Published in final edited form as:

Magn Reson Imaging. 2008 July ; 26(6): 847–850. doi:10.1016/j.mri.2008.01.050.

Optimal Phased Array Combination for Spectroscopy

Mark Bydder, PhD, Gavin Hamilton, PhD, Takeshi Yokoo, MD, PhD, and Claude B Sirlin, MD
Magnetic Resonance Research, Department of Radiology, University of California San Diego, 200 West Arbor Drive, San Diego CA 92103-8226, Phone: 619 471 0520, Fax: 619 471 0503

Mark Bydder: mbydder@ucsd.edu; Gavin Hamilton: ghamilton@ucsd.edu; Takeshi Yokoo: tyokoo@ucsd.edu; Claude B Sirlin: csirlin@ucsd.edu

Abstract

A method is described for making a weighted linear combination of the spectra acquired by a phased array coil. Unlike most previous combination methods, no special reference points in the data are chosen to represent the coil weights. Instead all the data points are used, which results in more reliable estimation. The method uses singular value decomposition to identify the coils weights and extract the principal component of variation in the signal. Subsequent processing of the combined signal (e.g. Fourier transform, baseline correction, phasing) may proceed as per a single coil acquisition.

Keywords

Spectroscopy; SVD; Phased Array; Coils

Introduction

Several ways to combine multiple spectra acquired by phased array coils have been proposed (1–9). The basic approach is to form a weighted linear combination of the spectra or free induction decays (fid) using weights that ensure constructive addition of signals with maximized signal to noise ratio (SNR). The various methods differ in how the weights are obtained. These may involve a short calibration scan or identification of special reference points that have high signal.

The model of the spectroscopy fid signal considered in the present study is comparable to those used in previous studies. However, for simplicity the coils are considered to be perfectly decoupled and subject to additive complex Gaussian noise with variance σ^2 that is independently and identically distributed. These ideal conditions can be approximately satisfied by hardware calibrations or by pre-processing the data using measurements of the noise correlation, although this problem is not addressed in the present study. Thus the fid signal $S_j(t)$ is described in Eq 1, where A_j is the coil amplitude, ϕ_j is the coil phase, $s(t)$ is the complex time-varying MR signal and $\varepsilon_j(t)$ is noise. The fid comprises time-points $t = 1, 2, \dots, m$ and coils $j = 1, 2, \dots, n$.

Correspondence to: Mark Bydder, mbydder@ucsd.edu.

Publisher's Disclaimer: This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

$$S_j(t) = A_j \exp(i\varphi_j) s(t) + \varepsilon_j(t) \quad [1]$$

The goal of combining the coils is to make a weighted linear sum of the n fids that takes maximum advantage of the SNR offered by the phased array, which can be expressed as in Eq 2 for complex weights w_j .

$$S(t) = \sum w_j S_j(t) \quad [2]$$

As discussed in Ref (7) and (10), the ideal choice should ensure that the signals are correctly aligned and that coils more sensitive to the spectroscopy voxel are weighted more heavily. This suggests using $w_j = A_j \exp(-i\varphi_j)$, however the challenge remains how to determine these weights in practice. Since A_j and φ_j include electronic gain and other scaling factors it is only the relative values of w_j that are important, i.e. a common scaling of the w_j has no effect on SNR. Therefore the signals themselves can be used to estimate the weights, since $S_j(t)$ may be viewed as $A_j \exp(-i\varphi_j)$ scaled by a common factor $s(t)$.

Ref (7) proposed using the first time point of the fids as weights, i.e. $w_j = S_j(1)$. The idea behind this choice is to ensure the noise terms are small relative to the signal terms, which undergo exponential decay and may be expected to be highest at the earliest time-point. Other authors have suggested looking in the spectral-domain for a few points near the largest peak (6), or the area of the largest peak (3) or a linear combination of several different peaks (4). Others have proposed using numerically modeled coil sensitivity profiles or performing a calibration scan (1,2,5) using a non-water saturated acquisition to ensure a large signal (9).

