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Ultrasonic focusing through inhomogeneous media by application of the inverse scattering problem

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Abstract

A new approach is introduced for self-focusing phased arrays through inhomogeneous media for therapeutic and imaging applications. This algorithm utilizes solutions to the inverse scattering problem to estimate the impulse response (Green's function) of the desired focal point(s) at the elements of the array. This approach is a two-stage procedure, where in the first stage the Green's functions is estimated from measurements of the scattered field taken outside the region of interest. In the second stage, these estimates are used in the pseudoinverse method to compute excitation weights satisfying predefined set of constraints on the structure of the field at the focus points. These scalar, complex valued excitation weights are used to modulate the incident field for retransmission. The pseudoinverse pattern synthesis method requires knowing the Green's function between the focus points and the array, which is difficult to attain for an unknown inhomogeneous medium. However, the solution to the inverse scattering problem, the scattering function, can be used directly to compute the required inhomogeneous Green's function. This inverse scattering based self-focusing is noninvasive and does not require a strong point scatterer at or near the desired focus point. It simply requires measurements of the scattered field outside the region of interest. It can be used for high resolution imaging and enhanced therapeutic effects through inhomogeneous media without making any assumptions on the shape, size, or location of the inhomogeneity. This technique is outlined and numerical simulations are shown which validate this technique for single and multiple focusing using a circular array.

INTRODUCTION

Ultrasound in the range of 500 kHz–13 MHz is gaining wider acceptance for a range of diagnostic and therapeutic applications.^{1–3} Phased arrays have become the standard in most diagnostic imaging systems since the mid 1980's. Recently, therapeutic systems that utilize phased arrays in hyperthermia^{4–7} and other high intensity applications have been introduced.^{8–10} One of the main advantages of phased arrays is their ability to scan at electronic speed thus eliminating the need for mechanical scanning. Coupled with advanced VLSI-based digital beamforming architectures, phased arrays promise to increase the utility of diagnostic systems by improving image quality, improving reliability, and increasing functionality. Similar improvements can be expected for therapeutic systems utilizing phased arrays.

A major promise of phased arrays that has not yet been fulfilled is their ability to noninvasively compensate for tissue inhomogeneity and, therefore, focus in the presence of such inhomogeneity.^{11–15} The successful implementation of such self-focusing arrays will have a major impact on ultrasonic imaging systems by allowing the use of large-aperture arrays which

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could help improve both spatial and contrast resolution in the next generation. Similarly, self-focusing arrays will provide essential components of a completely non-invasive therapeutic system capable of producing precise therapeutic dose at deep-seated tissue targets (for example, tumors) with a high degree of spatial localization.

Focusing is achieved by adjusting the phases of the ultrasound waves incident from each of the array elements so that they add constructively at the focus point of interest. In conventional (or geometrical) focusing, the phase associated with each transducer is computed from the known constant background speed of sound and the distance between that transducer and the desired focus point.¹⁶ This type of focusing is purely geometrical. In other words, it ignores the effects of tissue inhomogeneity on the speed of propagation of the ultrasonic waves. The phase distortions introduced by using the conventional focusing mechanism could be severe and may not be ignored. Phase aberrations can cause the energy at the focus to be dispersed and shifted from the intended location.¹⁵ Different invasive and noninvasive aberration correction mechanisms have been introduced and are being used for imaging^{13–15,17} and hyperthermia.^{5,7,18,19} Time delay estimation techniques, such as by cross-correlating signals from neighboring array elements, only correct for the effects of a thin aberrating layer near the phased array.¹³ Effectively, such techniques correct phase errors only. Recent extensions of this work attempt to correct for amplitude distortions as well as phase distortions.²⁰

Optimal pattern synthesis through homogeneous or inhomogeneous media can be attained using the pseudoinverse method.^{18,21–23} Using this technique, we solve for a scalar, complex valued excitation weight for each array transducer from the wave propagation matrix of that medium. This method was demonstrated to provide powerful results for field pattern synthesis within the region of interest (ROI) according to a predetermined collection of constraints on the structure of the total field at a set of points in the ROI (control points). The idea behind this method is to relate the excitation weights to the set of control points using a matrix operator for wave propagation in the homogeneous or inhomogeneous media. One can solve for the minimum-norm least-squares solution of the excitation weights using the pseudoinverse of the propagation matrix. A major advantage of this method is the ability to deliver precise power deposition levels to multiple points in space. Another advantage is the ability to optimize the solution procedure for increased array excitation efficiency and intensity gain, without introducing undesired hot spots.²³ The pseudoinverse pattern synthesis method requires knowing the Green's function of the desired focus point at the array.^{5,7,18} This cannot be easily achieved for the inhomogeneous case (in the presence of an aberrating medium).

The Green's function can be measured using a needle hydrophone implanted in the region at the point of interest.^{5,7,18,19} This measurement can also be used to compute the Green's function at other desired locations near the hydrophone. This method has been used successfully for hyperthermia, however, it has some limitations. First, it is invasive, requiring a hydrophone near the desired focus point. Second, it expects the desired point to be within the homogeneous medium surrounding the hydrophone. In other words, the region where focusing is to take place must be reasonably far from any aberrating medium (or inhomogeneity).

A time reversal mirror approach was demonstrated with several phased arrays and was shown to provide effective method of focusing in the presence of strong tissue inhomogeneities.¹² It was also shown that the iterative time reversal method converges to an estimate of the Green's function of the location of the dominant scatterer in the ROI to the array surface.^{24,25} This method has been extended to allow focusing near the dominant scatterer by estimating the Green's function to a desired point using the Green's function of the dominant scatterer.²⁶ Although these approaches are not limited to a thin aberrating layer near the transducer array, they do require a distinct dominant scatterer surrounded with a homogeneous medium.²⁶ Another extension of the time reversal mirror is the transfer matrix approach which allows for

selective focusing.^{24,25} A time reversal operator is computed whose eigenvectors provide the amplitudes and delays required to focus on selected scatterers. This approach requires that the scatterers be distinct (well resolved), and are distant from the aberrating medium. In fact, most approaches to aberration correction assume that the aberrating medium is a layer situated far from the desired focus point. In many cases, such as in breast tissue, this assumption is invalid.

We present a new self-focusing strategy which can prove successful in focusing through inhomogeneous media. In this strategy, we make use of the scattered field measured by the phased array to estimate the Green's function needed by the pseudoinverse pattern synthesis method. In particular, we use the distorted Born iterative (DBI) method to solve the nonlinear inverse scattering problem. The main idea behind the DBI method is that it alternates between (1) updating an estimate of the spatial structure of the inhomogeneity (the scattering function) given a current best estimate of the inhomogeneous Green's function and the total acoustic field in the ROI (i.e., solving an inverse scattering problem) and (2) updating the total field and Green's function given the previously generated estimate of the inhomogeneity (i.e., solving a forward scattering problem). The initial step of the algorithm corresponds to the zeroth order Born approximation, and the algorithm terminates when the Green's function is deemed to have converged. The convergence criterion is a percentage change in the estimated Green's function. Since the algorithm improves on the estimate of the kernel (the Green's function) at every iteration, it is referred to as the distorted Born iterative method (as opposed to the Born iterative method which updates only the total field at every iteration).^{27–30} Once convergence is achieved, we use the estimated Green's function of the desired focus pixel to construct the propagation matrix required by the pseudoinverse pattern synthesis method.^{21, 23} This self-focusing approach is noninvasive and it does not make any assumptions about the background medium, about the shape and location of the aberrator, and about the distribution of the scatterers. It simply requires measurements of the scattered field outside the region of interest.

