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Reconstructing uniformly attenuated rotating slant-hole SPECT projection data using the DBH method

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Abstract

This work applies a previously developed analytical algorithm to the reconstruction problem in a rotating multi-segment slant-hole (RMSSH) SPECT system. The RMSSH collimator has greater detection efficiency than the parallel-hole collimator with comparable spatial resolution at the expense of limited common volume-of-view (CVOV) and is therefore suitable for detecting low-contrast lesions in breast, cardiac and brain imaging. The absorption of gamma photons in both the human breast and brain can be assumed to follow an exponential rule with a constant attenuation coefficient. In this work, the RMSSH SPECT data of a digital NCAT phantom with breast attachment are modeled as the uniformly attenuated Radon transform of the activity distribution. These data are reconstructed using an analytical algorithm called the DBH method, which is an acronym for the procedure of differentiation backprojection followed by a finite weighted inverse Hilbert transform. The projection data are first differentiated along a specific direction in the projection space and then backprojected to the image space. The result from this first step is equal to a one-dimensional finite weighted Hilbert transform of the object; this transform is then numerically inverted to obtain the reconstructed image. With the limited CVOV of the RMSSH collimator, the detector captures gamma photon emissions from the breast and from parts of the torso. The simulation results show that the DBH method is capable of exactly reconstructing the activity within a well-defined region-of-interest (ROI) within the breast if the activity is confined to the breast or if the activity outside the CVOV is uniformly attenuated for each measured projection, while a conventional filtered backprojection algorithm only reconstructs the high frequency components of the activity function in the same geometry.

1. Introduction

The rotating multi-segment slant-hole (RMSSH) SPECT system is known for its high detection efficiency and a relatively small field-of-view (FOV). These two intrinsic properties make the RMSSH SPECT system applicable to breast, brain or cardiac imaging. In SPECT breast imaging, radioactive emissions from the breast are attenuated by the uniform breast tissue and captured by the gamma camera. The small FOV of an RMSSH SPECT system is large enough to contain the breast region, however, activities originating from other organs may cause artifacts if appropriate methods are not used to correct for the effects of truncation. In this paper, an RMSSH SPECT system for breast imaging was investigated through a simulation study. An analytically exact algorithm was utilized in reconstruction of simulated uniformly and nonuniformly attenuated data with truncation.

The SPECT system with an RMSSH collimator has been studied for some time (Edholm *et al* 1980, Holman *et al* 1981, Dale *et al* 1985, Wessell 1999, Bal *et al* 2000, ²⁰⁰⁶, Wagner *et al* 2001, ²⁰⁰², Baird *et al* 2003, Liu *et al* 2004, Xu *et al* 2006, Xu and Tsui 2006). Early development and application of RMSSH SPECT is reviewed in the dissertation of Wessell (1999). Recent interest in RMSSH imaging has been shown in cardiac (Bal *et al* 2000, 2006) and breast imaging (Baird *et al* 2003, Liu *et al* 2004). Reconstruction algorithms for RMSSH SPECT have also been developed (Wagner *et al* 2001, 2002, Xu *et al* 2004, Xu and Tsui 2006) for the case in which a complete tomographic data set is acquired. A projection data set is complete if the projections are sampled such that Orlov's condition is satisfied (Orlov 1975). Analytical algorithms for RMSSH SPECT with uniform attenuation correction were considered in the works of Wagner *et al* (2001, 2002) which assumed that the activity sources were fully contained within the FOV of the RMSSH SPECT system, i.e., there was no truncation. The 3D data set was rebinned to the conventional circular SPECT parallel geometry, so that a two-dimensional reconstruction algorithm could be used to compensate for the uniform attenuation. Xu *et al* (2004) presented an iterative algorithm to correct the non-uniform attenuation effect in breast imaging.

Recently, Xu and Tsui (2006) derived an analytical method for RMSSH SPECT breast imaging using a two-step procedure with differentiation backprojection (DBP) followed by a finite inverse Hilbert transform. The algorithm was an adaptation of the one developed by Noo *et al* (2004) in CT reconstruction to the RMSSH SPECT geometry. Although unable to correct for attenuation of gamma photon emissions, the algorithm was able to reconstruct a region-of-interest (ROI) from truncated projections in RMSSH SPECT geometry. A similar two-step algorithm on inverting the exponential Radon transform (Natterer 1986) was first presented by Rullgard (2004).

