. λ_{avg} and λ_{diff} are uncorrelated, orthogonal basis vectors spanning the linear subspace covered by λ_1 and λ_2 .

20 000 parameter combinations were generated and, in each case, six correlations were uniformly drawn from $[0, 0.9]$ and tested to ensure that the twelve first-order and six second-order partial correlations among X_1, Y_1, X_2, Y_2 were in $[-1, 1]$. This validity check was enforced by ensuring that the determinant of the 4×4 correlation matrix and the determinants of its four 3×3 symmetric submatrices were all ≥ 0 . That is, all five matrices were positive semidefinite [4]. Failure to meet this condition eliminated 6362 cases, resulting in 13 638 valid combinations. Linear models were fitted separately for positive and negative values of $r_{Y_1-X_1, Y_2-X_2}$ and are displayed in Table 3. Unlike convergent criterion correlations, positive and negative values of $r_{Y_1-X_1, Y_2-X_2}$ lack symmetry and are not functionally equivalent.

λ_{avg} and λ_{diff} are, respectively, the average standard deviation ratio across the MLDs and the difference between the two standard deviation ratios. $r_{X_1Y_1}$ and $r_{X_2Y_2}$ involve latencies within each MLD; $r_{X_1X_2}$ and $r_{Y_1Y_2}$ correlate latencies across the two MLDs that have the same sign (concordant); $r_{X_1Y_2}$ and $r_{X_2Y_1}$ correlate latencies that have opposed signs (discordant) across the MLDs. The qualifiers *within*, *concordant* and *discordant*, respectively, refer to these pairs of correlations.

The numerator of $r_{Y_1-X_1, Y_2-X_2}$ implies that the concordant coefficients are positive and the discordant coefficients are negative. Given that $\lambda_1\lambda_2 > \lambda_2 > \lambda_1 > 1$, the expected trend is $b_{r_{Y_1}Y_2} > b_{r_{X_1}Y_2} > b_{r_{X_2}Y_1} > b_{r_{X_1}X_2}$. Larger within correlations, $r_{X_1Y_1}$ and $r_{X_2Y_2}$, reduce the denominator and increase the magnitude of $r_{Y_1-X_1, Y_2-X_2}$. The sign of the within coefficients, $b_{r_{X_1}Y_1}$ and $b_{r_{X_2}Y_2}$, follows the numerator - positive when the concordant correlations dominate the discordant correlations, and negative when the discordant correlations dominate the concordant correlations.

These theoretical expectations were met in Table 3. In both sub-samples, the concordant coefficients were positive and the discordant coefficients were negative. For positive (negative) $r_{Y_1-X_1, Y_2-X_2}$, the concordant correlations were larger (smaller) than the discordant correlations. Correlations involving more variable latencies had greater influence. For positive $r_{Y_1-X_1, Y_2-X_2}$, $b_{r_{Y_1}Y_2} > b_{r_{X_1}Y_2} > b_{r_{X_2}Y_1} > b_{r_{X_1}X_2}$ and for negative $r_{Y_1-X_1, Y_2-X_2}$, $b_{r_{Y_1}Y_2} \approx b_{r_{X_1}Y_2} > b_{r_{X_2}Y_1} > b_{r_{X_1}X_2}$. λ_{diff} was more influential than λ_{avg} and these two parameters had opposed signs within the two models and reversed effects on the magnitude of $r_{Y_1-X_1, Y_2-X_2}$ across the two models. Suppose $\lambda_1 = \lambda_2 = 1$, that is $\lambda_{diff} = 0$ and $\lambda_{avg} = 1$. This constraint increased the mean correlation from 0.29 to 0.40 for $r_{Y_1-X_1, Y_2-X_2} > 0$. For $r_{Y_1-X_1, Y_2-X_2} < 0$, where the mean is -0.33 , this constraint had little or no effect. Thus, the magnitude of $r_{Y_1-X_1, Y_2-X_2}$ is attenuated only when two MLDs were positively associated.

2.7. Summary of parameter effects

Table 4 summarizes parameter impacts on difference score correlations identified in the preceding sections. The mean latency Y makes its presence felt directly via λ and indirectly via correlations involving Y . λ plays a large role in increasing both component and mean latency correlations. In the mean latency correlation, both the sum and the difference are increasingly dominated by Y and are highly correlated with each other.