Some of the above approaches use a fixed reference point where the signal is assumed to be large. This introduces a risk of poor estimation, since there are instances when any given point in the fid or spectrum may be degraded; e.g. when a symmetric echo or saturation/inversion pulses are employed or cancellation occurs due to J-coupling. The use of a single reference point also means that estimates are vulnerable to individual noise fluctuations in $\varepsilon_j(t)$, which may be large even though the variance σ^2 is small. A generic approach to reduce this vulnerability is to use multiple points or peak areas, as proposed in several of the studies. The logical end-point of this approach is to use all of the available data points and this should minimize the influence of noise.

A number of ways to do this have been proposed, notably in Ref (8) using a so-called multichannel nonparametric singular value decomposition (SVD) algorithm. Two other methods were described in Ref (8). For the case when the signals are uncorrelated but the noise variance is different on each channel, a minimization procedure was proposed for a specific error metric although it is known that this does not give a maximum likelihood estimate (11). For the case when the number of components in the spectrum is known, Ref (8) suggested a different approach however it is unclear what the value of such a method might be since it blurs the distinction between data acquisition and data analysis that might be better suited to dedicated tools (12).

By contrast, the SVD approach can be shown to produce the optimal SNR for the situation relevant to Eq 1; a theoretical proof is given in Theory. However the SVD method as presented in Ref (8) contains a scaling ambiguity, insofar as the sum of squares of the combined fid $S(t)$ must be unity. The present article describes a way to overcome the scaling problem and compares combined spectra using SVD and the method of Ref (7) over a range of SNR.

Theory

The fid data contained in $S_j(t)$ can be considered an $m \times n$ matrix, where m is the number of measured points in the fid and n is the number of coils. Denoting this as the data matrix $\mathbf{H}$ then, according to Eq 1, $\mathbf{H}$ comprises the sum of a signal term $\bar{\mathbf{H}}$ and a noise term $\mathbf{N}$.

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{N} \quad [3]$$

In the general case, there are a number of ways to estimate $\bar{\mathbf{H}}$ from $\mathbf{H}$ in this situation (13–15). In the present case a simplification is possible as it is known that the rows of $\bar{\mathbf{H}}$ are scalar multiples of the signal vector $s(t)$ and so $\bar{\mathbf{H}}$ is a rank-1 matrix. Ref (8) proposed making a rank-1 approximation of $\bar{\mathbf{H}}$ using SVD, $\bar{\mathbf{H}} = \mathbf{U}\mathbf{S}\mathbf{V}'$, then the estimate of the signal vector from the principal column of $\mathbf{U}$. The weights w_j associated with this combination method are contained in the principal column of $\mathbf{V}$.

Identical weights can be obtained from the eigenvector corresponding to the largest eigenvalue of $\mathbf{H}'\mathbf{H}$. As shown in Ref (16), this choice follows from a formal maximization of the SNR and so the SVD method of Ref (8) may be said to be optimal. A simplified proof is given below based on the comprehensive treatment in Ref (16). The optimization is posed a search for the n -vector $\mathbf{v}$ that maximizes the SNR metric defined in Eq 4.

$$\frac{\text{signal}}{\text{noise}} = \frac{\|\bar{\mathbf{H}}\mathbf{v}\|}{\|\mathbf{N}\mathbf{v}\|} = \frac{\mathbf{v}'\bar{\mathbf{H}}'\bar{\mathbf{H}}\mathbf{v}}{\mathbf{v}'\mathbf{N}'\mathbf{N}\mathbf{v}} \quad [4]$$

