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*THE ABUNDANCE OF LEAD IN THE SUN**

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The nuclear stability of lead and its role as the end product of decay schemes of naturally occurring radioactive elements lend this metal a peculiar importance in geochronology and cosmology. A datum of importance is the silicon/lead ratio. This ratio has been measured for chondrites and other meteorites. It is difficult to obtain a meaningful measurement for the earth because of the extreme fractionation suffered by terrestrial rocks.

Unless it has been affected by thermal and gravitational diffusion,¹ the solar abundance of lead presumably represents the primordial solar system abundance. Unfortunately, we cannot determine the solar isotope ratios by spectroscopic methods.

By a curve-of-growth procedure, which neglects the influence of the variation of temperature and pressure with depth in a stellar atmosphere, Unsöld² derived an abundance $\log N(\text{Pb}) = 2.55$ on the scale $\log N(\text{H}) = 12.00$. In their study of the composition of the solar atmosphere, Goldberg, Müller, and Aller³ obtained $\log N(\text{Pb}) = 1.33$ on the basis of unpublished f values by Aller and 1.25 using data by Khokhlov.⁴ Helliwell,⁵ using a model solar atmosphere by Claas,⁶ obtained $\log N(\text{Pb}) = 1.90$. Mutschlecner⁷ used Helliwell's f values but an improved model atmosphere; he also tried to take into account the effect of overlapping line wings. He obtained $\log N(\text{Pb}) = 1.64$. Very recently, Grevesse, Blanquet, and Boury,⁸ using excellent new observational material, obtained $\log N(\text{Pb}) = 2.00$.

The difficulty in the determination of the solar lead abundance arises not so much from uncertainties in the transition probabilities or f values, although these certainly stand in need of improvement, as in the character of the lines themselves. All solar spectral lines of lead fall in crowded regions of the near ultraviolet and are mostly heavily blended. Hence if one uses conventional methods, he must estimate the intensity of the line, or more particularly the width in equivalent angstroms it would have if it were unblended. The trouble with this procedure is that it is sometimes difficult to make reliable allowances for blending effects.

We have adopted a procedure called "spectrum synthesis" in which one attempts to reproduce the detailed intensity distribution over the relevant wavelength region of the solar spectrum. In order to carry out this technique one must know:

- (1) the wavelengths and identifications of all the lines in the region;
- (2) the absolute intensity at some point in the spectral region; and
- (3) the accurate, i.e., high spectral resolution, intensity distribution over the spectral region involved.

In addition, one should know the limb darkening in the nearby continuous spectrum in order to obtain a check on the continuous absorption coefficient and its variation with depth in the sun.

Let $\mu = \cos \theta$, where θ is the angle between the outward-directed normal and the ray. Then the emergent intensity is given by the well-known expression⁹

$$I_{\lambda}(o, \mu) = \int_0^{\infty} S_{\nu} e^{-\tau_{\nu}/\mu} d\tau_{\nu}/\mu, \quad (1)$$

where S_{ν} , the source fraction, is known for the line-forming regions of the solar atmosphere. The element of optical depth in the continuum is

$$d\tau_{\nu} = \frac{\kappa}{\kappa_0} d\tau_0, \quad (2)$$

where κ is the continuous absorption coefficient at the frequency ν and κ_0 is the corresponding quantity at the standard frequency ν_0 .

In the line,

$$dt = \frac{l_{\nu} + \kappa}{\kappa_0} d\tau_0, \quad (3)$$

where the line absorption coefficient, l_{ν} , varies rapidly across the line. The continuous absorption coefficient κ varies slowly with frequency.

The residual intensity in the line, r_{ν} , is defined as

$$r_{\nu} = I_{\nu}(\text{line})/I_{\nu}(\text{continuum}). \quad (4)$$

To calculate r_{ν} we must know $S_{\nu}(t_0)$, $t(t_0)_{\text{line}}$, $t(t_0)_{\text{contin.}}$. The continuous absorption coefficient is readily calculated once the temperature $T(t_0)$ and gas pressure $P_g(t_0)$ are given by the solar model atmosphere.