For the MLD correlation with an external criterion, r_{CY} has a larger influence than r_{CX} and the correlation between components, r_{XY} , increases it. This influence of r_{XY} can be inferred from its role in the denominator in the expression for $r_{C, Y-X}$. When X and Y correlate in opposed directions with the criterion but positively with each other, as in the context of convergent validity, the MLD removes common variance in X and Y that is unrelated to the criterion. λ attenuates $r_{C, Y-X}$ because, by giving more weight to Y , it fails to properly incorporate the validity of X in the MLD. In the case of divergent validity where the criterion is unrelated to the MLD, λ shifts the mean criterion correlation away from zero.

For the correlation between two MLDs, concordant and discordant correlations involving more variable latencies have larger impacts. The within correlations $r_{X_1Y_1}$, and $r_{X_2Y_2}$ increase the magnitude of $r_{Y_1-X_1, Y_2-X_2}$ as indicated by their negative coefficient in the square root of the denominator. λ 's that are greater than 1 attenuate the magnitude of positive $r_{Y_1-X_1, Y_2-X_2}$.

While correlational biases in the MLD undoubtedly exist, what is their significance for applied research? Kray and Lindenberger [15] reported that the general task switch cost MLD (difference between two blocks) was greater than the specific task switch cost MLD (difference between odd and even trials within the same block). They also observed that the correlation between a measure of general information processing rate and the general switch cost MLD was higher than the same correlation involving the specific switch cost MLD ($r = 0.57$ vs. $r = 0.35$). Their theoretical interpretation of this difference may be mistaken, as λ in the general switch cost MLD is likely larger than λ for the specific switch cost MLD. Likewise, the correlation between two MLDs need not be a veridical assessment of the relation between the corresponding constructs because it can be biased by artifacts such as MLD magnitude (which varies directly with λ) and the size of the within correlations.

3. Mean latency transformations

A number of researchers have, directly or indirectly, understood the limitations of the MLD and offered transformations that might address its weaknesses. Here we review some of the prominent proposed solutions and show that they do not redress the core deficiencies of the MLD.

3.1. Linear transformations

This section investigates (i) $E_{Y.X}$, the residual of Y regressed on X , (ii) $E_{Y-X, X+X}$, the residual of the mean latency difference regressed on the sum of the latencies, and (iii) $Y' - X$, where Y' is a linear, mean preserving transformation of Y that has the same variance as X . These measures can be expressed as linear combinations of X and Y . By applying (10), we obtain analytic expressions for their correlations with $Y + X$, $Y - X$, and a convergent criterion C .¹ Linear models of these expressions were examined in the domain $1 \leq \lambda \leq 2$, $0 \leq r_{XY} \leq 0.9$.

The residual $Y - E_{Y.X}$ or $E_{Y.X}$ was introduced by Cohen and Cohen [6, pp. 414–421] as an alternative to difference scores as it controlled for the level of X which varies across subjects.

The sample $\overline{E_{Y.X}}$ is zero and does not correlate with X . In the latency domain, the residual had a mean correlation of 0.93 with the MLD. The mean correlation between this residual and $Y + X$ was 0.60, considerably larger than the 0.41 mean correlation for the MLD. This strengthened

relation can be attributed to the standard deviation ratio $\frac{1}{r_{XY}}$ between the components of $E_{Y.X}$. This ratio always exceeds 1 and can be very large when r_{XY} is small. Unlike the MLD, the correlation of the residual with the criterion and the correlation between residuals are completely independent of the standard deviation ratios. The residual has a lower criterion correlation than the MLD (0.41 vs. 0.45) as it excludes the valid criterion variance in X . In the domain of positive correlations between MLDs, the mean correlation between residuals ($r = 0.46$) was much higher than that between MLDs ($r = 0.29$). $r_{Y_1 Y_2}$ uniquely accounts for 74.2% of the variance in $\gamma E_{Y_1.X_1}, E_{Y_2.X_2}$, dominated the numerator of the expression and, in 83% of cases, was greater than the correlation between MLDs. In the domain of negative correlations between MLDs (mean $r = 0.33$), the mean correlation between residuals was 0.07. Salthouse and Hedden [25, p. 865] criticize $E_{Y.X}$ as an alternative to MLDs on the grounds that it has reduced reliability. In contrast, the present analysis implies that, in fact, it typically exhibits higher reliability than the MLD. Even so, $E_{Y.X}$ should be considered to be inferior to the MLD as it has a higher mean latency correlation and a lower correlation with the criterion.