In this paper, a two-step algorithm which was developed in the works of Noo *et al* (2007) and Huang *et al* (2009) is applied to a complete RMSSH SPECT data set to reconstruct the activity in the breast with uniform attenuation correction. The algorithm reconstructs the same ROI as that developed by Xu and Tsui (2006) except the presented algorithm is more sensitive to numerical errors because of the modeling for attenuation and requires more careful numerical implementation to minimize reconstruction artifacts.

The analytical reconstruction algorithm developed in this paper for the correction of uniform attenuation is computationally more efficient than an iterative algorithm and may have potential applications in clinical SPECT. Analytical methods treat the projection data as perfect line integrals and the reconstruction can be carried out efficiently using mathematically explicit formulae. Analytical methods provide a quick reconstruction useful in quality control; for example, the evaluation of subject positioning and determination of whether an experiment needs to be redone, or the evaluation of whether geometrical calibrations, such as the center of rotation, are measured accurately. Analytical algorithms also provide theoretical consistency conditions. These conditions are extremely important in the evaluation and understanding of the performance of iterative reconstruction algorithms and for determining whether they may provide accurate solutions.

The paper is organized as follows: section 2 introduces the RMSSH SPECT system. Section 3 explains the principle of how the reconstruction is performed, section 4 presents the simulation results and section 5 concludes the paper.

2. The RMSSH SPECT system

The slant hole collimator is a variation of the parallel-hole collimator. For instance, the quadrant slant-hole collimator in figure 1(a) has four sets of parallel-holes angled toward a common volume of view (CVOV), with slant angle σ indicated in figure 1(b). The detector remains

fixed at different angular positions around the object being imaged. At each position, the collimator rotates around its central axis and detects photon emissions from four different directions simultaneously. This leads to a sensitivity improvement $4 \cos^3 \sigma$ times that of the conventional parallel-hole collimator with equivalent spatial resolution and equivalent acquisition time at the expense of a small detector FOV. Figure 1(b) shows the cross-sectional view of the collimator. The CVOV of these two segments shown in the figure is indicated by the shaded volume. The size of the CVOV, represented by the radius of the largest sphere that can project to all segments on the collimator without truncation, depends on the detector area, number of segments and the slant angle. For a specific imaging task, the ROI of the object should be located within the CVOV. The high sensitivity and small FOV of the RMSSH collimator ensure that it can be applied in dedicated SPECT systems, such as SPECT systems for breast, cardiac and brain imaging.

3. Methods

Within a segment of the slant-hole collimator, the slant-holes are parallel to each other. Thus the same mathematical tool used in the conventional parallel-hole collimator system can be used to model the data acquired with the RMSSH collimator. The exponential Radon transform (Natterer 1986) is used in this paper to describe the uniform attenuation of the gamma photons in breast tissue. The task of reconstruction is to restore the activity from its exponential Radon transform.

3.1. Exponential Radon transform

Activity at a point in the three-dimensional (3D) space $\vec{x} = (x, y, z)^T$ is denoted by $f(\vec{x})$. The projection detected by each segment has an angle $(\pi/2 - \sigma)$ to the detector plane, if the slant angle is σ . For simplicity, in this mathematical model the detector plane is assumed to be perpendicular to the projections and lie at the center of the field, as in figure 2. The emission ray is assumed to be along the direction $\vec{\theta} = \vec{\theta}(\varphi, \theta)$, which has the azimuth angle φ and the zenith angle θ . The projection $g(u, v, \vec{\theta})$ is the exponential Radon transform of the activity f :

$$g(u, v, \vec{\theta}) = \int_{-\infty}^{\infty} f(u\vec{\alpha} + v\vec{\beta} + t\vec{\theta}) e^{\mu t} dt, \quad (1)$$

where μ is the linear attenuation coefficient assumed to be known *a priori*. We assume that the breast tissue is adipose, which has an attenuation coefficient close to water. In the numerical study $\mu = 0.15 \text{ cm}^{-1}$ is chosen at the photon energy of 140 keV. The unit directional vectors in (1) are defined as

$$\begin{aligned} \vec{\alpha} &= (-\sin \varphi, \cos \varphi, 0)^T, \\ \vec{\beta} &= (-\cos \theta \cos \varphi, -\cos \theta \sin \varphi, \sin \theta)^T, \\ \vec{\theta} &= (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)^T. \end{aligned}$$