We present numerical simulations using a circular array with 128 line transducers which surround the region of interest as shown in Fig. 1. Only a subset, 32 transducers, is used for transmission in solving the inverse scattering problem. The results show the accuracy of the DBI method in estimating the Green's function as compared with the Green's function computed using the true spatial speed of sound. We use the pseudoinverse pattern synthesis method as the means of obtaining optimized (in the minimum-norm least-squares sense) excitation weights for retransmission using the 128 array transducers. We present simulation results of field patterns, which show the quality of focus as compared with geometrical focusing. We also compare the obtained field patterns with field patterns obtained using the true Green's function (computed from the true speed of sound). In addition, we present results of multiple focusing which show the additional advantage of this combined inverse scattering-based focusing and the pseudoinverse pattern synthesis methods.

In Sec. I, we present the theory and solution of the forward and inverse scattering problems. We also discuss the solution of the general Green's function in terms of the forward scattering problem, and present a numerical method for solving such a problem for an unknown ROI. In Sec. II, we discuss the pseudoinverse pattern synthesis method and focusing through inhomogeneous media using estimates of the Green's function. The quality of focus is discussed in relation with the geometrical focusing technique and with the optimal focusing technique obtained using the true Green's function. We show simulation results and discuss the advantages of inverse scattering-based self-focusing in Sec. III. Finally, in Sec. IV, we conclude with a summary and planned future work.

I. MATHEMATICAL FORMULATION

Consider an infinite space containing a homogeneous loss-free acoustic medium with a constant background speed of sound, c_0 . Embedded inside this space is a lossy, inhomogeneous medium with associated space-varying speed of sound, and constant density. Then the total field at a point, $\mathbf{r}$, due to a time-harmonic radiating source with $e^{-j\omega t}$ time dependence, can be described by the Helmholtz equation,

$$(\nabla^2 + k^2(\mathbf{r}))\Phi(\mathbf{r}) = 0, \quad (1)$$

and

$$\Phi(\mathbf{r}) = \Phi_i(\mathbf{r}) + \Phi_s(\mathbf{r}), \quad (2)$$

for $\mathbf{r} \in \mathcal{R}^2$, where Φ is the total acoustic field for the longitudinal motion, Φ_i is the incident field, and Φ_s is the scattered field which satisfies the Sommerfeld radiation condition uniformly in all directions. The spatial function $k^2(\mathbf{r}) = [\omega^2/c^2(\mathbf{r})] + j[2\omega\alpha(\mathbf{r})/c(\mathbf{r})]$ is the wave number at frequency ω , speed of sound $c(\mathbf{r})$, and attenuation factor $\alpha(\mathbf{r})$.

An integral form representation of the scattering problem, equivalent to Eqs. (1) and (2) is the Lippmann–Schwinger equation, defined as

$$\Phi(\mathbf{r}) = \Phi_i(\mathbf{r}) + \int_{\mathcal{R}^2} G_0(\mathbf{r}, \mathbf{r}') f(\mathbf{r}') \Phi(\mathbf{r}') d\mathbf{r}', \quad (3)$$

$$f(\mathbf{r}) = \omega^2 [1/c^2(\mathbf{r})$$

where $-1/c_0^2] + j[2\omega\alpha(\mathbf{r})/c(\mathbf{r})]$ is the unknown scattering function, and G_0 is the scalar homogeneous Green's function.^{28,31} The second term on the right hand side of Eq. (3) is the scattered field, $\Phi_s(\mathbf{r})$, which is measured at discrete points outside the ROI.

The scalar free-space (homogeneous) Green's function satisfies the wave equation,

$$(\nabla^2 + k_0^2)G_0(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}'), \quad (4)$$

k_0 where $k_0 = \omega/c_0$, and $\delta(\mathbf{r})$ is a two-dimensional unit impulse function which could represent a line source, or a line inhomogeneity. In other words, Green's function, $G_0(\mathbf{r}, \mathbf{r}')$ models the field at a point $\mathbf{r}$ resulting from a single line source at $\mathbf{r}'$, in the absence of any boundaries. The two dimensional free-space Green's function in cylindrical coordinates has a closed form solution in terms of the zeroth order cylindrical Hankel function of the first kind,^{28,32}

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{j}{4} H_0^{(1)}(k_0 |\mathbf{r} - \mathbf{r}'|). \quad (5)$$

In general, the Green's function for inhomogeneous media is unknown and must be computed numerically as shown below.

A. The forward scattering problem

We define the forward scattering problem as that of solving the scattering problem defined by Eq. (3) for the total field, $\Phi(\mathbf{r})$, at any point $\mathbf{r} \in \mathcal{R}^2$. We follow the method of moments to construct a matrix system representing Eq. (3),^{28,33} which is then solved by matrix inversion.

First, the unknown field $\Phi(\mathbf{r})$ is expanded as follows:

$$\Phi(\mathbf{r}) = \sum_{n=1}^N \varphi_n g_n(\mathbf{r}), \quad (6)$$

where φ_n are the expansion coefficients, the integer parameter N is the number of pixels covering the region of interest, and the function $g_n(\mathbf{r})$ is an appropriate basis function. In this paper, $g_n(\mathbf{r})$ is simply a two-dimensional rectangular pulse centered at the n th pixel. The region of interest is large enough so that it completely contains the scattering object. It must be noted that for an accurate recovery of the field from φ_n , N (for a given region of interest) must be selected to satisfy the sampling theorem.

Replacing $\Phi(\mathbf{r})$ from both sides of Eq. (3) by its expansion, and after some algebraic manipulations, gives

$$\sum_{n=1}^N \varphi_n \left[g_n(\mathbf{r}) - \int_{S_n} G_0(\mathbf{r}, \mathbf{r}') f(\mathbf{r}') g_n(\mathbf{r}') d\mathbf{r}' \right] = \Phi_i(\mathbf{r}), \quad (7)$$

where S_n is the region of support of the n th basis function, in this case, the square area of the n th pixel.

Upon multiplying both sides of Eq. (7) with a point matching function, $w_m = \delta_m$, $m = 1, \dots, N$, and integrating the result, we obtain

$$\begin{aligned} \sum_{n=1}^N \varphi_n \left[\delta_{mn} - \int_{S_n} G_0(\mathbf{r}_m, \mathbf{r}') f(\mathbf{r}') g_n(\mathbf{r}') d\mathbf{r}' \right] \\ = \Phi_i(\mathbf{r}_m). \end{aligned} \quad (8)$$

The two-dimensional impulse step function δ_m is unity at $\mathbf{r} = \mathbf{r}_m$, and is zero everywhere else. Similarly, the function δ_{mn} is unity at $\mathbf{r} = \mathbf{r}_n = \mathbf{r}_m$ (or when $m = n$), and is zero everywhere else.