The residual of the MLD regressed on $Y + X$ was proposed by Chapman et al. [5] as an index that is independent of the subject mean latency which, in turn, is taken as a measure of general information processing speed. The residual $E_{Y-X, Y+X}$ has a mean of zero and does not correlate with $Y + X$. $E_{Y-X, Y+X}$ had a mean correlation of 0.88 with the MLD and a mean correlation of 0.48 with the convergent criterion. Thus it was less similar to the MLD than $E_{Y.X}$ and had a higher criterion correlation than both $E_{Y.X}$ and the MLD. In the domain of positive correlations between MLDs, the mean correlation between residuals was 0.35. In the latency domain, $E_{Y-X, Y+X}$ was superior to both the MLD and $E_{Y.X}$.

Given the biasing role of λ , it is natural to examine methods that homogenize variance. Standardizing X and Y sets $\lambda = 1$ but also eliminates the mean difference. The following transformation preserves the sample average mean latency difference and the correlation between mean latencies while setting the standard deviations of X and Y to be equal:

¹These expressions are available in a web supplement at <http://faculty.washington.edu/agg/pdf/CorrelationalBiasesMLD.supplementAppendix.pdf>.

$$Y'_i = \frac{1}{\lambda}(Y_i + (\lambda - 1)\bar{Y}). \quad (11)$$

This transformation ensures that $\sigma_{Y'} = \sigma_X$, $\lambda' = 1$, $\bar{Y}' = \bar{Y}$, $r_{XY'} = r_{XY}$ and $\lambda' = 1$. The difference $Y' - X$ is closely related to the MLD with a mean correlation of 0.93. However, it is distinct in that it is largely independent of the average mean latency with which it has a mean correlation of 0.12. In the more typical latency sub-domain $1 \leq \lambda \leq 2$, $0.5 \leq r_{XY} \leq 0.9$, the mean $r_{Y'-X, Y-X}$ is 0.89 and the mean $r_{Y'-X, Y+X}$ is 0.08. The convergent criterion correlation $r_{C, Y'-X}$ (mean = 0.48) is independent of the standard deviation ratio and eliminates the bias associated with the MLD as it is identical to $r_{C, Y-X}$ in Table 1 when $\lambda = 1$. Likewise, $r'_{Y'_1 - X_1, Y'_2 - X_2}$ (mean = 0.36) does not depend on the standard deviation ratios at all; if the within correlations in the two MLDs are each 0.5, then $r'_{Y'_1 - X_1, Y'_2 - X_2}$ is the difference between the sums of the concordant and discordant correlations. The equalized score resembles the [5] residual score $E_{Y-X, Y+X}$ in that it is superior to the MLD and E_{Y-X} . Applying appropriate constraints to (10),

$$r'_{Y'-X, E_{Y-X, Y+X}} = \frac{(\lambda+1)\sqrt{1+r_{XY}}}{\sqrt{2(\lambda^2+2\lambda r_{XY}+1)}}. \quad (12)$$

The mean of the above correlation in the latency domain exceeded 0.99. Consequently, the equalizing transformation may be preferred over the Chapman et al. residual score on the grounds of computational and conceptual simplicity.

3.2. Limitations of linear transformations

Although the equalizing transformation does appear to address the biases reported in the earlier analyses, it is not a panacea. Consider a sample composed of two sub-samples that have MLDs with opposed signs; in the first sub-sample $Y > X$, whereas in the second sub-sample $X > Y$. The overall sample MLD is zero and the standard deviations of Y and X are similar. This dissociation emerges when a task that is easier for one group is harder for the other and vice versa, such as an Implicit Association Test measuring association strengths of *Democrats* with *good* (and *Republicans* with *bad*) in one condition, and *Republicans* with *good* (and *Democrats* with *bad*) in the other [22]. The equalizing transformation has no effect because $\lambda = 1$. In these cases, even as the average latency correlates zero with $Y - X$, it can have large positive correlations with $|Y - X|$. Likewise, within each sub-sample, correlations of the MLD with external criteria are biased and influence the overall sample correlation. Thus even though these transformations partially address the biases, they continue to be vulnerable in certain contexts.