3.2. Data acquisition

The activity function $f(\vec{x})$ can be exactly reconstructed if a complete tomographic set of projection data $g(u, v, \vec{\theta})$ is available. Using Orlov's condition (Orlov 1975), to acquire a complete tomographic projection set, the RMSSH detector head should stop at $\pi/(2\sigma)$ positions equally spaced on an arc of $\pi - 2\sigma$. At each position, the collimator rotates and records emissions at sufficient views.

The RMSSH SPECT system for breast imaging is shown in figure 1(a) for slant angle $\sigma = \pi/6$. The detector stops at three positions to acquire data. At each position, the collimator rotates

such that the trace of projection directions draws half a circle on the unit sphere seen as solid arcs in figure 3 and denoted by Ω_1 , Ω_2 and Ω_3 , respectively. The representation of all the projection directions on the unit sphere is then a continuous curve consisting of these three half circles.

The measured data are assumed to have the form

$$p(u, v, \vec{\theta}) = \int_{-\infty}^{\infty} f(u\vec{\alpha} + v\vec{\beta} + t\vec{\theta}) e^{-\int_t^{\infty} \mu d\tau} dt. \quad (2)$$

where in practice, the detector plane is located at a finite distance outside the patient. As is implied in (1), the data used in the new algorithm assume that the detector is placed at the center of rotation, which is the center of the ROI. Thus the acquired data is modified to form

the exponential Radon transform in (1) by multiplying by a factor $e^{\int_0^{\infty} \mu d\tau}$ to obtain the modified projections $g(u, v, \vec{\theta})$ before the step of differentiation. With the attenuation map known, it is easy to calculate this factor. Along each projection ray, the factor is the integral of the attenuation map from the imaginary central detector plane to the edge of the attenuation map. This requires the boundary of the attenuation map to be known and a proper coordinate system to be set up. For instance, the Cartesian coordinates for Ω_2 are shown in figure 3. Similarly, for Ω_1 and Ω_3 , the y - z plane should be rotated around the x -axis by 2σ counterclockwise and clockwise.

3.3. Differentiation backprojection

To reconstruct the image, the two-step method first calculates the differentiation backprojection (DBP) of the modified projections $g(u, v, \vec{\theta})$. Without loss of generality, the data $g(u, v, \vec{\theta})$ are assumed to be modified from the data measured at the second camera position. The differentiation is carried out along the u -axis, which is the tangential direction of the self-rotating orbit of the collimator:

$$\begin{aligned} \frac{\partial g}{\partial u}(u, v, \vec{\theta}) &= \int_{-\infty}^{\infty} \frac{\partial}{\partial u} [f(u\vec{\alpha} + v\vec{\beta} + t\vec{\theta}) e^{\mu t}] dt, \\ &= \int_{-\infty}^{\infty} [\vec{\alpha} \cdot \nabla f(u\vec{\alpha} + v\vec{\beta} + t\vec{\theta})] e^{\mu t} dt, \end{aligned} \quad (3)$$

where ∇ is the gradient operator. For a ray passing through $\vec{x}$, the differentiation is

$$\begin{aligned} \frac{\partial g}{\partial u}(u, v, \vec{\theta})|_{u=\vec{x} \cdot \vec{\alpha}, v=\vec{x} \cdot \vec{\beta}} &= \int_{-\infty}^{\infty} [\vec{\alpha} \cdot \nabla f((\vec{x} \cdot \vec{\alpha})\vec{\alpha} + (\vec{x} \cdot \vec{\beta})\vec{\beta} + t\vec{\theta})] e^{\mu t} dt, \\ &= \int_{-\infty}^{\infty} [\vec{\alpha} \cdot \nabla f(\vec{x} + (t - \vec{x} \cdot \vec{\theta})\vec{\theta})] e^{\mu t} dt, \\ &= \int_{-\infty}^{\infty} [\vec{\alpha} \cdot \nabla f(\vec{x} + \tau\vec{\theta})] e^{\mu(\tau + \vec{x} \cdot \vec{\theta})} d\tau. \end{aligned}$$