The line absorption coefficient is given by

$$l_{\nu} = n_{r,s} \alpha_0 H(a, \nu), \quad (5)$$

where $n_{r,s}$ is the number of atoms capable of absorbing the line in question (ionization stage r , excitation level s). It is calculated by the Saha and Boltzmann equation. The effective absorption coefficient at the line center is

$$\alpha_0 = \frac{\sqrt{\pi} \epsilon^2}{M_e C} f \frac{c}{\nu_0 V} = \frac{0.01497 f \lambda}{\sqrt{\frac{8.38 \times 10^{11}}{M \theta} + \xi^2}} \quad (6)$$

where $\theta = 5040/T$, V is the most probable atomic velocity, M is the atomic weight, and ξ is the most probable turbulent velocity. Here ξ depends on the model atmosphere adopted. The line-broadening function

$$H(a, \nu) = H\left(\frac{0.07958\Gamma}{\lambda V}, \frac{\Delta\lambda}{\lambda} \frac{c}{V}\right) \quad (7)$$

is symmetrical about the central frequency ν_0 and depends on the damping constant Γ . For weak to moderately strong lines, the value of Γ has little influence on the total energy absorbed by the line, but the line profile depends strongly on Γ . In the solar atmosphere, the major contribution to Γ comes from the van der Waal's interactions with neutral hydrogen atoms. After O'Mara we may write¹²

$$\Gamma = \Gamma_{\text{coll}} \times 10^{3\theta^{0.7} P_g}. \quad (8)$$

In the simplest instance a single line is formed in an unperturbed region of the spectrum. The known sources of opacity then usually suffice to account for the observed intensity of the continuum. One adjusts the abundance of the element or more particularly the quantity $gf\lambda A$, where g is the statistical weight, f is the f value, λ the wavelength, and $A = N(\text{element})/N(\text{H})$ until a fit to the line profile is obtained. It is often necessary to try several values of Γ_{coll} to obtain a satisfactory fit. Note that one obtains the product gfA , not the abundance itself.

If two or more lines develop at a given point, then

$$I_\nu = \sum n_{r,s(i)} a_0(i) H(a(i), \nu(\nu - \nu_i)), \quad (9)$$

where ν_i is the center of the i th line and the summation is taken over all relevant lines. The problem is now more complicated in that one has two parameters to adjust for each line, $gf\lambda A$ and Γ_{coll} , and is required to fit a pattern that may be fairly complex.

The observational data consists of high-resolution spectrum scans made with the vacuum spectrograph at the McMath Hulbert Observatory of the University of Michigan. Later, these were supplemented by scans made with the McMath solar telescope at Kitt Peak National Observatory.

Figure 1 shows the fitting of the profile of the 3683.476 line. Figure 2 shows the representation of the Michigan data for the 3639.583 line, while Figure 3 compares the Kitt Peak and Michigan traces. Note that the theoretical points denoted by circles can be made to agree with the Michigan observations more closely than the observations agree among themselves. A few discrepancies remain; the center of the strong Fe—Co—V blend near $\lambda 3683.1$ is not as deep as predicted. Since we have assumed local thermodynamic equilibrium which is known to fail at the centers of strong lines, a discordance here is not unexpected. Alternately, the line actually may be deeper than has been depicted. Faint, as yet unknown lines may fall near $\lambda 3683.2$. The $\lambda 3639.583$ line falls in the wing of a complex blend; the representation is accordingly more difficult.

The symbol $\times$ denotes a line of unknown origin. These transitions present a difficult problem for the methods of spectral synthesis. Some of them are undoubtedly metals; other weak features may be of molecular origin. We handle

TABLE 1. *Data for synthesis of Pb I 3639.583, 3683.476*

λ	Element	Γ_{coll}	$\log(gf\lambda A)$	$\log(gf)$	Ref.	$\log N$	Notes
3639.277	X	550	-12.348	NA	—	—	(1)
3639.321	FeI	550	-12.098	NA	—	—	—
3639.450	CoI	700	-12.448	-0.02	CB	4.01	—
3639.521	FeI	600	-11.808	NA	—	—	—
3639.583	PbI	1000	-15.138	-0.554	H	1.85	—
3639.691	FeI	500	-13.418	NA	—	—	—
3639.802	CrI	700	-11.138	1.19	CB	4.11	—
3683.045	CoI	1000	-12.553	0.34	CB	3.54	(2)
3683.092	FeI	1000	-13.643	0.01	CB	2.78	(2)
3683.130	VI	1000	-12.833	-0.45	CB	4.05	(2)
3683.380	X	1000	-12.863	NA	—	—	(3)
3683.476	PbI	400	-15.123	-0.305	H	1.62	—
3683.623	FeI	300	-11.853	NA	—	—	(4)
3683.623	FeI	300					

Notes.