Equation (8) can be written in matrix form as

$$\mathbf{P}\boldsymbol{\varphi} = \boldsymbol{\varphi}_i, \quad (9)$$

where $\mathbf{P}$ is an $N \times N$ nonsingular matrix, $\boldsymbol{\varphi}$ is an $N \times 1$ vector containing the unknown total field coefficients, and $\boldsymbol{\varphi}_i$ is an $N \times 1$ vector of elements from the incident field evaluated at $\mathbf{r}_m$ for $m = 1, \dots, N$. Once the matrix $\mathbf{P}$ is computed, it may be inverted to solve for the coefficients of the total field. Normally, a solution of this forward scattering problem is obtained for each transmit position, using the scattering function, $f(\mathbf{r})$, or its estimate.

B. The inverse scattering problem

The inverse scattering problem is that of solving Eq. (3) for the scattering function, $f(\mathbf{r})$, from measurements of the scattered field outside a ROI. As in the forward scattering problem, we use the method of moments to obtain a matrix representation of the inverse scattering problem. In this case, however, the unknown parameter is $f(\mathbf{r})$. Hence, we use the expansion

$f(\mathbf{r}) = \sum_{n=1}^N f_n g_n(\mathbf{r})$ in the integral formula of the scattered field $\Phi_s(\mathbf{r})$,

$$\Phi_s(\mathbf{r}) = \int_{\mathbb{R}^2} G_0(\mathbf{r}, \mathbf{r}') f(\mathbf{r}') \Phi(\mathbf{r}') d\mathbf{r}', \quad (10)$$

to produce

$$\begin{aligned} \sum_{n=1}^N f_n \left[\int_{s_n} G_0(\mathbf{r}_m, \mathbf{r}') g_n(\mathbf{r}') \Phi(\mathbf{r}') d\mathbf{r}' \right] \\ = \Phi_s(\mathbf{r}_m), \end{aligned} \quad (11)$$

for $m = 1, \dots, M$ and $n = 1, \dots, N$, where M is the number of array elements used for reception, and N is the number of pixels in the ROI. In matrix form, we have

$$\mathbf{Q}\mathbf{f} = \boldsymbol{\varphi}_s, \quad (12)$$

where $\mathbf{Q}$ is an $M \times N$ matrix, $\mathbf{f}$ is an $N \times 1$ vector containing the unknown scattering function coefficients, and $\boldsymbol{\varphi}_s$ is an $M \times 1$ vector containing measurements of the scattered field as observed at all M points.

We note, by observing Eq. (11), that Eq. (12) is dependent on the total field inside the ROI which is normally unknown. This requires approximating the wave equation using either the Born or Rytov methodologies. Both have certain advantages and disadvantages.³⁴ The Rytov approximation is potentially useful at high operating frequencies. Whereas, the Born approximation is more appropriate at low operating frequencies^{34,35} and is, therefore, the method of choice throughout our current work. The zeroth order Born approximation assumes that the total field in the ROI is equal to the incident field, which is acceptable only for weak scattering. In other words, it assumes that $\Phi_s(\mathbf{r})$ is small compared to the incident field, $\Phi_i(\mathbf{r})$.^{28,34} In applications of interest, such as image-guided therapy in the breast, the regions are characterized as highly scattering,³⁶ thus corrections are usually needed to obtain reasonable inverse solutions.^{27–30,35} Also, the kernel matrix of the inverse scattering problem is under-determined and ill-posed.³⁷ We resolve these problems by using the singular value decomposition to obtain a regularized minimum-norm least-squares solution.^{37–40} The inverse solution obtained from Eq. (12) is due to one transmit position. Normally, the inverse scattering problem is solved for multiple transmit position, where the individual inverse solution vectors are vector-averaged to produce a reasonable final solution vector.

C. Estimation of the Green's function

We utilize the distorted Born iterative method for solving the nonlinear inverse scattering problem. This method, unlike the Born iterative method, attempts to correct for the integral equation kernel (Green's function) as well as the total field. At every iteration, the algorithm

uses the latest estimate of the scattering function to compute the Green's function and the total field at every pixel in the ROI. These estimates are used in the inverse scattering problem to update the scattering function incrementally. The advantages of the DBI method are that it is not limited to a simple homogeneous background and that it has a larger range (in scattering strength) of applicability.^{29,35} Although this algorithm has a faster convergence rate relative to the simpler Born iterative method, it is computationally intensive and, at this stage, cannot be used for real time applications. Also, the algorithm is sensitive to measurement noise, therefore careful treatment of updating and inverting the inverse problem kernel matrix is essential.^{39–41} Finally, in the case of strongly scattering media, the DBI method could diverge or converge to an erroneous inverse solution, thus producing wrong estimates of the inhomogeneous Green's function. This is caused by a misleading initial Born approximation. Another aspect of our research deals with improving the DBI method for solving the inverse scattering problem of large, strongly scattering regions. These techniques can still be used in place of the DBI method for self-focusing.⁴¹ The DBI method was developed and used extensively for electromagnetic inverse scattering applications.^{27–29} Its applicability and utility in ultrasound is obvious and appears to be promising.⁴¹

In estimating the inhomogeneous Green's function, we make use of the fact that this general (inhomogeneous) Green's function is the impulse response of the inhomogeneous wave equation. Consequently, we can write the Green's function at any point $\mathbf{r}$ due to some fictitious source at $\mathbf{r}_s$ as the solution to the forward scattering problem,

$$G(\mathbf{r}, \mathbf{r}_s) = \Phi_i(\mathbf{r}) + \int_{\mathbb{R}^2} G_0(\mathbf{r}, \mathbf{r}') f(\mathbf{r}') G(\mathbf{r}', \mathbf{r}_s) d\mathbf{r}'. \quad (13)$$

Equation (13) is a forward scattering problem which, upon discretization using the method of moments, translates into a matrix system equivalent to Eq. (9),

$$\mathbf{P} \underbrace{\begin{bmatrix} G(\mathbf{r}_1, \mathbf{s}_1) & G(\mathbf{r}_1, \mathbf{s}_2) & \cdots & G(\mathbf{r}_1, \mathbf{s}_M) \\ \vdots & \vdots & \vdots & \vdots \\ G(\mathbf{r}_N, \mathbf{s}_1) & G(\mathbf{r}_N, \mathbf{s}_2) & \cdots & G(\mathbf{r}_N, \mathbf{s}_M) \end{bmatrix}}_{\mathbf{G}} = \underbrace{\begin{bmatrix} G_0(\mathbf{r}_1, \mathbf{s}_1) & G_0(\mathbf{r}_1, \mathbf{s}_2) & \cdots & G_0(\mathbf{r}_1, \mathbf{s}_M) \\ \vdots & \vdots & \vdots & \vdots \\ G_0(\mathbf{r}_N, \mathbf{s}_1) & G_0(\mathbf{r}_N, \mathbf{s}_2) & \cdots & G_0(\mathbf{r}_N, \mathbf{s}_M) \end{bmatrix}}_{\mathbf{G}_0}, \quad (14)$$

where $\mathbf{P}$ is as defined in Eq. (9), $G(\mathbf{r}_n, \mathbf{s}_m)$ and $G_0(\mathbf{r}_n, \mathbf{s}_m)$ for $n = 1, \dots, N$ and $m = 1, \dots, M$ are the expansion coefficients of the true inhomogeneous Green's function and the homogeneous Green's function, respectively, M is the number of scattered field measurements, and N is the total number of pixels covering the ROI. The expansion coefficients, which are similar to those defined for the total field in Eq. (6), reduce to samples of the Green's functions when using the simple rectangular pulse expansion function.⁴¹