The assumption made in the Introduction that noise is uncorrelated and identically distributed means $\mathbf{N}'\mathbf{N} = \sigma^2\mathbf{I}$ and so the denominator is constant. Therefore the optimization amounts to maximizing $\mathbf{v}'\bar{\mathbf{H}}'\bar{\mathbf{H}}\mathbf{v}$ subject to $\mathbf{v}'\mathbf{v} = 1$, which is a Rayleigh quotient with known maximum when $\mathbf{v}$ is the eigenvector corresponding to the largest eigenvalue of $\bar{\mathbf{H}}'\bar{\mathbf{H}}$ (17). In practice $\bar{\mathbf{H}}'\bar{\mathbf{H}}$ is not available and $\mathbf{H}'\mathbf{H}$ must be used. For a statistically large sample ($m \gg n$) the cross-terms vanish and $\mathbf{H}'\mathbf{H} \rightarrow \bar{\mathbf{H}}'\bar{\mathbf{H}} + \sigma^2\mathbf{I}$; since the second term is constant with respect to $\mathbf{v}$ the location of the maximum is unchanged. This concludes the proof.

Note that the definition of SVD requires the columns of $\mathbf{U}$ and $\mathbf{V}$ to have unit norm, which leads to the scaling ambiguity mentioned in Ref (8). This is troublesome for comparing spectra acquired at different TE (say, to correct for T2 decay) because the signals are scaled into the same dynamic range. The scaling problem can be overcome by constructing

$$\mathbf{H}_1 = \mathbf{U}\mathbf{S}_1\mathbf{V}' \quad [5]$$

where $\mathbf{S}_1$ has all but the largest singular value set to zero. The matrix $\mathbf{H}_1$ contains denoised fids with the same scaling as the original data matrix $\mathbf{H}$ and any single column or linear combination of columns may be used as the SNR-optimal fid.

Methods

Spectra were acquired on a 3T Signa TwinSpeed scanner (General Electric Healthcare, Milwaukee, WI) using an 8-channel head coil and a spectroscopy head phantom. The phantom

contained the main brain metabolites: NAA (2.02 ppm), choline (3.02 and 3.91 ppm), creatine (3.2 ppm) and myo-inositol (3.55 ppm). A $10 \times 10 \times 10 \text{ mm}^3$ voxel was selected near the centre of the phantom and identical voxel positions and shim values were used for all acquisitions. A water-suppressed stimulated echo acquisition mode (STEAM) protocol was performed: TR 3000 ms, TE 10 ms, TM 13.7 ms, 5 kHz bandwidth, 2048 sampled points and with 2–64 averages (NEX) to vary the SNR. Raw fid signals from all coils were processed offline using MATLAB (The Mathworks, Natick, MA). Reconstruction times were less than one second on a 3GHz personal computer.

Results

Figures 1 and 2 show plots of the spectra produced by the two methods for the 16 NEX and 64 NEX data sets; in both Figures panel A shows the method of Ref (7) and panel B shows the method of the present study. Visually it is difficult to assess the differences in noise level therefore two quantitative approaches were used to provide a more definitive comparison.

The construction of $\mathbf{H}_1$ facilitates comparisons with the method of Ref (7), which requires a specific linear combination of the columns of $\mathbf{H}$ to generate the estimate. The same linear combination can be for the SVD estimate but of the columns of $\mathbf{H}_1$, which eliminates differences due to scaling or phasing. Results of an SNR comparison are given in Table I. In all cases, the SNR is higher with SVD than the method of Ref (7).

A second way to compare the methods is to look at the consistency of the weights w_j in different data sets. Since there is no reason to expect any difference in the weights other than from noise, the vulnerability to individual noise fluctuations should show up as increased variability. Defining the *Error* as the variance of the weights over all data sets (NEX 2 to 64), then the *Error* for each coil can be computed for the two methods.

Tables II and III show examples of the weights (relative to the first coil) determined by the two methods for 16 and 64 NEX data sets. The amplitudes and angles for the method of Ref (7) exhibit large differences whereas those determined by the SVD method are more similar. These qualitative observations are confirmed by the *Error* given in Table IV. The *Error* is substantially lower for the SVD method indicating weights are more consistently estimated from experiment to experiment.