3.3. Non-linear transformations

If linear transformations are not sufficient, perhaps non-linear transformations of mean latency, including the square root, logarithm, and reciprocal can be a more complete corrective. These transformations have an advantage over linear transformations because unlike linear residuals that depend on sample statistics, non-linear difference scores do not have this dependence and are invariant across samples. Unfortunately, it is not possible to derive general, distribution free, expressions for correlations concerning differences of non-linearly transformed mean latencies. Consequently, to investigate the impact of non-linear transformations, we used an

empirical relationship between average mean latency and standard deviation of mean latency to simulate mean latencies.

In classic papers, Curtiss [7] formulated a mathematical theory of monotonic transformations and Bartlett [3] applied them in data analysis. Monotonic or rank preserving transformations play an important role in testing hypotheses concerning means. For example, many significance tests assume independent, normally distributed variates wherein the mean and variance are not associated. However, if the mean is related to the variance, transformations may be applied to the mean latency variate to achieve homogeneous variances.

What is the impact of using monotonic transformations such as the square root, logarithm, and reciprocal on the correlational structure of the difference score? When the mean latencies are transformed non-linearly, obtaining a general expression for difference score correlations, in terms of λ and r_{XY} , is not possible. Consequently a simulation was used to investigate the influence of the square root, logarithm and reciprocal transformations on the correlational structure of the transformed latency difference score. Hale et al. [12, p.409] described an empirical relationship between the mean and the standard deviation of the mean response latency. Their model for samples of younger subjects was $SD = 0.354RT - 43$, $R^2 = 0.885$; for samples of older subjects it was $SD = 0.317RT - 30$, $R^2 = 0.861$.

The simulation generalized their model so that the slope and intercept were, for a given sample, selected from the ranges [0.2, 0.4] and [30, 50] respectively. The simulation included 1000 samples, with 100 subjects in each. For each sample, the grand average mean latency across two conditions was selected from [500, 1000] and the deviation of each of the two condition average mean latencies from the grand mean latency was selected from [25, 250]. This procedure ensured that the mean latencies were in [250, 1250] and the mean latency differences in [50, 500], a plausible range in practice for both. For each sample, the SD of X and Y for the sample were computed using the linear equation relating the mean and SD. The correlation between X and Y for each sample was selected from [0, 0.9]. Given the mean and SD for X and Y , and the correlation between them, 100 individual subject mean latencies for the X and Y conditions were random variates from two correlated normal distributions with the assigned means and SDs. 707 samples were uniformly chosen from the latency domain $1 \leq \lambda \leq 2$ and positive $0 \leq r_{XY} \leq 0.9$ and formed the basis for subsequent analyses and plots. A convergent criterion was simulated as a normal deviate that correlated in the range $[-0.5, 0]$ with X and in $[0, 0.5]$ with Y . Prior to generating the mean latencies for each sample, the population values of r_{XY} , r_{CY} , and r_{CX} were checked to be valid combinations. Across samples, the correlation between the mean and SD of mean latency was 0.67, considerably less than the 0.93 correlation in [12]. This is so because the linear equations relating the mean and SD vary across samples. If the relationship between the mean and SD was held constant across samples, a correlation of 0.9 emerged.

The effect of the square root monotonic transformation on r_{XY} was negligible (see Fig. 4) in that the correlation remained unchanged after the transformation. The logarithm transformation results in only slightly more variability. Compared to the square root and logarithm transformations, the reciprocal has some impact on r_{XY} as there is about 12% error variance. Unlike r_{XY} , these transformations have a marked effect on λ . As expected, the square root transformation increases homogeneity of variance by reducing λ . However the error variance of the transformed λ was discernible at 16% and appeared to increase with the value of λ . The logarithm transformation had a strong effect on λ . Like the square root, the logarithm reduced the magnitude of λ and shifted it towards 1. However, the error variance was high at 79%. The reciprocal transformation had an even stronger effect on λ (for the reciprocal, λ is $\frac{\sigma_x}{\sigma_y}$), with 99% error variance. Thus the transformations have much more impact on λ than r_{XY} and the

convergence between the original difference score and the transformed difference score was maximal for the square root transformation and least for the reciprocal transformation. In each sample the correlation between the MLD and the transformed difference scores was obtained as an index of convergence. The square root difference was most closely related to the MLD (mean $r = 0.98$), followed by the logarithmic difference (mean $r = 0.92$); the reciprocal difference was quite dissimilar to the MLD (mean $r = 0.71$).