Figure 2 shows a flowchart description of the DBI method for estimating the Green's function. At every iteration we solve the inverse scattering problem for an average incremental correction vector, $\Delta \mathbf{f}$, which is added to the scattering function vector. This correction vector is a function of the kernel matrix, $\mathbf{Q}(\boldsymbol{\varphi}^{k,l-1}, \mathbf{G}^{l-1})$, and the difference between the measured scattered field,

φ_s^k , and the $(l-1)$ th estimate of the scattered field, $\widehat{\varphi}_s^{k,l-1}$, averaged over K transmit positions. Effectively, we solve K inverse problems whose average solution comprise the incremental correction vector, where K is a subset of the array transducers used for transmission. Initially, $\widehat{\varphi}_s^{k,0}=0$ for $k=1,\dots,K$, thus the incremental change in the first step is the zeroth order Born inverse solution which is added to an initial scattering function solution of zero.

Each incremental inverse scattering problem is of the form:

$$\begin{aligned} \Phi_s(\mathbf{r}) - \widehat{\Phi}_s^{k,l-1}(\mathbf{r}) \\ = \int_{\mathbb{R}^2} G^{l-1}(\mathbf{r}, \mathbf{r}') \Delta f^{k,l}(\mathbf{r}') \Phi_s^{k,l-1}(\mathbf{r}') d\mathbf{r}', \end{aligned} \quad (15)$$

which is discretized using the method of moments to produce

$$\begin{aligned} \mathbf{Q}(\varphi^{k,l-1}) \\ , \mathbf{G}^{l-1} \Delta \mathbf{f}^{k,l} = (\varphi_s^k - \widehat{\varphi}_s^{k,l-1}). \end{aligned} \quad (16)$$

Also, at every iteration we compute the total field in the ROI due to the incident field from each of the K transmitters independently. The incident field vectors, $\{\varphi_i^k\}_{k=1}^K$, and the total field vectors, $\{\varphi^k\}_{k=1}^K$, are of size $N \times 1$, where N is the total number of pixels in the ROI. This solution uses the latest estimate of the scattering function, $\mathbf{f}^l$, in forming the $N \times N$ matrix $\mathbf{P}(\mathbf{f}^l)$. Finally, the inhomogeneous Green's function is computed at every iteration using $\mathbf{P}(\mathbf{f}^l)$ in Eq. (14). That is, $\mathbf{P}(\mathbf{f}^l)$ is computed once per iteration and is used to solve for the total field and the inhomogeneous Green's function.

The algorithm terminates when ΔG falls below a preset threshold parameter, η , where ΔG is a normalized distance measure between the current estimate and previous estimate of the Green's function averaged over the N pixels and the M measurements. Currently, we use

$$\Delta G = \frac{\sum_{n=1}^N \sum_{m=1}^M |G(\mathbf{r}_n, \mathbf{s}_m)^l - G(\mathbf{r}_n, \mathbf{s}_m)^{l-1}|^2}{\sum_{n=1}^N \sum_{m=1}^M |G(\mathbf{r}_n, \mathbf{s}_m)^l|^2}. \quad (17)$$

This measure can be easily modified to account for the Green's function due to specific focus points in the ROI or due to the pixel with the largest change, for example,

$$\Delta G = \max_{n=1,\dots,N} \left\{ \frac{\sum_{m=1}^M |G(\mathbf{r}_n, \mathbf{s}_m)^l - G(\mathbf{r}_n, \mathbf{s}_m)^{l-1}|^2}{\sum_{m=1}^M |G(\mathbf{r}_n, \mathbf{s}_m)^l|^2} \right\}. \quad (18)$$

II. PATTERN SYNTHESIS METHOD

The incident field pattern delivered by the M time-harmonic array elements can be written as

$$\Phi_i^f(\mathbf{r}) = \sum_{m=1}^M G_0(\mathbf{r}, \mathbf{r}_m) u_m, \quad (19)$$

where it is assumed that the incident field is generated using infinite line sources modeled by the homogeneous Green's function, which has been defined by Eq. (5), Φ_i^f is the focused incident field pattern, and u_m is a scalar, complex valued excitation weight associated with the m th array element.

In geometric focusing, the excitation weights are determined using the known background speed of sound and the distance between the single focus point and the transducers.¹⁶ Mathematically,

$$\Phi_i^f(\mathbf{r}) = \sum_{m=1}^M G_0(\mathbf{r}, \mathbf{r}_m) e^{-j(\omega/c_0)d_m}, \quad (20)$$

where d_m is the distance between the focus point and the m th transducer, ω is the operating frequency, and c_0 is the known speed of sound in the homogeneous medium. Equivalently, we can use the homogeneous Green's function for geometrical focusing, which, in the far field assumption, reduces to Eq. (20).

Following the theoretical development of the pseudoinverse pattern synthesis method,^{21,23} we write Eq. (19) in matrix form,

$$\begin{bmatrix} \Phi_i^f(\mathbf{r}_1) \\ \vdots \\ \Phi_i^f(\mathbf{r}_N) \end{bmatrix} = \begin{bmatrix} G_0(\mathbf{r}_1, \mathbf{s}_1) & G_0(\mathbf{r}_1, \mathbf{s}_2) & \cdots & G_0(\mathbf{r}_1, \mathbf{s}_M) \\ \vdots & \vdots & & \vdots \\ G_0(\mathbf{r}_N, \mathbf{s}_1) & G_0(\mathbf{r}_N, \mathbf{s}_2) & \cdots & G_0(\mathbf{r}_N, \mathbf{s}_M) \end{bmatrix} \times \begin{bmatrix} u_1 \\ \vdots \\ u_M \end{bmatrix}, \quad (21)$$

where the $N \times 1$ vector $\boldsymbol{\varphi}_i^f = [\Phi_i^f(\mathbf{r}_1) \cdots \Phi_i^f(\mathbf{r}_N)]^T$ contains samples of the focused incident field, $G_0(\mathbf{r}_n, \mathbf{s}_m)$ for $n = 1, \dots, N$ and $m = 1, \dots, M$ are samples of the homogeneous Green's function, and $\mathbf{u} = [u_1 \cdots u_M]^T$ is an $M \times 1$ excitation-weight vector.

The homogeneous media does not change the focused incident field pattern, φ_i^f , equivalently, the P matrix of the forward scattering problem is equal to the identity matrix. In this case, the actual total field pattern delivered to the ROI is the focused incident field pattern, φ_i^f . On the other hand, when the medium is inhomogeneous, this field pattern goes through a distortion operation which can be defined in terms of the forward scattering problem. Therefore, by application of the forward scattering problem, Eq. (21) becomes

$$\mathbf{P}^{-1}\boldsymbol{\varphi}_i^f = \mathbf{P}^{-1}\mathbf{G}_0\mathbf{u}, \quad (22)$$

where $\mathbf{P}$ is the $N \times N$ forward scattering problem matrix, and $\mathbf{G}_0$ is the $N \times M$ homogeneous Green's function matrix encountered in Eqs. (14) and (21). Note that $\boldsymbol{\varphi} = \mathbf{P}^{-1}\boldsymbol{\varphi}_i^f$ is the total field vector due to the focused (modulated) incident field, $\boldsymbol{\varphi}_i^f$, and $\mathbf{G} = \mathbf{P}^{-1}\mathbf{G}_0$ is the inhomogeneous Green's function matrix defined by Eq. (14). The $\mathbf{P}$ matrix distorts the focused incident field due to the presence of inhomogeneities in the media. This could result in shifting the focus point and in widening the main lobe. Such distortions could be dramatic and must be accounted for when computing the excitation weights.