The relation $\vec{x} = (\vec{x} \cdot \vec{\alpha})\vec{\alpha} + (\vec{x} \cdot \vec{\beta})\vec{\beta} + (\vec{x} \cdot \vec{\theta})\vec{\theta}$ was used in the second equality. The last equality of (4) holds for $\tau = t - \vec{x} \cdot \vec{\theta}$. Then the exponential backprojection is performed over the entire Ω_2 with $d\omega = \sin \theta d\theta d\varphi$:

$$\begin{aligned}
b_2(\vec{x}) &= \iint_{\Omega_2} \frac{\partial g}{\partial u}(u, v, \vec{\theta}) \Big|_{u=\vec{x} \cdot \vec{\alpha}, v=\vec{x} \cdot \vec{\beta}} e^{-\mu \vec{x} \cdot \vec{\theta}} d\vec{\omega}, \\
&= \iint_{\Omega_2} \left(\int_{-\infty}^{\infty} [\vec{\alpha} \cdot \nabla f(\vec{x} + t\vec{\theta})] e^{\mu(t + \vec{x} \cdot \vec{\theta})} dt \right) e^{-\mu \vec{x} \cdot \vec{\theta}} d\vec{\omega}, \\
&= \iint_{\Omega_2} \left(\int_{-\infty}^{\infty} [\vec{\alpha} \cdot \nabla f(\vec{x} + t\vec{\theta})] e^{\mu t} dt \right) d\vec{\omega}.
\end{aligned} \tag{5}$$

The expression

$$[\vec{\alpha} \cdot \nabla f(\vec{x} + t\vec{\theta})] = \left[\frac{1}{t \sin \theta} \frac{\partial}{\partial \varphi} f(\vec{x} + t\vec{\theta}) \right]$$

holds by the chain rule

$$\frac{\partial}{\partial \varphi} f(\vec{x} + t\vec{\theta}) = \nabla f(\vec{x} + t\vec{\theta}) \cdot \frac{\partial}{\partial \varphi} (\vec{x} + t\vec{\theta})$$

and

$$\frac{\partial}{\partial \varphi} (\vec{x} + t\vec{\theta}) = (-\sin \theta \sin \varphi, \sin \theta \cos \varphi, 0) = (t \sin \theta) \vec{\alpha}.$$

Thus after rearrangement, (5) can be expressed as

$$b_2(\vec{x}) = \int_{-\infty}^{\infty} \frac{e^{\mu t}}{t} \left(\iint_{\Omega_2} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} f(\vec{x} + t\vec{\theta}) \right] d\vec{\omega} \right) dt. \tag{6}$$

Define two unit directions $\vec{\theta}_1 = \vec{\theta}_1(\varphi_1, \sigma)$ and $\vec{\theta}_2 = \vec{\theta}_2(\varphi_2, \sigma)$ illustrated in figure 3. The integral in the parenthesis in (6) is

$$\begin{aligned}
&\iint_{\Omega_2} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} f(\vec{x} + t\vec{\theta}) \right] d\vec{\omega}, \\
&= \int_{\varphi_1}^{\varphi_2} \int_0^{\pi/2} \delta(\theta - \sigma) \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} f(\vec{x} + t\vec{\theta}) \right] \sin \theta d\theta d\varphi, \\
&= \int_{\varphi_1}^{\varphi_2} \frac{\partial}{\partial \varphi} f(\vec{x} + t\vec{\theta}) \Big|_{\theta=\sigma} d\varphi, \\
&= f(\vec{x} + t\vec{\theta}_2) - f(\vec{x} + t\vec{\theta}_1).
\end{aligned}$$

