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Issues in Intraindividual Variability: Individual Differences in Equilibria and Dynamics Over Multiple Time Scales

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Introduction

This special issue of *Psychology in Aging* is an illustration of the developmental trajectory of the methodology associated with the study of intraindividual variability. Six very high quality articles address within--person processes using a wide variety of methods and coming from several different theoretical viewpoints. These articles and the special issue itself are the result of a workshop held in Dölln, Germany hosted by the Max Planck Institute for Human Development in the summer of 2008.

The study of intraindividual processes has reached its adulthood. The concerns are now less about *whether* variability within an individual should be studied than it is about *how* to make use of this important source of information about psychological processes. No longer is the argument about individual differences versus individual variability, now authors such as Almeida, Piazza, & Stawski (2009); MacDonald, Li, & Bäckman (2009); and Newell, Mayer-Kress & Liu (2009) are writing about how to best use both sources of information together to illuminate both how short term variability over time can differ between people in diagnostically interesting ways.

In fact, it may be time to make the case that the developmental trajectory of the study of intraindividual variability has entered into a period of parenthood. The *amount* of variability is less a focus of these seven papers than is the *time dynamics* of the variability. Ram & Gerstorf (2009) make the case that the time structure of intraindividual variability needs to be considered along with what they call the net variability. Intraindividual variability is, in some sense, the intellectual parent of dynamical systems analysis in psychology. Sliwinski, Almeida, Smyth, & Stawski (2009) use age-related changes in patterning of intraindividual variability to make theoretical distinctions concerning negative affect and reactivity. When all is said and done, probably the most important contribution of any methodological advance is to sharpen the focus of the language with which we describe our theories.

It is far too early to say that the study of intraindividual variability has entered into its middle age. There are still a great number of methodological questions that remain to be addressed in this field. Schmiedeck, Lövdén, & Lindenberger (2009) address one of these problems: the assumption of no relation between means and variances.

We will first expand on three methodological issues that address needs required to improve research in intraindividual variability: (1) The need to consider the relationship between the time scale of a process and the time scale of its measurement, (2) The need to first model both

the deterministic and the stochastic components of psychological processes at the intraindividual level then at a second level model the variation in these deterministic and stochastic components in samples of individuals, and (3) The need to expand one's thinking beyond individual differences in variance and covariance of latent variables given measurement invariance in order to consider the opposite possibility: idiosyncratic measurement models with invariance applied to the variance and covariance of latent variables.

Multiple Time Scales

While writing this commentary, one of the authors was listening to Anner Bylsma perform Bach's Suites for Cello. These six suites for solo cello each have six sections and are highly structured in time. Bylsma's performances are fluid and highly personal renditions of the musical scores. As a thought experiment, imagine solo cello music playing in the background while we consider a few ways that we might measure the variability of a performance.

Each section is between two and seven minutes long. Suppose we played all 36 sections back to back on a CD player and sampled the audio for 50 ms once each minute. From each of these short audio samples we use an FFT to determine the frequency of the note currently being played. Even though the music is highly structured in time and our time samples are equal interval, the resulting time series of notes would bear no resemblance to Bach. It would be an apparently random sequence of notes because we were missing all of the structure that occurred at a shorter time scale. On the other hand, the mean and standard deviation of the frequency of the notes in our sample would be an unbiased estimate of the mean and standard deviation of the frequency of all the notes in the six Bach Cello Suites.

The mean and variance of the audio frequency of the notes are not affected by the interval between samples, while the time patterning of the notes that distinguishes Bach from a random sample of cello notes is highly dependent on the time-scale of your measurements. If we were to sample again with samples drawn at intervals of 75ms, the time series would be immediately recognizable as a Bach Cello Suite. But if the interval between samples is increased to two seconds, the sequence would become difficult, if not impossible, to recognize since the length of each note in the Bach Cello Suites is short relative to a two second sampling interval. Not only does the sampling interval matter, it may make all the difference if one is wondering whether a time series has determinism.

Anner Bylsma's performance of the Bach Cello Suites is considered by cellists to be idiosyncratic. Why? Because Bach's music is generally interpreted as being played with nonvariable timing. Bylsma plays with changes in timing that are interesting and emotionally involving, but are not faithful to the timing of the musical score. If one listens to Mstislav Rostropovich play the same solo Cello Suites in his accurate style, the difference is immediately apparent. But, one would be unlikely to be able to tell the difference between Bylsma and Rostropovich in the best sampling from the previous paragraph. The reason is that the timing differences are on a much finer time scale than the notes themselves. If one is interested in the individual differences in these performances, one must design a study with very short intervals between very short samples.

This thought experiment was devised to highlight the importance of choosing an appropriate time scale for measurement of human processes. Measurement at the wrong time scale may give accurate information about individual means and variances, but may miss deterministic dynamics entirely. Once sufficient measurement has been accomplished to capture the time dynamics of a selected behavior, one will want to fit a model that estimates the deterministic part of the within-person process and separates it from stochastic components. But bear in mind that what looks stochastic measured at one time scale may turn out to be deterministic at another time scale.