For a given $n \leq M$ predetermined set of control points on the field pattern at some pixels in the ROI defined by the control pattern vector $\mathbf{p}$, we can write Eq. (22) as

$$\mathbf{p} = \mathbf{H}\mathbf{u}, \quad (23)$$

where the $n \times M$ matrix, $\mathbf{H}$ is the propagation matrix, and $\mathbf{u}$ is the excitation-weight vector. For simplicity and without loss of generality, the $n \times 1$ control pattern vector, $\mathbf{p}$, is set to $[1 \dots 1]^T$ throughout our simulations presented in Sec. III. It should be clear from Eq. (22) that using an estimate of $\mathbf{P}$, or equivalently $\mathbf{G}$, in constructing the propagation matrix, $\mathbf{H}$, compensates for the inhomogeneities in the media. Therefore, the rows of the $n \times M$ matrix $\mathbf{H}$ are the n rows of the estimated Green's function matrix, $\mathbf{G}$, corresponding to the pixels referenced by the control pattern vector. In other words,

$$\mathbf{H} = \begin{bmatrix} G(\mathbf{r}_{i_1}, \mathbf{s}_1)^\infty & G(\mathbf{r}_{i_1}, \mathbf{s}_2)^\infty & \dots & G(\mathbf{r}_{i_1}, \mathbf{s}_M)^\infty \\ \vdots & \vdots & & \vdots \\ G(\mathbf{r}_{i_n}, \mathbf{s}_1)^\infty & G(\mathbf{r}_{i_n}, \mathbf{s}_2)^\infty & \dots & G(\mathbf{r}_{i_n}, \mathbf{s}_M)^\infty \end{bmatrix}, \quad (24)$$

where $i_1 \dots i_n$ are the pixels corresponding to the focus locations of interest, and $G(\mathbf{r}_i, \mathbf{s}_m)^\infty$ for $m = 1, \dots, M$ is the i th row from the best estimate (after convergence of the estimation algorithm as indicated by the $(\cdot)^\infty$) of the $N \times M$ matrix $\mathbf{G}$.

Minimum-norm least-squares optimized excitation weights, which can be used for synthesizing the field patterns of inhomogeneous wave propagation, are obtained by taking the pseudoinverse of $\mathbf{H}$,^{21,23}

$$\hat{\mathbf{u}} = \mathbf{H}^{*T} (\mathbf{H}\mathbf{H}^{*T})^{-1} \mathbf{p}, \quad (25)$$

where the $(\cdot)^{*T}$ denotes the conjugate transpose, and the operator $\mathbf{H}^\dagger = \mathbf{H}^{*T}(\mathbf{H}\mathbf{H}^{*T})^{-1}$ is the pseudoinverse of $\mathbf{H}$. For example, in the single focus case with a unity control point, the propagation matrix becomes a row vector equal to $\mathbf{h} = [G(\mathbf{r}_f, \mathbf{s}_1) \ G(\mathbf{r}_f, \mathbf{s}_2) \dots G(\mathbf{r}_f, \mathbf{s}_M)]$ for the M -element phased array, where $\mathbf{r}_f$ is the focus point of interest. The excitation solution vector reduces to

$$\hat{\mathbf{u}} = \frac{\mathbf{h}^{*T}}{(\mathbf{h}\mathbf{h}^{*T})}. \quad (26)$$

The advantages of the pseudoinverse pattern synthesis method include the ability to produce optimal field patterns for multiple focusing through homogeneous and inhomogeneous media.^{5,19} This scheme also has the capability of optimizing the solution for increased gain and array excitation efficiency, without introducing undesired hot spots.^{17,21–23}

This inverse scattering-based focusing incorporates information about the true nature of the ROI in the pseudoinverse pattern synthesis method. We estimate the inhomogeneous Green's function from the scattered field measured outside the ROI. Unlike geometrical focusing, this approach does account for the spatial variation in the speed of sound throughout the ROI. Also, this technique is noninvasive; it does not make any assumption about the location of the inhomogeneity (or aberrating medium) relative to the transducer array; and it does not rely on the existence of distinct dominant or pointlike scatterers near the focus point. Finally, we benefit from the advantages of the pseudoinverse pattern synthesis method such as producing optimal multiple focus field patterns through the inhomogeneous media, and optimizing solutions for increased array excitation efficiency and intensity gain.

III. SIMULATION RESULTS AND DISCUSSION

We simulated the DBI algorithm for the circular array shown in Fig. 1. It consists of 128 receive elements which are equally distributed over a circle of radius $10\lambda_0$. Only 32 equally spaced elements are used, one at a time, to transmit a continuous wave at a frequency of 500 kHz. The background medium is assumed loss-free with a known constant speed of sound equal to 1.484 mm/ μ s, thus $\lambda_0 = 2.968$ mm. We consider two cases for the region of interest. In the first case, a two-dimensional region of interest of size $6\lambda_0 \times 6\lambda_0$, centered at the center of the circular array, is discretized at pixel width of $\lambda_0/5.5 \times \lambda_0/5.5$. Hence, the dimension of the $\mathbf{Q}$ matrix computed in the inverse scattering problem is 128×1089 , and the dimension of the $\mathbf{P}$ matrix computed in the forward scattering problem is 1089×1089 . In the second case, the two-dimensional region of interest (shown in Fig. 12) is of size $12\lambda_0 \times 12\lambda_0$, discretized at pixel width of $\lambda_0/6 \times \lambda_0/6$. This produces a $\mathbf{Q}$ matrix of size 128×5184 and a $\mathbf{P}$ matrix of size 5184×5184 . This intensive memory requirement is the limiting factor of the method of moments for solving the forward scattering problem. Other techniques, such as finite element methods, produce symmetric and sparse matrices of which we can take advantage. The rank reduction strategy is used to obtain a regularized pseudoinverse operator^{37–41} for solving the inverse scattering problem at every iteration of the DBI algorithm.

Figure 3 shows a gray scale image of a true region of interest, which displays the contrast as a percentage increase in the speed of sound inside the scattering objects relative to the speed of sound in the background medium. Although this figure shows percentage contrasts of 2%, 3%, and 5%, we vary the contrast on the two smaller object [marked (1) and (2)] in the simulation results. Figure 4 shows the squared error between the estimated Green's function and the true Green's function (computed using the true scattering function) averaged over all pixels in the ROI versus transducer index for low contrast case (using contrasts 2%, 3%, and 5%), and high contrast case (object contrasts increased to 2%, 13%, and 15%, respectively). Figure 5 shows the squared error of these estimates, averaged over all pixels in the ROI and over all transducers, versus the iteration number of the DBI algorithm for both, the low and the high contrast cases. Note that the true Green's function is computed via the forward scattering problem using the true spatial speed of sound in the ROI. The simulations show excellent estimation of the true Green's function even for the stronger inhomogeneous media.