Accordingly, equation (5) (the DBP of data measured at the second camera position) can be written as

$$\int_{-\infty}^{\infty} \frac{e^{\mu t}}{t} (f(\vec{x} + t\vec{\theta}_2) - f(\vec{x} + t\vec{\theta}_1)) dt.$$

The equation shows that the DBP is the difference of the image blurred along two directions. These two directions are determined by the two ends of the trajectory (i.e., Ω_2) of the projection directions. The DBP for the first camera position and the third camera position can be similarly

obtained, although the coordinate system should be rotated so that the z -axis aligns with the self-rotating axis of the detector. The three DBP's add up to

$\int_{-\infty}^{\infty} \frac{e^{\mu t}}{t} (f(\vec{x} + t\vec{v}_2) - f(\vec{x} + t\vec{v}_1)) dt$. Direction $\vec{v}_1$ is the starting point of Ω_1 in the unit sphere. Direction $\vec{v}_2$ is the ending point of Ω_3 in the unit sphere. With the system configuration in figure 1(a), the two directions $\vec{v}_1$ and $\vec{v}_2$ are opposite to each other along the y -axis, i.e., $\vec{v}_1 = -\vec{v}_2$ and $\vec{v}_2 = (0, 1, 0)$. The sum of all three DBP's is

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{e^{\mu t}}{t} [f(\vec{x} + t\vec{v}_2) - f(\vec{x} + t\vec{v}_1)] dt, \\ &= \int_{-\infty}^{\infty} \frac{e^{\mu t}}{t} [f(\vec{x} + t\vec{v}_2) - f(\vec{x} - t\vec{v}_2)] dt, \\ &= \int_{-\infty}^{\infty} \frac{e^{\mu t} + e^{-\mu t}}{t} f(\vec{x} + t\vec{v}_2) dt, \\ &= \int_{-\infty}^{\infty} \frac{e^{\mu t} + e^{-\mu t}}{t} f(x, y+t, z) dt, \\ &= -2\pi \frac{\cosh \mu y}{\pi y} * f(x, y, z) \end{aligned} \quad (7)$$

$$\triangleq \tilde{f}(x, y, z). \quad (8)$$

The DBP of the projection data is then a one-dimensional (1 D) convolution of the activity function with a hyperbolic-cosine-weighted Hilbert kernel along the y -axis. The image can be reconstructed by inverting this convolution.

3.4. Inversion of the finite weighted Hilbert transform

Care must be taken while numerically inverting the hyperbolic-cosine-weighted Hilbert transform. The kernel $(\cosh \mu y)/y$ has a singularity as in the conventional Hilbert transform. However, with constant attenuation coefficient μ , the difference is that the kernel is not bounded at infinity and truncation is also present in the data which makes the problem more complicated.

For $\tilde{f}(x, y, z)$ in (8), the goal is to restore the function $f(x, y, z)$ from $\tilde{f}(x, y, z)$, which is the finite weighted Hilbert transform of $f(x, y, z)$. The function $f(x, y, z)$ can be obtained from the measured data via differentiation and backprojection. However, with truncation, only the part in some finite region (the FOV, for instance) is accurate. In our previous work (Huang *et al* 2009), we showed that $f(x, y, z)$ could be exactly reconstructed from $\tilde{f}(x, y, z)$ if $\tilde{f}(x, y, z)$ was available in the same support region of $f(x, y, z)$. Here we sketch how this is performed.

Since $f(x, y, z)$ and $\tilde{f}(x, y, z)$ are related by a 1 D operator, we simplify notations by $h(y) = f(x, y, z)$ and $H_{\mu}(y) = \tilde{f}(x, y, z)/(-2\pi)$. Assume $f(x, y, z) = 0$ for $y \in (-\infty, -q) \cup (q, \infty)$. This is justifiable since radioactive activity is always restricted in some organs or patient body. Then the finite weighted Hilbert transform is

$$\begin{aligned} H_{\mu}(\tilde{y}) &= \int_{-\infty}^{\infty} \frac{\cosh(\mu(\tilde{y} - y))}{\pi(\tilde{y} - y)} h(y) dy, \\ &= \int_{-q}^q \frac{\cosh(\mu(\tilde{y} - y))}{\pi(\tilde{y} - y)} h(y) dy. \end{aligned} \quad (9)$$