When correlations of the various scores with the sum $Y + X$ were considered, the MLD itself had the highest mean correlation at 0.40, followed by the square root, logarithm, and reciprocal (mean r 's = 0.30, 0.18, and -0.03 respectively). While the difference of mean reciprocals was independent of the average latency, it also had the lowest correlation with the convergent criterion (mean $r = 0.41$); the MLD, square root and logarithm predicted the convergent criterion better (mean r 's = 0.45, 0.47, and 0.47 respectively).

The square root transformation had minimal effect on the structure of the MLD. On average, the logarithm difference score had about 80% shared variance with the MLD and the reciprocal difference score had 50% shared variance with the MLD. The variability of the transformed λ , particularly in the case of the reciprocal, was excessive and indicates that this transformation is unsuitable for mean response latencies. In comparison, the logarithm is better as it reduced λ on average, and did so with less variability. The logarithm transformation has been advocated [17] but requires restrictive assumptions to motivate it theoretically; e.g., the assumption that subject mean latencies are the multiplicative product of a subject scaling factor (reciprocal of general speed) with the informational demands of the task [10].

3.4. Summary

Among the linear transformations, the equalized score was found to be superior and among the non-linear transformations the logarithm performed better. In the above simulation, the equalized score was computed and had mean correlations of 0.94, 0.96, 0.96, and 0.82 with the MLD, square root, logarithm, and reciprocal difference scores. On average, it correlated 0.11 with $Y + X$ and had a mean criterion correlation of 0.49, suggesting that the equalized score is slightly superior to the logarithmic difference which, unlike the equalized score, has the advantage of being independent of sample statistics.

4. Discussion

The mean latency difference is asymmetrically influenced by the more variable latency. The MLD magnitude has an inverse association with cognitive fluency, attenuates correlation with valid criteria, has non-zero correlations with invalid criteria, and has biased correlations with other convergent MLDs. Transformations of mean latencies do not satisfactorily resolve the problems inherent in the MLD.

Another source of potentially artifactual results pertains to interactions involving the MLD. The Simon effect contrasts a slower, more variable incongruent task with a faster, less variable congruent task. Bialystok et al. [2] reported that bilinguals outperformed monolinguals in the Simon task and that the superiority of bilinguals was larger in older participants. Compared to younger subjects, older subjects have larger latencies that are more variable. Consequently, the $Age \times Bilingualism$ interaction here could be amplified by the expected negative correlation between cognitive fluency and Simon task MLD and should be treated with circumspection.

Semantic priming, the latency in identifying a word as a function of presence-absence of a semantic relation to a preceding word, has been found to vary with word frequency and extent of visual degradation of the stimuli [18]. Tasks using low frequency words or visually degraded words have greater mean latencies and greater variance of mean latencies compared to tasks

using high frequency words or non-degraded stimuli. Because of the potential for artifacts in comparisons of MLDs, a claim that semantic priming is larger for low frequency or visually degraded words should be treated with caution.

Thus far, our analyses have been restricted to the aggregated level of mean latencies, glossing over the individual response latency distributions underlying the mean latency. These within-subjects, individual response latency distributions are positively skewed. They are depicted by probability distributions such as the ex-gaussian², which is specified using gaussian parameters μ , σ , an exponential parameter τ and has mean $= \mu + \tau$, standard deviation $= \sqrt{\sigma^2 + \tau^2}$ [14]. The exponential parameter causes the mean and variance to be positively correlated. Consider a set of latency distributions generated by a single subject from an experiment with four conditions, each incorporating either a simple task or a choice task under low cognitive load or high cognitive load (see Fig. 5).