Intraindividual Deterministic and Stochastic Modeling

The analysis of intraindividual variability has begun to shift focus from the amount of variability to the patterning of variability. This shift in focus has been driven both by substantive theory and methodological advances. In this section, we will present a very general framework for fitting models for the patterning of intraindividual variability. But first, let us consider why modeling the deterministic and stochastic dynamics (i.e., the time-related pattern of intraindividual variability) is important to substantive theories of behavioral and developmental processes.

Processes can have both a deterministic and a stochastic component. Thus, we must build models that can account for the patterned time sequence of data. And in addition, our models of intraindividual variability must be able to partition the within-person variance into a part accounted for by deterministic processes and a part accounted for by stochastic, non-time-dependent “innovation”. The deterministic part of the model is especially interesting because this is where the substantive theorist’s work comes into play. Careful thought about mechanisms for a particular process of interest should be translated into a model (either as a forward prediction equation or differential equation) that represents the mechanistic part of the theory. This work is difficult, but extremely rewarding. Not only does such thoughtful consideration result in a model that can be tested against data, but also the theorist is required to express ideas with a precision that may otherwise have been lacking. One fruitful way of proceeding is to carefully examine physical analogies that are used to describe psychological processes and then see how the analogous processes are modeled in physics.

The remainder of this section will describe one of the methods that can be used to fit models to intensively measured within-person data while simultaneously taking into account the possibility that the parameters of the model may be changing within each person across time and that these within-person parameters may differ between individuals.

Dynamic Developmental Models with Multiple Time Scales

A theme which has a nonzero intersection with all six papers is the combination of a) dynamic systems, b) different time scales, and c) multilevel models. Stated in this way, this theme would seem to consist of a loose conjunction of three different subthemes. We would like to argue, though, that these three subthemes can be integrated into a whole which has surprisingly rich theoretical and methodological implications.

In recent work it has become possible to apply dynamic systems models which have the following schematic form (to ease the presentation, time t is considered to proceed in discrete steps of equal length):

$$\mathbf{y}(t) = \mathbf{g}[\boldsymbol{\eta}(t), \boldsymbol{\theta}(t)] + \boldsymbol{\epsilon}(t) \quad (1a)$$

$$\boldsymbol{\eta}(t+1) = \mathbf{f}[\boldsymbol{\eta}(t), \boldsymbol{\theta}(t)] + \boldsymbol{\zeta}(t+1) \quad (1b)$$

$$\boldsymbol{\theta}(t+1) = \mathbf{h}[\boldsymbol{\theta}(t)] + \boldsymbol{\xi}(t+1) \quad (1c)$$

In (1a), $\mathbf{y}(t)$ denotes an observed p -variate time series, $\boldsymbol{\eta}(t)$ denotes a q -variate latent factor series (also called a latent state process), $\boldsymbol{\epsilon}(t)$ is measurement error and $\mathbf{g}[\boldsymbol{\eta}(t), \boldsymbol{\theta}(t)]$ is a smooth linear or nonlinear function linking the latent factor series $\boldsymbol{\eta}(t)$ to the observed series $\mathbf{y}(t)$. $\mathbf{g}[\boldsymbol{\eta}$

(t), $\theta(t)$] depends upon unknown parameters in the r-variate parameter vector $\theta(t)$. Importantly, the parameter vector $\theta(t)$ is itself allowed to be time-varying in unknown, arbitrary ways. For instance, if $g[\eta(t), \theta(t)]$ is linear, i.e., $g[\eta(t), \theta(t)] = G[\theta(t)]\eta(t)$, then $G[\theta(t)]$ is a (p,q)-dimensional matrix of factor loadings, where the unknown factor loadings are allowed to vary in time. (1^b) explains the evolution in time of the latent factor (state) process. $f[\eta(t), \theta(t)]$ is a smooth linear or nonlinear function mapping $\eta(t+1)$ into $\eta(t)$. $f[\eta(t), \theta(t)]$ depends upon unknown parameters in the r-variate time-varying parameter vector $\theta(t)$. $\zeta(t)$ denotes a q-variate noise process associated with $\eta(t)$. Finally, (1^c) describes the evolution of the r-variate vector of time-varying parameters $\theta(t)$; $\xi(t)$ is the r-variate noise process associated with this evolution. $h[\theta(t)]$ is a given smooth linear or nonlinear function mapping $\theta(t+1)$ into $\theta(t)$.

(1^a) - (1^c) represent a rather general dynamic systems model (although certainly not the most general). It can be fitted to observed p-variate time series $y(t)$, obtained with a single subject (Molenaar, Sinclair, Rovine, Ram & Corneal, 2009) or with multiple subjects in a replicated time series design, under the following conditions: a) the functional forms of $g[.]$, $f[.]$ and $h[.]$ are specified a priori, and b) the rate of change of the time-varying parameter vector $\theta(t)$ is an order of magnitude smaller than the rate of change of the latent factor series $\eta(t)$ or, equivalently, the rate of change of the observed series $y(t)$. Convenient mathematical criteria are available to compare rates of change, but in actual fits of (1^a) - (1^c) to the data these are superfluous: estimates of the time-varying parameter vector always obey this soft criterion. The model also can be straightforwardly extended in several ways, for instance with the inclusion of time-varying covariates affecting the evolution of the latent factor process and/or with time-varying mean trends.