The inversion can be symbolically put in the following equation:

$$h(y) = M^{-1} \int_{-q}^q \frac{H_{\mu}(\tilde{y})}{\pi(\tilde{y} - y)} \sqrt{\frac{q^2 - y^2}{q^2 - \tilde{y}^2}} d\tilde{y}, \quad (10)$$

where M^{-1} is an operator performed on the entire integral, which can be implemented as a matrix multiplication if both $h(y)$ and $H_{\mu}(\tilde{y})$ are column vectors. The matrix $M = M(y, \tilde{y})$ is obtained as follows:

$$\begin{aligned} M(y, \tilde{y}) &= K(y, \tilde{y}) + \delta(y - \tilde{y}), \\ K(y, \tilde{y}) &= \int_{-q}^q \frac{A_{\mu}(s - \tilde{y})}{\pi^2(s - y)} \sqrt{\frac{q^2 - y^2}{q^2 - s^2}} ds, \\ A_{\mu}(y) &= \frac{\cosh(\mu y) - 1}{y}. \end{aligned}$$

More details on implementation can be found in our previous work (Huang *et al* 2009).

4. Numerical study

Let us first outline the four simulations which were performed. In each simulation, the 3D NCAT phantom (Segars *et al* 1999, Segars 2001) with an attached breast and with a 1 cm breast lesion was used for the numerical study. The contrast of breast, lesion, heart, liver and lungs to the body background was set to 2, 20, 75, 75 and 4 to 2. The voxel size was 1.56 mm. Figure 4 shows one transaxial slice of the emission distribution, in the phantom. The first simulation generated attenuated projections without noise assuming a uniform attenuation coefficient of 0.15 cm^{-1} throughout the thorax. This simulation was repeated with Poisson noise added to the projections. A third simulation was performed using the same emission distribution in figure 4 but with a variable attenuation distribution provided by the NCAT phantom. This was repeated for noise by adding Poisson noise to the projections.

The simulated slant-hole collimator had a slant angle $\sigma = 30^\circ$ and a CVOV with a diameter of 12.5 cm. The detector plane was chosen to be a 256×256 array with a bin size of 1.56 mm. Projection data were simulated at three detector positions. The rotating axes of the collimator at these three positions were 60° apart in a transaxial plane. At each detector position projections were simulated at 90 angular steps. Thus, a set of projection data were formed with view angle steps of 1° . Examples can be seen in figure 5 where the detector is located at the second position. The uniform attenuation coefficient was chosen to be 0.15 cm^{-1} , the breast tissue attenuation at 140 keV.

The direction of differentiation is along vector $\vec{a}$ shown in figure 2, which is equivalent to the tangential direction of the trajectory of the projection directions drawn on Orlov's sphere. In the numerical implementation, the central difference operator for the first derivative based on three points was performed on the projections along the directions indicated by the arrows in figure 5. After backprojecting the differentiated data to a 3D mesh in the image space, the 1D inversion of the finite weighted Hilbert transform was performed within the CVOV.

In the backprojection, the activity at each point in the image space was calculated by adding the differentiated data for all the views. For computational simplicity, differentiated projections at the nearest detector bins were used. Then, a 1D processing of the DBP data was performed within the CVOV. This was to invert the finite-weighted Hilbert transform. High activity from outside of the breast, such as heart and liver, contributed to the projection data at some views

but not all views. The algorithm presented in this work reconstructs the activity in the ROI accurately even with this data inconsistency if the attenuation distribution is uniform.

Figure 6 displays the reconstructed image with attenuation correction. Three slices passing through the center of the lesion are shown. The same gray scale is used for both true images and reconstructed images. Figure 6(d) shows profiles along lines indicated in figure 6(a). The profiles of the reconstructed image using the presented algorithm match those of the phantom, except for the end portions of the vertical profiles where the truncation artifacts are severe.