Notice that the (Choice Latency–Simple Latency) MLD is exactly 100 ms at both low and high cognitive load. However, if we consider the degree of separation between the distributions using Cohen's d , one may be persuaded that the contrast is more compelling under low ($d = 0.81$) than high cognitive load ($d = 0.47$). Should such an approach prove productive, effect size measures could be used, in lieu of the MLD, to contrast latency distributions. One such measure, the D score, has been applied successfully for the Implicit Association Test [11]. Our preliminary investigations [27] suggest that such scale invariant or distribution free measures (cf. the mean of the relative distribution [13]) have the potential to increase statistical power, reduce correlational biases, and improve construct validity.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

References

1. Balota DA, Chumbley JI. Are lexical decisions a good measure of lexical access? The role of word frequency in the neglected decision stage. *JEP: Human Perception and Performance* 1984;10:340–357.
2. Bialystok E, Craik FIM, Klein R, Viswanathan M. Bilingualism, aging, and cognitive control. *Psychology and Aging* 2004;19:290–303. [PubMed: 15222822]
3. Bartlett MS. The use of transformations. *Biometrics* 1947;3:39–52. [PubMed: 20240416]
4. Budden, M.; Hadavas, P.; Hoffman, L.; Pretz, C. Generating valid 4×4 correlation matrices. (software). 2005. Retrieved March 15, 2006, from Carnegie Mellon University, StatLib website: <http://lib.stat.cmu.edu/>
5. Chapman LJ, Chapman JP, Curran TE, Miller MB. Do children and the elderly show heightened semantic priming? How to answer the question. *Developmental Review* 1994;14:159–185.
6. Cohen, J.; Cohen, P. *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences*. 2nd. Lawrence Erlbaum; Hillsdale, NJ: 1983.
7. Curtiss JH. On Transformations used in the Analysis of Variance. *The Annals of Mathematical Statistics* 1943;14:107–122.
8. Fan J, McCandliss BD, Sommer T, Raz A, Posner MI. Testing the efficiency and independence of attentional networks. *Journal of Cognitive Neuroscience* 2002;14:340–347. [PubMed: 11970796]

²Alternatives to the ex-gaussian include the log-normal, displaced gamma, and the reciprocal gaussian. The estimation of statistical parameters from latency data has been examined (e.g. [28]) as has theoretical modeling of response latencies and errors in specific tasks [19,23]. These distributional approaches have yet to provide a replacement for the MLD that could be applied across a variety of tasks, but such a measure, in contradistinction to the distribution free approach that we are investigating [27], remains a distinct possibility.