$N = 10^{12} n_{\text{a1}}/n_{\text{H}}$.

NA: gf values not available.

References to f values:

CB = Corliss, C., and W. R. Bozman, *U. S. Natl. Bur. Stand. Mono. no. 53* (1962).

H = experimental values by Helliwell, T. M., *Ap. J.*, **133**, 566 (1961).

- (1) Synthesized as though it were an FeI line with lower excitation potential of 2.0 volts.
- (2) This complex blend cannot yield any meaningful abundances.
- (3) Synthesized as though it were an FeI line with lower excitation potential of 3.0 volts.
- (4) This iron line is a blend of two lines of unknown f values but excitation potentials of 2.48 and 3.30 eV; for the purpose of this calculation we took the f values as equal.

them in the analysis as though they were metallic lines, and treat them in detail as though they were FeI lines. The procedure appears to be adequate unless the line is severely blended with the transition of interest.

Table 1 shows the detailed parameters required in the synthesis for $\lambda 3639.583$ and $\lambda 3683.476$. The process yields the value of Γ_{coll} and $\log gf\lambda A$ for each line. Once a given region is matched to good observations, adjustment in the abundance to allow for new f value determinations is made immediately. A further advantage of the spectrum synthesis method is that if a line is blended with some heretofore unrecognized feature, it usually will be impossible to get a good fit to the profile.

Metallic lines behaved in somewhat similar fashions. It was possible to get a good fit of the 3639 region by treating 3639.52 as though it were the lead line and 3639.583 as an iron line. The lead abundance was false, of course. This possibility comes about from freedom in choosing the gfA product and Γ_{coll} . Note that in the procedure the latter parameter is derived from the observations and may be falsified if an incorrect value for the mean turbulent velocity is used.

TABLE 2. *The solar abundance of lead.*

λ	Γ_{coll}	$\log gf$	Source of f value	$\log gf\lambda A$	$\log N$ (Pb)/(N(H) + 12.00)
3639.583	1000 ± 300	-0.554	H, exp	-15.138	1.85 ± 0.10
3683.476	400 ± 100	-0.305	H, exp	-15.123	1.62 ± 0.05
3740.944	900 ± 300	+0.70	Ref. 3	-14.197	1.50 ± 0.20
4057.813	1400 ± 300	+0.262	H, exp	-14.36	1.80 ± 0.10

H, exp = Helliwell's experimental f values (ref. 5).

Ref. 3 = f value due to Allen, quoted by Goldberg *et al.*

FIG. 1.

FIG. 2.

FIG. 3.

The tabulated lead abundances are used on Helliwell's experimental f values. If we use Penkin's results,¹⁰ we obtain $\log N = 2.03$ from $\lambda 3639.6$ and 1.76 from $\lambda 3683.48$.

Table 2 summarizes the results for all four of the lead lines. The data for $\lambda 3740$ and $\lambda 4057$ are less accurate because of difficulty in synthesizing extremely crowded regions of the solar spectrum. The uncertainties arising from the goodness of fit are indicated in the last column. Before we derive a "most probable" abundance, two remarks are in order. First, the continuous absorption coefficient in the region of the 3740 and 3683 lines has been underestimated because of the difficulty in taking into account the overlapping hydrogen lines. The

tabulated values are lower limits; when this hydrogenic blending effect is taken into account, the abundances are raised by about $\delta \log N = 0.1$. Secondly the lines must be assigned weights depending on the complexity of the region synthesized. When this is done, we find $\log N(\text{Pb}) = 1.76$ ($\log N(\text{H}) = 12.00$), using Helliwell's experimental f values. If we use the values recommended recently by Brown,¹¹ the lead abundance should be increased by about a factor of 2.

Adopting O'Mara's¹² value for the silicon abundance, $\log N(\text{Si}) = 7.30$, we obtain for the lead/silicon ratio 2.9×10^{-6} as compared with about 1.6×10^{-6} for the Orgueil meteorite, which is a carbonaceous chondrite.¹³ If Brown's f values are adopted, the ratio is about 6×10^{-6} .

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