Two distinct time scales are involved in the dynamic systems model (1^a) - (1^c). A fast time scale associated with the dynamics of the observed series $y(t)$ and the latent factor process $\eta(t)$ and a slower time scale associated with the time-varying parameter vector $\theta(t)$. These two time scales also define two levels of a hierarchy, with (1^a) and (1^b) describing processes at the lowest (first) level having fast time scales and (1^c) describing a process at the next (second) level describing a process at a slower time scale. Consequently this model already integrates the three subthemes of dynamic systems, time scales and hierarchical organization. But (1^a) - (1^c) can be straightforwardly extended with additional levels. In (1^c) the evolution of the time-varying parameter vector is explained in terms of a given smooth function $h[\theta(t)]$ mapping $\theta(t+1)$ into $\theta(t)$. Notice that $h[\theta(t)]$ itself does not depend upon unknown parameters. But this is an unnecessary restriction: we can define the function mapping $\theta(t+1)$ into $\theta(t)$ as itself depending upon an s-variate time-varying parameter vector $\beta(t)$. Then (1^c) is redefined as:

$$\theta(t+1) = h[\theta(t), \beta(t)] + \xi(t+1), \quad (1c')$$

and a third level is added to describe the evolution of the time-varying parameter vector $\beta(t)$:

$$\beta(t+1) = k[\beta(t)] + v(t+1) \quad (1d)$$

where $k[\beta(t)]$ is a given smooth linear or nonlinear function mapping $\beta(t+1)$ into $\beta(t)$ and $v(t)$ denotes the process noise associated with this evolution. Again the soft constraint mentioned above should apply: the rate of change of $\beta(t)$ should be an order of magnitude smaller than the rate of change of $\theta(t)$ at the previous (second) level associated with (1^c). Hence (1^a) - (1^d) defines a dynamic systems model with three different time scales which give rise to a 3-tiered hierarchy.

It is clear that more and more higher-order levels can be added in this way, where each next level is associated with a time-varying vector of parameters changing at increasingly slower

time scales. At the highest levels parameters would slowly change across the entire life span of a subject. Of course this would require intensive repeated measurement of this subject throughout its life span; something which is practically not (yet) feasible. Considered from a more theoretical perspective, however, we have here a principled modeling scheme with which microscopic development (e.g., day-to-day fluctuations) can be lawfully coupled to macroscopic development (e.g., changes across major phases of the life span). Moreover, it is clear that there is no a priori logical necessity that the ways in which time-varying parameters change at different levels are lawfully related to each other. Stated more specifically, the function $\mathbf{h}[\boldsymbol{\theta}(t), \boldsymbol{\beta}(t)]$ in (1^c) mapping $\boldsymbol{\theta}(t+1)$ into $\boldsymbol{\theta}(t)$ can in principle be of a completely different form than the function $\mathbf{k}[\boldsymbol{\beta}(t)]$ in (1^d) mapping $\boldsymbol{\beta}(t+1)$ into $\boldsymbol{\beta}(t)$. Consequently, parameters changing at different time scales may obey completely different dynamic laws.

To reiterate, in the dynamic systems model under consideration, the parameter vector $\boldsymbol{\theta}(t)$ at the second level obeys the soft criterion that its rate of change is an order of magnitude smaller than the rate of change of the observed series $\mathbf{y}(t)$. Nothing has been said, however, about the sampling rate with which repeated observations of $\mathbf{y}(t)$ are obtained. If the discrete time step is very small, say one millisecond, then $\boldsymbol{\theta}(t)$ can vary dramatically within a second and still obey the soft criterion. If, however, the sampling rate is much larger, say one week, then only variations of $\boldsymbol{\theta}(t)$ across months can be detected. Hence the sampling rate with which repeated observations of $\mathbf{y}(t)$ at the first level are obtained generates an upper limit to the rate of change of $\boldsymbol{\theta}(t)$ which can be detected at the second level.

The dynamic systems model under consideration explains a wide range of developmental processes. If it is linear then $\mathbf{g}[\boldsymbol{\eta}(t), \boldsymbol{\theta}(t)] = \mathbf{G}[\boldsymbol{\theta}(t)]\boldsymbol{\eta}(t)$ in (1^a). The model then can track smoothly time-varying factor loadings in $\mathbf{G}[\boldsymbol{\theta}(t)]$. That is, it can track the waxing and waning of factor loadings, possibly leading to the emergence and/or disappearance of latent factor series. If the model is nonlinear, in particular if $\mathbf{f}[\boldsymbol{\eta}(t), \boldsymbol{\theta}(t)]$ in (1^b) is nonlinear, the smoothly changing parameter vector $\boldsymbol{\theta}(t)$ can give rise to bifurcations (phase transitions). A bifurcation is a sudden qualitative reorganization of the configuration of equilibria of a nonlinear system as the result of smoothly changing parameters. Old equilibria suddenly disappear and new ones emerge. The stage transitions occurring in cognitive development have been successfully modeled by bifurcations (van der Maas & