Dose and detectability for a cone-beam C-arm CT system revisited

Arundhuti Ganguly,^{a)} Sungwon Yoon, and Rebecca Fahrig

Department of Radiology, Lucas MRS Center, Stanford University, 1201 Welch Road, Palo Alto, California 94305

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Purpose: The authors had previously published measurements of the detectability of disk-shaped contrast objects in images obtained from a C-arm CT system. A simple approach based on Rose's criterion was used to scale the data, assuming the threshold for the smallest diameter detected should be inversely proportional to $(\text{dose})^{1/2}$. A more detailed analysis based on recent theoretical modeling of C-arm CT images is presented in this work.

Methods: The signal and noise propagations in a C-arm based CT system have been formulated by other authors using cascaded systems analysis. They established a relationship between detectability and the noise equivalent quanta. Based on this model, the authors obtained a relation between x-ray dose and the diameter of the smallest disks detected. A closed form solution was established by assuming no rebinning and no resampling of data, with low additive noise and using a ramp filter. For the case when no such assumptions were made, a numerically calculated solution using previously reported imaging and reconstruction parameters was obtained. The detection probabilities for a range of dose and kVp values had been measured previously. These probabilities were normalized to a single dose of 56.6 mGy using the Rose-criteria-based relation to obtain a universal curve. Normalizations based on the new numerically calculated relationship were compared to the measured results.

Results: The theoretical and numerical calculations have similar results and predict the detected diameter size to be inversely proportional to $(\text{dose})^{1/3}$ and $(\text{dose})^{1/2.8}$, respectively. The normalized experimental curves and the associated universal plot using the new relation were not significantly different from those obtained using the Rose-criterion-based normalization.

Conclusions: From numerical simulations, the authors found that the diameter of detected disks depends inversely on the cube root of the dose. For observer studies for disks larger than 4 mm, the cube root as well as square root relations appear to give similar results when used for normalization. © 2010 American Association of Physicists in Medicine.

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I. INTRODUCTION

In our previous work,¹ we investigated the relationship between image noise, dose, kVp, and contrast detectability for a C-arm CT system. To allow for a more meaningful comparison of the system performance at various dose settings, we introduced the concept of an “equivalent diameter” that can be used for normalizing detectability profiles. It predicts the diameter of the smallest size disk that is visible at a given dose when the result at any reference dose value is known. The derivation of the relationship was based on a simple approach, using Rose's criterion for threshold signal-to-noise ratio in 2D projection images.² In this note, we aim to find a more accurate relationship for the detectability of objects in images from such a system given a more complete description of the signal and noise power spectra for C-arm CT systems.³

II. METHOD

For images with quantum limited noise, the detectability of objects with identical image contrast following Rose's relation results in the following:¹

$$d_1 \cdot \sqrt{\bar{D}_1(0)} = d_2 \cdot \sqrt{\bar{D}_2(0)}, \quad (1)$$

$$d_{\text{ref}} = d \cdot \sqrt{\frac{\bar{D}(0)}{\bar{D}_{\text{ref}}(0)}}. \quad (2)$$

Here, d is the object diameter and $\bar{D}(0)$ is the mean dose at isocenter computed using the area-averaging approximation for CTDI_W measurements. The relation in Eq. (1) shows how the diameter of the smallest visible object scales with dose. The rearranged form in Eq. (2) allows for normalization of detectability plots to a common reference dose. This is particularly important in evaluation of C-arm CT images where the presence of automatic exposure control during imaging makes it difficult to specify the desired image dose *a priori*.

While the above approach was developed for projection images, it is likely not valid for 2D image slices reconstructed from 3D volumes. The correct approach would be to use the more general relationship that connects the detectability to the noise equivalent quanta (NEQ) weighted by the object's spatial frequencies. Recently, the theoretical expres-

TABLE I. Definition of terms used and values of terms used for the detectability analysis.

Term	Definition	Value used
T_j	Transfer function at j th stage	
S_j	Noise power spectrum at j th stage	
$\bar{q}_0$	Mean incident photon fluence at the detector for a given projection	
$\bar{g}_1$	Mean fraction of incident photons interacting with the detector	
$\bar{g}_2$	Mean number of optical photons produced per incident photon, including k -shell interactions	
P_k	Transfer function for k -fluorescence	
T_3	Transfer function due to spread of optical photons	
$\bar{g}_4$	Coupling efficiency of photodiode	
T_5	Transfer function due to photodiode aperture	
a_i	Detector sampling interval in i direction	0.154 mm
III	Sampling function (comb, Shaw)	
A_i	Number of pixels binned in i direction	2
B_i	Downsampling factor in direction i	1
T_8	Transfer function due to 2D binning aperture	
T_{10}	Filter	Modified Shepp–Logan
T_{11}	Apodization filter	
h_{win}	Smoothing parameter	
T_{12}	Interpolation filter	
m	Number of projections per sweep	543
M	Geometric magnification	1.4
T_{13}	Transfer function associated with backprojection	
Θ_{13}	Transfer function associated with backprojection of noise	
b_i	3D sampling interval indirection i	0.85 mm

sion for the signal, the noise power spectrum (NPS), and hence the NEQ for 3D images reconstructed from a C-arm CT system was published by Tward and Siewerdsen.³ The detectability of an object is given by

$$d'^2 = \int \int \int_0^\infty \text{NEQ}(f_x, f_y, f_z) |\Delta H|^2 df_x df_y df_z$$

$$= \int \int \int_0^\infty \frac{\text{MTF}^2(f_x, f_y, f_z)}{\text{NPS}(f_x, f_y, f_z)} |\Delta H|^2 df_x df_y df_z. \quad (3)$$

Here, d' is the detectability, $\text{MTF}(f_x, f_y, f_z)$ is the 3D modulation transfer function, and $\text{NPS}(f_x, f_y, f_z)$ is the normalized noise power spectrum. Parameter ΔH is the Fourier transform of the signal above the background, which in our case is a circular disk of diameter d_i and height equal to slice thickness w_z . Its Fourier transform is a jinc function in the X-Y plane and a sinc function in Z. This can be written as

$$d'^2 = \int \int \int_0^\infty \frac{T_3^2 T_5^2 T_{11}^2 T_{12}^2}{\text{NPS}} |\Delta H|^2 df_x df_y df_z. \quad (4)$$

Here, T_j denotes the MTF at each stage in the imaging chain as described by Tward and Siewerdsen.³

If it is assumed that the system has low electronic noise and if the resampling and rebinning are ignored, then for an image reconstructed using a ramp filter, the NPS can be expressed as

$$\text{NPS} = \frac{\pi f}{\bar{q}_0 G m M^2} T_3^2 T_5^2 T_{11}^2 T_{12}^2, \quad (5)$$

where $f = \sqrt{f_x^2 + f_y^2}$.

The definitions of the different terms in Eq. (5) are explained in Table I. The gain G is given by $G = \bar{g}_1 \bar{g}_2 \bar{g}_4 P_k$ with the assumption that $\bar{g}_4 P_k T_3^2 \gg 1$ and the spatial variation in P_k is small. Here, m is the number of projections and M is the geometric magnification. Plugging these and the jinc and sinc functions into the detectability formula

$$d'^2 = \frac{\bar{q}_0 G m M^2}{\pi} \int \int \int_0^\infty \frac{1}{f} \times \left| 2\pi d_i^2 \frac{J_1(2\pi d_i f)}{(2\pi d_i f)} \cdot \frac{w_z \cdot \sin(\pi w_z f)}{(\pi w_z f)} \right|^2 df_x df_y df_z. \quad (6)$$

Changing to radial coordinates and assuming all in-plane parameters are independent of z in the given slice,

$$d'^2 = \frac{\bar{q}_0 G m M^2}{\pi} \int_0^\infty \frac{2\pi}{f} \left| 2\pi d_i^2 \frac{J_1(2\pi d_i f)}{(2\pi d_i f)} \right|^2 f df \times \int_0^\infty \left| w_z \frac{\sin(\pi w_z f)}{(\pi w_z f)} \right|^2 df_z. \quad (7)$$

Using the substitution $x = 2\pi d_i f$ and the fact that $\int_0^\infty |w_z \cdot \sin c(\pi w_z f)|^2 df_z = w_z/2$,

$$d'^2 = 2\pi\bar{q}_0 GmM^2 d_i^3 w_z \int_0^\infty \left| \frac{J_1(x)}{x} \right|^2 dx, \quad (8)$$

$$d'^2 = 2\pi\bar{q}_0 GmM^2 d_i^3 w_z \frac{4}{3\pi}, \quad (9)$$

$$d'^2 = \frac{8}{3}\bar{q}_0 GmM^2 d_i^3 w_z, \quad (10)$$

$$d' \propto \sqrt{\bar{D}(0)} d_i^{3/2}, \quad (11)$$

since

$$q_0 \propto \bar{D}(0). \quad (12)$$

The above relationship was derived for a simple case of 3D reconstruction assuming no aliasing, no rebinning, and no resampling, with ramp filter, and with no additive electronic noise. We further investigated the effect of CT reconstruction by considering binning, sampling, and a reconstruction filter that is relevant to imaging with our C-arm CT system. For this purpose, the detectability versus dose-diameter relation was calculated numerically. The system parameters used were the same as those listed in our previous work¹ and are listed again here in Table I for convenience. For the analysis, 10 mm thick reconstructed slices were used. We assumed an ideal 2D noise transfer characteristic for the detector, i.e., $NPS_{2D} \propto MTF_{2D}^2 * III_6$. This assumption was used because upgrades to the detector on our C-arm system since acquisition of the observer data reported by Fahrig *et al.*¹ precluded measurement of the appropriate 2D NPS. This approach, however, allows us some insight into the effects of the reconstruction process on the detectability. This assumption is valid when the additive noise is small and $\bar{g}_4 P_K T_3^2 \gg 1$. The detectability in this case is then expressed as

$$d'^2 = \int \int \int_0^\infty \frac{T_3^2 T_5^2 T_8^2 T_{10}^2 T_{11}^2 T_{12}^2 \Theta_{13}^2}{NPS} |\Delta H|^2 df_x df_y df_z, \quad (13)$$

where

$$NPS = S_{15} = \frac{\pi}{\bar{q}_0 GmM^2} \{ [(T_3^2 T_5^2) * * III_6] T_8^2 * * III_8 T_{10}^2 T_{11}^2 T_{12}^2 \Theta_{13}^2 * * * III_{14} \} T_{15}^2 * * * III_{15}. \quad (14)$$

This equation is obtained from Eq. (12a) in Ref. 3. From the above relation again we see that

$$d' \propto \sqrt{\bar{D}(0)} \quad (15)$$

since

$$q_0 \propto \bar{D}(0). \quad (16)$$

Therefore, numerically, we evaluated the relationship between the detectability d'^2 [which is proportional the dose $\bar{D}(0)$, as per Eq. (15)] and the contrast disk diameter d_i since this relation is not obvious from the equation. To ensure that

FIG. 1. Numerically calculated results for squared detectability for different disk diameters. The open circles are results for the no rebinning and no resampling with low additive noise and a ramp filter. The filled circles are the results for the case when these assumptions are not made. Power law fits to the data show a power value of 3.06 and 2.80 for the two cases, respectively. The dotted lines show the fit to the Rose-criteria-based relation in both cases.

the numerical calculations were correctly implemented, we first verified the relationship for the no binning, no resampling, and low additive noise assumption. The numerical calculations were implemented using MATLAB (The Mathworks, Natick, MA) using a 3072×3072 grid in 2D with the sampling frequency going out to eight times the Nyquist frequency for the presampled data, which was as far as memory limitations would allow.