The algorithm can be permitted to iterate further for better estimate of the Green's function. However, at signal-to-noise ratios (SNR) lower than 30 dB, the DBI method could show increasing mean squared error after many iterations due to the accumulation of error. A modified DBI method has been investigated for use with low SNR measurements.^{40,41}

It is expected the quality of Green's function estimate to degrade for increased scattering media (either due to increase in the size of the scatterers, or due to the increase in contrast levels). Figure 6 shows a plot of the average squared error of the estimated Green's function versus contrast level, where the true Green's function is used as the reference. The contrast level cases (x-axis in Fig. 6) correspond to the contrasts of objects (1) and (2) in Fig. 3 increased from 5% and 3% to 15% and 13%, respectively, in increments of 1%. Although the error measure increases as a function of contrast, we can allow the algorithm to iterate further so as to provide the needed accuracy for the Green's function estimate. The results discussed so far show that the DBI provide excellent estimates for the Green's function, and therefore, can be used as the tool for focusing the ultrasonic waves through inhomogeneous media.

We simulate the Green's function focusing for the same circular array shown in Fig. 1. In the estimation of the Green's function, we use 128 receive elements, of which only 32 elements are used for transmission. As for the focusing procedure, we retransmit the continuous ultrasound wave from all 128 elements. The focused incident beam is computed as given by Eq. (19), and the excitation weights are computed as described in Sec. II. As a reference, we compute the focused incident field using excitation weights derived from the true Green's function (computed from the true speed of sound), and excitation weights derived from the homogeneous Green's function to simulate geometrical focusing through inhomogeneous media. We operate on the focused incident field with the true $\mathbf{P}$ matrix (computed using the true scattering function) to compute the total field in the ROI. Also, we show results of the total field in the ROI obtained for geometrical focusing through homogeneous media. In this case, the $\mathbf{P}$ matrix is replaced with identity matrix, operating on the geometrically focused incident field. These results show the effects of the true inhomogeneity on each of the focusing schemes relative to the homogeneous case.

In the first simulation study, we use the true ROI shown in Fig. 3 with contrast level equal to 15% for object (1), and contrast level equal to 13% for object (2). The focus point of interest is the pixel marked in black in Fig. 3. In Fig. 7, we display intensity images with 16-dB dynamic range for the total field in the ROI. These results are computed for the geometrical focusing through homogeneous media, the geometrical focusing through inhomogeneous media, the true Green's function focusing through inhomogeneous media, and the estimated Green's function focusing through inhomogeneous media. We show a cross-section line plot of the total fields of Fig. 7 in Fig. 8. These results show the effects of the inhomogeneity on the total field, which are particularly visible in the geometrically focused field pattern. These results also show that the geometrically focused pattern has a slightly wider main lobe and a shift in the focal point, which are somewhat corrected for in the Green's function focusing. Finally, the energy outside the main lobe is apparently larger, and is unevenly distributed in the field patterns obtained from the geometrical focusing scheme. This uneven distribution of energy and focal point shift are dependent on the scattering properties of inhomogeneous media in the ROI. Therefore, for increased size inhomogeneities, as in breast tissue, the distortions in the field pattern are expected to increase. On the other hand, the estimation of the Green's function is independent of the strength of the inhomogeneity as long as it is within the applicability of the algorithm used for solving the inverse scattering problem.

We compute the mean squared error for the total fields resulting from geometrical focusing and from the inverse scattering-based focusing. The total field distribution computed using the true Green's function focusing is used as the reference field. Figure 9(a) shows such results as

a function of contrast for the same focus point, where the contrast level for objects (1) and (2), respectively, is increased from 5% and 3% to 15% and 13% in steps of 1%. Figure 9(b) presents the absolute error in focal point shift (in mm) at each contrast level for the geometrical focusing and the inverse scattering based focusing. The results presented in this simulation show the field pattern produced using the inverse scattering-based focusing has at least a 10-dB error improvement over the geometrically focused pattern. Also, it is apparent that the technique does not suffer from problems of shifting the focal point even at high contrast levels.

Figure 10 shows the gray scale image of the spatial distribution for the second case. This region is larger with higher contrast than the previously simulated case. Also, the SNR level is 30 dB, which is well within the capabilities of our measurement systems. Figure 11 shows the magnitude and phase of the true Green's function (solid), the estimated Green's function (dash-dotted), and the homogeneous Green's function (dotted) versus transducer index for the pixel at the center of the array. The apparent large phase difference between the estimated Green's function and the true Green's function around the 20-th transducer index is approximately 2π , which is a negligible phase error. Figure 12 displays the field pattern intensity in the region of interest on a 16-dB scale using the geometrical focusing in (a) and (b) and the Green's function focusing in (c) and (d). The pixel location of the focal point of interest is shown, in white, on the region in Fig. 10. We also show cross-section line plots through the field patterns at $x = -0.25$ mm in Fig. 13, and at $y = -9.27$ mm in Fig. 14. This simulation result, once again, demonstrates the accuracy of estimating the Green's function for highly scattering media, and in the presence of noise. It is apparent from the field pattern of the geometrical focusing through inhomogeneous media [Fig. 12(b)] that geometrical focusing is inappropriate in this more realistic situation. As for focusing using the estimated Green's function, the focused field pattern is preserved and the field pattern is almost exactly the same as that produced using the true Green's function. With the use of the pseudo-inverse pattern synthesis method, we can further constrain the solution to produce better distributed interference patterns outside the focal spots.

In the second simulation result using the region shown in Fig. 10, we make use of the capability of the pseudoinverse method to synthesize multiple focus field patterns. One advantage of this inverse scattering based focusing is the ability of incorporating it in the pseudoinverse pattern synthesis method. The pixel locations of the points of interest are marked in white on Fig. 10. In Fig. 15, we display an intensity image of the resulting field pattern with 16-dB dynamic range for the multiple focusing. Figures 16 and 17 shows cross-sectional plots for the y-axis planes through the two focus points. This simulation study demonstrates the capability to produce multiple focus through highly scattering media. It also shows that the field patterns produced using the estimated Green's function are almost identical to the patterns produced using the true Green's function. This simulation shows the superiority of this inverse scattering-based focusing approach over geometrical focusing. In fact, the field pattern produced using geometrical focusing is not acceptable since it is almost impossible to locate both foci. Finally, Fig. 15 shows relatively high intensity levels outside the focus points even for the Green's function focusing. The estimated Green's function and the spatial structure of the inhomogeneous media, also obtained by solving the same inverse scattering problem, can be exploited in the pseudo-inverse pattern synthesis method to solve a constrained optimization problem for better distribution of the energy outside the focal spots.²³

The simulation results presented in this section are for loss-free acoustic media. However, additional simulations we carried out for lossy media show that there is not a major difference from the results presented in this section. The advantages of the Green's function focusing still hold under such circumstances.