The NCAT phantom with the right breast attached is used in figure 7(a) to illustrate the region of interest that can be reconstructed accurately. With the aforementioned data acquisition system set up, the 1 D filtration on the DBP is performed along parallel lines. The finite Hilbert transform requires the original function to be defined in a finite region and vanishes outside the region. Hence, if the intersection of the filtration lines with the patient body falls within the CVOV, the breast activity can be exactly reconstructed, even when emissions from the heart and the liver also contribute to the measured data. The hexagon in figure 7(a) indicates the CVOV and the region within the hexagon and above the dashed line can be exactly reconstructed with the current system configuration.

Simulation results show that with the presented algorithm, the breast activity in some region can be restored accurately. It is not the case if a traditional backprojection filtering (BPF) algorithm is applied. We designed the filter according to Wessell's dissertation (1999) and show the result in the last column of figures 6(a)–(c). Severe artifacts can be observed in these reconstructed images using the BPF algorithm. The profiles in figure 6(d) also show that the reconstructed image using the BPF algorithm does not agree with the true image quantitatively.

In SPECT imaging, acquired data are always contaminated by noise. Thus the algorithm was evaluated with simulated noisy data. The data without noise were first used to generate data with Poisson noise and then modified as described in section 3.2. To simulate the practical case, the average total count of events was chosen to be 45 000 for the four segments at one view. A reconstructed image from these simulated noisy data is shown in figure 8. In order to illustrate the effect of noise in this algorithm, no filter was used to suppress the noise. The result looks noisy since the differentiation magnifies noise. However, the lesion is evident in the reconstructed image with the current noise level and activity contrast.

In this algorithm, the attenuation is assumed to be uniform everywhere there is activity. However, projections passing through the lungs and the bones are not attenuated in the same way as those only passing through the breast tissues. The image in figure 9 shows the reconstructed image from nonuniformly attenuated data while the reconstruction was performed assuming a uniform attenuation map. The assumption causes quantitative inaccuracy in the image. The activity outside the region of interest and the nonuniformity of attenuation map outside the region of interest both contribute to the inaccuracy. Figure 10 shows the reconstruction with noise at a clinical level. The image is as noisy as that in figure 8 where uniformly attenuated data were reconstructed.

5. Discussions and conclusion

The RMSSH SPECT system is applicable in breast, brain and heart imaging. The DBH method applied to uniformly attenuated SPECT projection data as previously presented (Huang *et al* 2009) can be used in the reconstruction of RMSSH SPECT data with uniform attenuation. The DBH method is a new direction in the development of analytical reconstruction algorithms for tomographic imaging. It provides important insight into understanding the limitations and possibilities for reconstructing truncated projections with uniform attenuation correction. For a well-defined region of the breast the method suppresses the effects of truncation due to the

small CVOV of the RMSSH collimator. Results from using this method are superior to those obtained with a conventional BPF algorithm, which is not capable of reconstructing the image from truncated projections. Furthermore, the implementation of the conventional BPF method requires a three-dimensional filtration, while in the proposed method only a 1D derivative and a 1D de-convolution are required. Even though these results are very encouraging, methods still need to be developed to better quantify in the case of activity included in the projection measurements that is attenuated by the variable attenuation in the thorax.

The region that has an exact and stable reconstruction is determined by the geometry. For instance, if the detectors are located as shown in figure 11(a) instead of as those shown in figure 1(a), then the 1D weighted Hilbert transform would be performed parallel to the line indicated in figure 11(a). As a consequence, the region that can be reconstructed exactly is different and larger as shown in figure 11(b). We would like to point out that for non-attenuated data, Defrise *et al* (2006) generalized the sufficiency condition for an exact and stable reconstruction so that an even larger region can be reconstructed accurately. The question whether the same theory works for uniformly attenuated data or not remains for future work.