Discussion

The method proposed in Ref (8) for spectroscopy coil combination using SVD has been investigated and compared with another previously described method (7). It is experimentally verified that the SNR is higher with the SVD method and that the estimates of the coil weights are more consistent from experiment to experiment.

Although only two of the many published methods for spectroscopy coil combination have been compared (1–9), it is meaningful to distinguish between those that use a small number of data points to estimate the weights and those that use all of the data points. Whereas the methods of Ref (1–7,9) are representative of the former, Ref (8) and the present study are representative of the latter. The reason for making this distinction is because the data points are subject to random noise and so the more points used to estimate the weights the greater the reliability of the estimates. A proof of the SNR optimality of using all the data points can be made, based on arguments described in (16,17).

A limitation of the SVD combination method is the requirement for noise to be uncorrelated and identically distributed for the optimality criterion to hold. There are standard ways of pre-processing the signals so that they conform to this model using noise correlation measurements,

as reviewed in Refs (7,16) and elsewhere. Noise correlation measurements require separate calibrations or an assumption that some portion of the acquired data is representative of the noise only. A second limitation, shared with most MR acquisitions, is how to compensate for eddy currents, motion during and other sources of non-Gaussian error during the scan. The choice of weights that yield maximum SNR under a Gaussian noise model is unlikely to produce a maximum likelihood estimate with respect to other types of error, although to make any definitive statement would require the error distribution to be defined and the maximum likelihood estimator for that distribution to be calculated.

Acknowledgments

The authors thank General Electric Healthcare for research support.

References

1. Hardy CJ, Bottomley PA, Rohling KW, Roemer PB. An NMR phased array for human cardiac 31P spectroscopy. *Magn Reson Med* 1992;28:54. [PubMed: 1435221]
2. Wald LL, Moyher SE, Day MR, Nelson SJ, Vigneron DB. Proton spectroscopic imaging of the human brain using phased array detectors. *Magn Reson Med* 1995;34:440. [PubMed: 7500884]
3. Maril N, Lenkinski RE. An Automated Algorithm for Combining Multivoxel MRS Data Acquired With Phased-Array Coils. *J Magn Reson Imaging* 2005;21:317. [PubMed: 15723370]
4. Wright S, Wald LL. Theory and application of array coils in MR spectroscopy. *NMR Biomed* 1997;10:394. [PubMed: 9542737]
5. Schaffter T, Bornert P, Leussler C, Carlsen IC, Leibfritz D. Fast 1H spectroscopic imaging using a multi-element head-coil array. *Magn Reson Med* 1998;40:185. [PubMed: 9702700]
6. Prock T, Collins DJ, Dzik-Jurasz ASK, Leach MO. An algorithm for the optimum combination of data from arbitrary magnetic resonance phased array probes. *Phys Med Biol* 2002;47:N39. [PubMed: 11841052]
7. Brown MA. Time-Domain Combination of MR Spectroscopy Data Acquired Using Phased-Array Coils. *Magn Reson Med* 2004;52:1207. [PubMed: 15508170]
8. Sandgren N, Stoica P, Frigo FJ, Selen Y. Spectral analysis of multichannel MRS data. *J Magn Reson* 2005;175:79. [PubMed: 15949751]
9. Dong Z, Peterson B. The rapid and automatic combination of proton MRSI data using multi-channel coils without water suppression. *Magn Reson Imaging* 2007;25:1148. [PubMed: 17905247]
10. Roemer PB, Edelstein WA, Hayes CE, Souza SP, Mueller OM. The NMR Phased Array. *Magn Reson Med* 1990;16:192. [PubMed: 2266841]
11. Stoica P, Li J. On nonexistence of the maximum likelihood estimate in blind multichannel identification. *IEEE Signal Process Mag* 2005;22:99.
12. Vanhamme L, van den Boogaart A, Van Huffel S. Improved Method for Accurate and Efficient Quantification of MRS Data with Use of Prior Knowledge. *J Magn Res* 1997;129:35.
13. De Moor B. The Singular Value Decomposition and Long and Short Spaces of Noisy Matrices. *IEEE Transactions on Signal Processing* 1993;41:2826.
14. Van Huffel S. Enhanced resolution based on minimum variance estimation and exponential data modeling. *Signal Processing* 1993;33:333.
15. Bydder M, Du J. Noise Reduction in Multiple Echo Data Sets using Singular Value Decomposition. *Magn Reson Imaging* 2006;24:849. [PubMed: 16916702]
16. Walsh DO, Gmitro AF, Marcellin MW. Adaptive Reconstruction of Phased Array MR Imagery. *Magn Reson Med* 2000;43:682. [PubMed: 10800033]
17. Engeln-Müllges, G.; Uhlig, F. *Numerical Algorithms with C*. Schon, M.; Uhlig, F., translators. Springer-Verlag; Berlin, Heidelberg, New York: 1996. p. 166