IV. CONCLUSION

We introduced a self-focusing approach which uses the scattered field, measured by the array outside a region of interest, to estimate the Green's function from the desired focal point(s) to the array. We use these estimates in constructing the propagation matrix of the pseudoinverse pattern synthesis method to compute the excitation weights required for retransmission with the same array. We formulate the Green's function estimation problem in terms of the ultrasound inverse scattering problem, which is solved using the distorted Born iterative method. In this DBI method we iteratively solve the inverse scattering problem for the scattering function and the forward scattering problem for the total field and the Green's function. Convergence criterion is specified in terms of the normalized mean squared change in the Green's function of the focus point between one iteration and the next. This technique results in an excellent estimation of the true Green's function as shown in the simulation results provided in Sec. III. We also show simulation results for self-focusing through inhomogeneous media. These results indicate improved quality focusing compared to the geometrical focusing. Also, the quality of field patterns obtained using geometrical focusing is expected to degrade with increased size and contrast of the inhomogeneities.

Although, we presented the DBI algorithm as the method of choice, other approaches to solving the nonlinear inverse scattering problem can be used instead. Our recent research efforts are directed toward developing DBI-based robust algorithms for large size, high contrast inhomogeneities. Also, the use of finite element methods (FEM) will make the computational complexity and memory requirements of the DBI method reasonable for image-guided therapy systems for breast tissue, which is characterized as strongly scattering. In our simulations, we only consider circular arrays suitable for breast imaging and therapy. We plan to extend this strategy to work with limited angle (circular section) arrays for applications with limited access regions. We also plan to extend this approach using time-domain inverse scattering for real-time applications with pulse-mode phased arrays.

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References

1. Driller J, Lizzi FL. Therapeutic applications of ultrasound: A review. *IEEE Engineering in Medicine and Biology Magazine* 1987;6:33–40. [PubMed: 19493845]
2. Macovski A. Ultrasonic imaging using arrays. *Proc IEEE* 1979;67:484–495.
3. Sanghvi NT, Fry FJ, Foster R, Chua R, Chua G. System design considerations for high intensity focused ultrasound device for the treatment of tissue *in vivo*. *Med Biol Eng Comput* 1991;29:748.
4. Buchanan MT, Hynynen K. Design and experimental evaluation of an intracavitary ultrasound phased array system for hyperthermia. *IEEE Trans Biomed Eng* 1994;41:1178–1187. [PubMed: 7851919]
5. Seip R, VanBaren PD, Ebbini ES. Dynamic focusing in ultrasound hyperthermia treatments using implantable hydrophone arrays. *IEEE Trans Ultrason Ferroelectr Freq Control* 1994;41:706–713. [PubMed: 18263259]
6. Seip, R.; Ebbini, ES. In: Levy, M.; Schneider, SC.; McAvoy, BR., editors. *Invasive and noninvasive feedback for ultrasound phased array thermometry*; IEEE Ultrasonics Symposium Proceedings; Seattle, WA. New York: IEEE; 1995.
7. VanBaren, PD.; Seip, R.; Ebbini, ES. In: Levy, M.; Schneider, SC.; McAvoy, BR., editors. *Real-time dynamic focusing through tissue inhomogeneities during hyperthermia treatments with phased arrays*; IEEE Ultrasonics Symposium Proceedings; Cannes, France. New York: IEEE; 1994. p. 1815-1819.

8. Hutchinson, EB.; Buchanan, MT.; Hynenen, K. In: Levy, M.; Schneider, SC.; McAvoy, BR., editors. Evaluation of an aperiodic phased array for prostate thermal therapies; IEEE Ultrasonics Symposium Proceedings; Seattle, WA. New York: IEEE; 1995. p. 1601-1604.
9. Ebbini, ES.; Seip, R.; VanBaren, PD.; Simon, C.; Cain, C. In: Ermert, H., editor. Noninvasive temperature measurement for feedback control of thermal therapy using ultrasound phased arrays; Proceedings of the 9th International Symposium on Acoustical Imaging; Bochum, Germany. New York: Plenum; 1992.
10. VanBaren, PD.; Simon, C.; Seip, R.; Solf, T.; Cain, C.; Ebbini, ES. In: Levy, M.; Schneider, SC.; McAvoy, BR., editors. Image-guided phased array system for ultrasound thermotherapy; IEEE Ultrasonics Symposium Proceedings; New York: IEEE; 1996.
11. Fink, MA.; Prada, C.; Wu, F.; Cassereau, D. In: McAvoy, BR., editor. Self-focusing in inhomogeneous media with time reversal acoustic mirrors; IEEE Ultrasonics Symposium Proceedings; Montreal. New York: IEEE; 1989. p. 681-686.
12. Fink MA. Time reversal of ultrasonic fields—Part I: Basic principles. IEEE Trans Ultrason Ferroelectr Freq Control 1992;39:555–566. [PubMed: 18267667]
13. Flax SW, O'Donnell M. Phase-aberration correction using signals from point reflectors and diffuse scatterers: Basic principles. IEEE Trans Ultrason Ferroelectr Freq Control 1988;35:758–767. [PubMed: 18290213]
14. Mallart, R.; Fink, MA. In: Franconi, C.; Arcangeli, G.; Cavaliere, R., editors. Sound speed fluctuations in medical ultrasound imaging: Comparison between different correction algorithms; Proceedings of the 19th International Symposium on Acoustical Imaging; Rome, Italy: Tor Vergata; 1991.
15. Trahey GE, Zhao D, Miglin JA, Smith SW. Experimental results with a real-time adaptive ultrasonic-imaging system for viewing through distorting media. IEEE Trans Ultrason Ferroelectr Freq Control 1990;37:418–427. [PubMed: 18285059]
16. Ebbini ES, Umemura S, Ibbini MS, Cain C. A cylindrical-section ultrasound phased-array applicator for hyperthermia cancer therapy. IEEE Trans Ultrason Ferroelectr Freq Control 1988;35:561–572. [PubMed: 18290188]
17. Nock L, Trahey GE, Smith SW. Phase aberration correction in medical ultrasound using speckle brightness as a quality factor. J Acoust Soc Am 1989;85:1819–1833. [PubMed: 2732378]
18. Ebbini, ES.; Wang, H.; O'Donnell, M.; Cain, CA. In: McAvoy, BR., editor. Acoustic feedback for hyperthermia phased-array applicators: Aberration correction, motion compensation and multiple focusing in the presence of tissue inhomogeneities; IEEE Ultrasonics Symposium Proceedings; New York: IEEE; 1991. p. 1343-1346.
19. Wang H, Ebbini ES, O'Donnell M, Cain CA. Phase aberration correction and motion compensation for ultrasonic hyperthermia phased arrays: Experimental results. IEEE Trans Ultrason Ferroelectr Freq Control 1994;41:34–43.
20. Krishnan S, Li PC, O'Donnell M. Adaptive compensation of phase and magnitude aberrations. IEEE Trans Ultrason Ferroelectr Freq Control 1996;43:44–55.
21. Ebbini ES, Cain C. Multiple-focus ultrasound phased-array pattern synthesis: optimal driving signal distribution for hyperthermia. IEEE Trans Ultrason Ferroelectr Freq Control 1989;36:540–548. [PubMed: 18290231]
22. Ebbini, ES.; Ngo, F.; Cain, C. In: McAvoy, BR., editor. The pseudoinverse pattern synthesis method: Experimental verification using a prototype cylindrical-section ultrasound hyperthermia phased-array applicator; IEEE Ultrasonics Symposium Proceedings; Montreal. New York: IEEE; 1989.
23. Ebbini, ES. PhD dissertation. University of Illinois; Urbana–Champaign: 1990. Deep localized hyperthermia with ultrasound phased arrays using the pseudoinverse pattern synthesis method.
24. Prada, C.; Lartillot, N.; Fink, MA. In: Levy, M.; McAvoy, BM., editors. Selective focusing in multiple-target media: The transfer matrix method; IEEE Ultrasonics Symposium Proceedings; New York: IEEE; 1993. p. 1139-1142.
25. Prada C, Fink MA. Eigenmodes of the time reversal operator: A solution to selective focusing in multiple-target media. Wave Motion 1994;20:151–163.
26. Dorne C, Fink MA. Ultrasonic beam steering through inhomogeneous layers with a time reversal mirror. IEEE Trans Ultrason Ferroelectr Freq Control 1996;43:167–175.