Furthermore, compared to the case without attenuation, the simulation and reconstruction with uniform attenuation are more vulnerable to numerical errors. For instance, the non-attenuated projections with detector bins of $3.12 \times 3.12 \text{ mm}^2$ and angular steps of 3° could be sufficient for a good reconstruction; while with the attenuation, finer angular sampling and smaller detector bin size are preferred. Also, for quantitative accuracy especially when the lesion is small, a dense grid is needed for backprojection. In our simulation for example, without attenuation, the voxel size could be 3.12 mm, while with attenuation, the voxel should be as small as 1.56 mm for comparable resolutions.

This work extends our previous analytical work to an application in breast SPECT imaging and provides a fully three-dimensional reconstruction from truncated projections. The performance of the algorithm is compromised in the presence of noise. Data processing procedures could be added to improve the signal-to-noise ratio. The nonuniformity of the attenuation map also results in quantitative inaccuracies. Further work will be focused on variable attenuation correction in the same scenario presented in this paper.

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Figure 1.

Rotating multi-segment slant-hole collimator. (a) A quadrant slant-hole collimator for breast imaging. The detector head collects data at three positions indicated in the figure. At each position, the collimator rotates around its own central axis. (b) The side view of the quadrant slant-hole collimator. The detector field-of-view is determined by the common volume-of-view of the four segments with the slant angle σ .

Figure 2.

Coordinate systems. The imaginary detector plane (shaded) is assumed to be perpendicular to the projection direction $\vec{\theta}$ and contains the origin O . The projection direction is indicated by a unit vector $\vec{\theta}$ with the azimuth angle φ and the zenith angle θ . Unit vectors $\vec{\alpha}$ and $\vec{\beta}$ indicate the directions of the u -axis and the v -axis of the detector plane.

Figure 3. Projection directions on a unit sphere. Each point on the sphere indicates a unit vector. The RMSSH SPECT system in figure 1(a) has projection directions constituting three arcs Ω_1 , Ω_2 and Ω_3 shown in solid lines for positions 1, 2 and 3, respectively.

Figure 4. Sample phantom slice of the emission distribution in a transaxial plane. The contrast of breast, lesion, heart, liver and lungs to the body background was set to 2, 20, 75, 75 and 4 to 2, respectively. Projections from this distribution are simulated both for uniform and for nonuniform attenuation.

Figure 5.
The simulated projections on the detector at position 2 (in figure 3) from two view angles. The arrows indicate the direction of differentiation on each of the four quadrants of the slant-hole collimator.

(a) Transaxial slices.

(b) Coronal slices.

(c) Sagittal slices.

(d) Profiles along lines 1, 2, and 3 indicated in (a).

Figure 6.

Simulation results. (a)–(c) Slices passing through the center of the lesion. The left column is for the true images, the central column is for the reconstructed images with the algorithm presented and the right column is for the reconstructed images using conventional BPF algorithm. All images are displayed in the same gray scale. (d) Profiles along lines shown in the phantom image in (a). The part along the vertical profiles that can be reconstructed exactly with the DBH method is marked as 'ROI' in the figure. With the conventional BPF method, no region can be reconstructed accurately from truncated projections.

Figure 7.

The region of interest that can be reconstructed accurately. (a) A transaxial slice of the NCAT phantom with the right breast attached. With the RMSSH system in figure 1(a), the finite Hilbert transform should be inverted along lines parallel to the dashed line. Within the hexagonal CVOV the part above the dashed line can be reconstructed accurately. (b) A sagittal slice of the same phantom in the CVOV. The part of the breast to the left of the dashed line can be reconstructed accurately.

Figure 8.
The reconstructed images from noisy data displayed with the same gray scale as in figure 6. Only the region which can be reconstructed exactly is shown. No filter was used to suppress the noise effect in the reconstruction and display.

Figure 9. The reconstructed images from nonuniformly attenuated data. The reconstruction was performed assuming a uniform attenuation map, which results in quantitative inaccuracy in the image.

Figure 10.

The reconstructed images from nonuniformly attenuated data with noise. The average number of total counts of the four segments at each view is around 45 000.

(a)

(b)

Figure 11.

(a) A different system orientation. The detectors are located such that the 1 D weighted Hilbert transform is performed parallel to the solid line in the figure. (b) A different available region for exact reconstruction. Within the hexagonal CVOV the part above the dashed line can be reconstructed exactly.