III. RESULTS

For the case of no binning, aliasing, or resampling, using Eq. (11), the diameter of the detectable object and the dose are related as follows:

$$d_{\text{ref}} = d \cdot \left[\frac{\bar{D}(0)}{\bar{D}_{\text{ref}}(0)} \right]^{1/3}. \quad (17)$$

Instead of the $\frac{1}{2}$ power as in the relationship derived from the Rose criterion, the above results show a $\frac{1}{3}$ power dependence of the detectable diameter to the dose ratio.

Figure 1 shows the results from the numerical calculations with a power law fit relating d'^2 [which is proportional to $\bar{D}(0)$ and d_i^n]. The test case for the no rebinning/resampling with low additive noise produced a fit value of $n=3.05$, which is similar to the expected value of 3. For the calculation with the real imaging parameters, the best-fit value of n was 2.80, which is still closer to the theoretically calculated n of 3 for the simple case than it is to the value of 2 which is predicted using Rose's criterion. (dotted curves).

The detectability data that we had reported in our previous work¹ when normalized using Eq. (2) and Eq. (17) are shown in Fig. 2. The data prior to normalization, for different kVp values, is shown in Figs. 2(a), 2(d), 2(g), and 2(j). Results of normalization using Eq. (2) are shown in Figs. 2(b), 2(e), 2(h), and 2(k) and using Eq. (17) are shown in Figs.

FIG. 2. Detectability-diameter relationship for various doses at different kVp values: [(a), (d), (g), and (j)] Original data reported previously (Ref. 1); [(b), (e), (h), and (k)] normalized data using Eq. (2); and [(c), (f), (g), and (l)] normalized data using Eq. (17).

2(c), 2(f), 2(i), and 2(l). Following normalization, the curves appear to form a universal curve in both cases. For the set of kVp values used, the convergence appears to improve significantly with increase in kVp for both types of normalization. A comparison of universal curves for both types of normalization is shown in Fig. 3.

IV. CONCLUSIONS AND DISCUSSION

From the above theoretical and simulated results, we found that for our C-arm CT system, the dose is inversely proportional to the cube root of the diameter of the smallest disk detected. However, for the data set that was analyzed, a

square root relation based on the Rose criteria for 2D projections with white noise also appears to adequately model the measured data. This observation is based on the fact that the average standard deviation of the mean normalized detectability data for the four kVp values (70, 81, 109, and 125 kVp) for the square root relation and cube root relation are: 13.1% and 10.0%, 17.2% and 18.9%, 16.0% and 19.0%, and 4.0% and 4.5%, respectively. Thus, data normalized with either relation has a similar spread about the mean.

The theoretical calculations and numerical simulations suggest that the detectability squared should increase much faster, i.e., as the cube of the disk diameter rather than as the

FIG. 3. Mean normalized curves for different kVp values. (a) Data normalized using Eq. (2) and (b) data normalized using Eq. (17).

square of the diameter; the squared relationship would apply if the noise in the images was white. However, we see that both relations give similar results with respect to reduction in spread when they are used to normalize our measured data. This implies that either the range of measurements we made is not sufficient to distinguish between the two relations, or that the measurements are noisy and include observer, focal spot, and other system effects, and therefore for this data set, either relationship can be used to normalize the observed detectability.

Similar observations have been made in regular CT where, a simplified theoretical estimation of the detectability has been established by other authors.^{4,5} Yet for clinical CT data, the relationship between diameter and dose is approxi-

mately equivalent to that of white noise as described in Eq. (2).^{4,5} This was found to be true across several CT scanners from several manufacturers as well as from different models from the same manufacturers. The anomaly between the theoretical and experimental relation is explained by Cohen *et al.*,^{6,7} who state that “the system and observer limitations combine to yield pixel averaging more nearly characteristic of white noise.” Therefore, despite CT noise being correlated, the final reconstructed images when viewed by observers appear to possess noise that is fairly white.

From the above results, we see that a cube root relation between dose and diameter should be used for normalizing detectability data from a C-arm CT system. However, if the diameters involved are larger than 4 mm (which is the smallest detected disk in our study), the simpler square root relation based on the Rose formulation is adequate.

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^{a)}Electronic mail: aganguly@stanford.edu

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