27. Chew WC, Wang YM. Reconstruction of two-dimensional permittivity distribution the using distorted Born iterative method. *IEEE Trans Med Imaging* 1990;9:218–225. [PubMed: 18222767]
28. Chew, WC. *Waves and Fields in Inhomogeneous Media*. Van Nostrand Reinhold; New York: 1992.
29. Habashy TM, Mittra R. On some inverse methods in electromagnetics. *J Electromagn Waves Appl* 1987;1:25–58.
30. Wang YM, Chew WC. An iterative solution of two-dimensional electromagnetic inverse scattering problem. *Int J Imaging Syst Technol* 1989;1:100–108.
31. Colton, D.; Kress, R. *Inverse Acoustic and Electromagnetic Scattering Theory*. Springer-Verlag; New York: 1992.
32. Vardan, VV.; Lakhtakia, A.; Vardan, VK. *Field Representations and Introduction to Scattering*. Elsevier Science; New York: 1991.
33. Harrington, RF. *Field Computation by Moment Methods*. IEEE with Oxford U.P; New York: 1993.
34. Kak, AC.; Slaney, M. *Principles of Computed Tomographic Imaging*. IEEE; New York: 1988.
35. Habashy TM, Groom RW, Spies BR. Beyond the Born and Rytov approximations: A nonlinear approach to electromagnetic scattering. *J Geophys Res* 1993;98:1759–1775.
36. Zhu Q, Steinberg BD. Large-transducer measurements of wavefront distortion in the female breast. *Ultrason Imaging* 1992;14:276–299. [PubMed: 1448892]
37. Engl, HW.; Groetsch, CW., editors. *Inverse and Ill-posed Problems*. Academic; New York: 1987.
38. Demoment G. Image reconstruction and restoration: Overview of common estimation structures and problems. *IEEE Trans Acoust, Speech, Signal Process* 1989;37:2024–2036.
39. Golub, GH.; VanLoan, CF. *Matrix Computations*. Johns Hopkins U.P; Baltimore, MD: 1983.
40. Haddadin, OS.; Ebbini, ES. In: Delogne, P., editor. *Adaptive regularization of a distorted Born iterative algorithm for diffraction tomography*; *IEEE Proceedings of the International Conference on Image Processing*; New York: IEEE; 1996. p. 725-728.
41. Haddadin, OS. PhD Dissertation. University of Michigan; Ann Arbor: 1997. Ultrasonic inverse scattering for tomographic imaging and self-focusing arrays.

FIG. 1.
The circular receive/transmit array and region of interest.

FIG. 2.

A flow chart description of the DBI estimation of the Green's function. The $(\cdot)^\dagger$ denotes the generalized inverse.

FIG. 3.
The true speed of sound in the ROI shown as a percentage increase over the background speed of sound.

FIG. 4.

The squared error in estimating the Green's function averaged over all pixels in the region of interest for the low contrast case (+) and the high contrast case (○), where the reference is the true Green's function.

FIG. 5. The mean squared error of the Green's function estimates versus iteration number for the low contrast case (+) and the high contrast case (o), where the reference is the true Green's function.

FIG. 6. The mean squared error of the estimated Green's function (*) and the homogeneous Green's (o) function versus average contrast level percentage, where the reference is the true Green's function.

FIG. 7.

The magnitude of the total field intensity in the region for (a) geometrical focusing through homogeneous media, (b) geometrical focusing through inhomogeneous media, (c) true Green's function focusing through inhomogeneous media, and (d) estimated Green's function focusing through inhomogeneous media.

(a)

(b)

FIG. 8.

A slice at the diagonal of the total field pattern for geometrical focusing through homogeneous media (solid), geometrical focusing through inhomogeneous media (dotted), true Green's function focusing through inhomogeneous media (dashed), and estimated Green's function focusing through inhomogeneous media (dash-dotted).

FIG. 9.

(a) The mean squared error of the total field in the region of interest, and (b) the error shift of the focus point for the geometrical focusing through inhomogeneous media (○) and for the estimated Green's function focusing (*) as a function of contrast.

FIG. 10.
The true speed of sound in the ROI shown as a percentage increase over the background speed of sound.

FIG. 11. (a) Magnitude and (b) phase of the true Green's function (solid), the estimated Green's function (dash-dotted), and the homogeneous Green's function (dotted) for the center pixel.

FIG. 12.

The magnitude of the total field intensity in the region for (a) geometrical focusing through homogeneous media, (b) geometrical focusing through inhomogeneous media, (c) true Green's function focusing through inhomogeneous media, and (d) estimated Green's function focusing through inhomogeneous media.

FIG. 13.

A slice at $x = -9.27$ mm of the total field pattern (a) for geometrical focusing through homogeneous media (solid) and geometrical focusing through inhomogeneous media (dotted), and (b) for true Green's function focusing (dashed), estimated Green's function focusing (dash-dotted) and geometrical focusing (dotted) through inhomogeneous media.

(a)

(b)

FIG. 14.

A slice at $y = -0.25$ mm of the total field pattern (a) for geometrical focusing through homogeneous media (solid) and geometrical focusing through inhomogeneous media (dotted), and (b) for true Green's function focusing (dashed), estimated Green's function focusing (dash-dotted), and geometrical focusing (dotted) through inhomogeneous media.

FIG. 15.

The magnitude of the total field intensity in the region for (a) geometrical focusing through homogeneous media, (b) geometrical focusing through inhomogeneous media, (c) true Green's function focusing through inhomogeneous media, and (d) estimated Green's function focusing through inhomogeneous media.

FIG. 16.

A slice at $x = 8.78 \text{ mm}$ of the total field pattern (a) for geometrical focusing through homogeneous media (solid) and geometrical focusing through inhomogeneous media (dotted), and (b) for true Green's function focusing (dashed), estimated Green's function focusing (dash-dotted), and geometrical focusing (dotted) through inhomogeneous media.

(a)

(b)

FIG. 17.

A slice at $x = -9.27$ mm of the total field pattern (a) for geometrical focusing through homogeneous media (solid) and geometrical focusing through inhomogeneous media (dotted), and (b) for true Green's function focusing (dashed), estimated Green's function focusing (dash-dotted), and geometrical focusing (dotted) through inhomogeneous media.