Figure 1. Spectra for the 16 NEX data, following FFT of the combined fids using the methods of Ref (7) and the SVD method.

Figure 2.
Spectra for the 64 NEX data, following FFT of the combined fids using the methods of Ref (7) and the SVD method.

Table I

SNR estimates for the two combination methods: Ref (7) and SVD of Ref (8) for various NEX. The ratio of the SNR values is also shown – values less than indicate superior SNR for the SVD method. The mean SNR efficiency = SNR/sqrt(NEX) is 2.9 for the method of Ref (7) and 3.3 for the SVD method.

NEX	SNR [†] : Ref (7)	SNR [†] : SVD	Ratio
2	4.34	4.77	0.91
4	6.10	6.95	0.88
8	7.96	9.15	0.87
16	10.67	12.93	0.83
32	14.94	18.00	0.83
64	23.73	25.17	0.94

[†] SNR was estimated as the average of the three closest points to the peak at 2.02 ppm divided by the standard deviation of the noise between 5 and 15 ppm. For the reasons noted in the Introduction, the use of only a small number of points means this measure is subject to noise error however it does provide an approximate estimate.

The amplitude and angle of the weights determined by the method of Ref (7) for two different data sets: one with 16 NEX and one with 64 NEX. Ideally the weights should be the same, however large differences are observed between experiments.

Table II

Coil <i>j</i>	1	2	3	4	5	6	7	8
$ w_j $ (NEX=16)	1	1.44	0.47	0.76	0.59	1.23	0.43	0.26
$ w_j $ (NEX=64)	1	0.99	1.06	0.47	0.97	1.34	0.79	0.45
$\angle w_j$ (NEX=16)	0	-1.36	-1.44	-1.16	0.71	-0.70	-0.92	2.60
$\angle w_j$ (NEX=64)	0	-1.25	-0.83	-1.08	1.16	-0.99	-0.68	-0.68

Table III

The amplitude and angle of the weights determined by the SVD method for the same two data sets as in Table II. The weights are relatively consistent for the two experiments.

Coil j	1	2	3	4	5	6	7	8
$ w_j $ (NEX=16)	1	1.30	1.38	1.35	1.23	1.21	1.08	0.80
$ w_j $ (NEX=64)	1	1.31	1.26	1.25	1.25	1.23	1.03	0.82
$\angle w_j$ (NEX=16)	0	-0.80	-1.92	-2.39	0.57	-1.07	-0.80	-1.25
$\angle w_j$ (NEX=64)	0	-0.77	-1.88	-2.37	0.63	-0.98	-0.70	-1.25

Error, defined as the variance of (complex) coil weights over all data sets (NEX 2 to 64), determined for the two methods. The *Error* gives a measure of the sensitivity to noise in the estimation of coil weights and a lower value indicates less sensitivity.

Table IV

Coil <i>j</i>	1	2	3	4	5	6	7	8
<i>Error</i> : Ref (7)	0	0.48	1.07	2.17	1.00	1.35	0.78	1.05
<i>Error</i> : SVD	0	0.05	0.03	0.10	0.09	0.05	0.07	0.17