9. Fazio RH, Sanbonmatsu DM, Powell MC, Kardes FR. On the automatic activation of attitudes. *Journal of Personality and Social Psychology* 1986;50:229–238. [PubMed: 3701576]
10. Faust ME, Balota DA, Spieler DH, Ferraro, Individual differences in information processing and amount: Implications for group differences in response latency. *Psychological Bulletin* 1999;125:777–799. [PubMed: 10589302]
11. Greenwald AG, Nosek BA, Banaji MR. Understanding and using the Implicit Association Test: I. An improved scoring algorithm. *Journal of Personality and Social Psychology* 2003;85:197–216. [PubMed: 12916565]
12. Hale S, Myerson J, Smith GA, Poon LW. Age, variability, and speed: Between-subjects diversity. *Psychology and Aging* 1988;4:407–410. [PubMed: 3268286]
13. Handcock, MS.; Morris, M. *Relative Distribution Methods in the Social Sciences*. Springer-Verlag; New York: 1999.
14. Heathcote A, Brown S, Mewhort DJK. Quantile maximum likelihood estimation of response time distributions. *Psychonomic Bulletin and Review* 2002;9:394–401. [PubMed: 12120806]
15. Kray J, Lindenberger U. Adult age differences in task switching. *Psychology and Aging* 2000;15:126–147. [PubMed: 10755295]
16. MacLeod CM. Half a century of research on the Stroop effect: An integrative review. *Psychological Bulletin* 1991;109(2):163–203. [PubMed: 2034749]
17. Meiran N, Chorev Z, Sapir A. Component processes in task switching. *Cognitive Psychology* 2000;41(3):211–253. [PubMed: 11032657]
18. Madden DJ. Adult age differences in the effects of sentence context and stimulus degradation during visual word recognition. *Psychology and Aging* 1988;3:167–172. [PubMed: 3268255]
19. Melara RD, Algom D. Driven by information: A tectonic theory of Stroop effects. *Psychological Review* 2003;3:422–471. [PubMed: 12885110]
20. Meyer DE, Osman AM, Irwin DE, Yantis S. Modern mental chronometry. *Biological Psychology* 1988;26:3–67. [PubMed: 3061480]
21. Neely, JH. Semantic priming effects in visual word recognition: A selective review of current findings and theories. In: Besner, D.; Humphreys, GW., editors. *Basic processes in reading: Visual word recognition*. Lawrence Erlbaum Associates, Inc.; Hillsdale, NJ, England: 1991. p. 264–336.
22. Nosek BA, Smyth FL, Hansen JJ, Devos T, Lindner NM, Ranganath KA, Smith CT, Olson KR, Chugh D, Greenwald AG, Banaji MR. Pervasiveness and correlates of implicit attitudes and stereotypes. *European Review of Social Psychology* 2007;18:36–88.
23. Ratcliff R, Gomez P, McKoon G. A diffusion model of the lexical decision task. *Psychological Review* 2004;1:159–182. [PubMed: 14756592]
24. Ratcliff R, Spieler DH, McKoon G. Explicitly modeling the effects of aging on response time. *Psychonomic Bulletin and Review* 2000;1–25. [PubMed: 10780018]
25. Salthouse TA, Hedden T. Interpreting reaction time measures in between-group comparisons. *Journal of Clinical and Experimental Neuropsychology* 2002;24:858–872. [PubMed: 12647765]
26. Spieler DH. Modeling age-related changes in information processing. *European Journal of Cognitive Psychology* 2001;13:217–234.
27. Sriram, N.; Nosek, BA.; Greenwald, AG. Scale invariant contrasts of response latency distributions. University of Virginia; 2009. Unpublished manuscript
28. Van Zandt T. How to fit a response time distribution. *Psychonomic Bulletin & Review* 2000;7:424–465. [PubMed: 11082851]

Fig. 1.
Component correlation $r_{y, y-x}$.

Fig. 2.
Component correlation $r_{x|y-x}$.

Fig. 3.
Mean latency correlation $r_{y+x, y-x}$.

Fig. 4.
The effect of monotonic transformations on r_{xy} and λ .

Fig. 5.
Mean–variance correlation in individual latency distributions.

Table 1

Correlation of the MLD with components, sum, criterion and another MLD.

Variable	Correlation	Constraints	Expression
Y	$r_{Y, Y-X}$	$X_1 = X_2 = X$	$\frac{\lambda - r_{XY}}{\sqrt{\lambda^2 - 2\lambda r_{XY} + 1}}$
		$Y_1 = Y_2 = Y$	
		$\alpha_1 = 0, \beta_1 = 1$	
		$\alpha_2 = -1, \beta_2 = 1$	
		$\lambda_1 = \lambda_2 = \lambda$	
X	$r_{X, Y-X}$	$X_1 = X_2 = X$	$\frac{\lambda r_{XY} - 1}{\sqrt{\lambda^2 - 2\lambda r_{XY} + 1}}$
		$Y_1 = Y_2 = Y$	
		$\alpha_1 = 1, \beta_1 = 0$	
		$\alpha_2 = -1, \beta_2 = 1$	
		$\lambda_1 = \lambda_2 = \lambda$	
Y + X	$r_{Y+X, Y-X}$	$X_1 = X_2 = X$	$\frac{\lambda^2 - 1}{\sqrt{\lambda^4 - 4\lambda^2 r_{XY}^2 + 2\lambda^2 + 1}}$
		$Y_1 = Y_2 = Y$	
		$\alpha_1 = 1, \beta_1 = 1$	
		$\alpha_2 = -1, \beta_2 = 1$	
		$\lambda_1 = \lambda_2 = \lambda$	
C	$r_{C, Y-X}$	$X_1 = C$	$\frac{\lambda r_{CY} - r_{CX}}{\sqrt{\lambda^2 - 2\lambda r_{XY} + 1}}$
		$X_2 = C$	
		$Y_2 = Y$	
		$\alpha_1 = 1, \beta_1 = 0$	
		$\alpha_2 = -1, \beta_2 = 1$	
MLD	$r_{Y_1 - X_1, Y_2 - X_2}$	$\lambda_2 = \lambda$	$\frac{r_{X_1 X_2} - \lambda_2 r_{X_1 X_2} - \lambda_1 r_{X_2 Y_1} + \lambda_1 \lambda_2 r_{Y_1 Y_2}}{\sqrt{\lambda_1^2 - 2\lambda_1 r_{X_1 Y_1} + 1} \sqrt{\lambda_2^2 - 2\lambda_2 r_{X_2 Y_2} + 1}}$
		$\alpha_1 = \alpha_2 = -1$	
		$\beta_2 = \beta_2 = 1$	

Table 2

Regression models of MLD correlations.

MLD correlation	R^2	λ		r_{XY}		r_{CX}		r_{CY}	
		b	β	b	β	b	β	b	β
$r_Y, Y-X$	0.84	0.35	0.87	-0.13	-0.29				
$r_X, Y-X$	0.94	0.49	0.48	0.97	0.84				
$r_{Y+X}, Y-X$	0.94	0.65	0.93	0.23	0.29				
$r_{C,Y-X} > 0$	0.94	-0.08	-0.11	0.36	0.46	-0.71	-0.52	1.07	0.78
$r_{C,Y-X} \approx 0$	0.85	0.25	0.55	0.14	0.27	-0.82	-1.4	1.2	2

Table 3

Linear model of the correlation between two MLDs.

Parameter	$r_{Y_1-X_1,Y_2-X_2} > 0, M = 0.29, SD = 0.20, Intercept = -0.18 R^2 = 0.86, n = 6136$				$r_{Y_1-X_1,Y_2-X_2} < 0, M = -0.33, SD = 0.23, Intercept = -0.08 R^2 = 0.85, n = 7502$			
	<i>M</i>	<i>SD</i>	<i>b</i>	β	<i>M</i>	<i>SD</i>	<i>b</i>	β
λ_{avg}	1.52	0.22	0.11	0.11	1.49	0.20	0.23	0.20
λ_{diff}	0.29	0.22	-0.36	-0.39	0.37	0.24	-0.23	-0.25
$r_{X_1Y_1}$	0.42	0.24	0.23	0.27	0.43	0.25	-0.28	-0.31
$r_{X_2Y_2}$	0.43	0.24	0.19	0.23	0.43	0.24	-0.22	-0.24
$r_{X_1X_2}$	0.48	0.24	0.47	0.55	0.37	0.23	0.49	0.51
$r_{Y_1Y_2}$	0.55	0.22	0.86	0.92	0.32	0.22	0.83	0.80
$r_{X_1Y_2}$	0.31	0.22	-0.77	-0.82	0.51	0.23	-0.83	-0.83
$r_{X_2Y_1}$	0.35	0.22	-0.65	-0.72	0.50	0.24	-0.63	-0.66

Table 4

Parameter effects on MLD correlations.

MLD correlation	Parameter sequence
$r_{Y, Y-X}$	$\lambda - r_{XY}$
$r_{X, Y-X}$	$r_{XY} + \lambda$
$r_{Y+X, Y-X}$	$\lambda + r_{XY}$
$r_{C, Y-X} > 0$	$r_{CY} - r_{CX} + r_{XY} - \lambda$
$r_{C, Y-X} \approx 0$	$r_{CY} - r_{CX} + \lambda + r_{XY}$
$r_{Y_1-X_1, Y_2-X_2} > 0$	$r_{Y_1Y_2} - r_{X_1Y_2} - r_{X_2Y_1} + r_{X_1Y_2} - \lambda_{diff} + r_{X_1Y_1} + r_{X_2Y_2} + \lambda_{avg}$
$r_{Y_1-X_1, Y_2-X_2} < 0$	$-r_{X_1Y_2} + r_{Y_1Y_2} - r_{X_2Y_1} + r_{X_1X_2} - r_{X_1Y_1} - r_{X_2Y_2} - \lambda_{diff} + \lambda_{avg}$

Note: The parameters are ordered, from most important to least important (left to right).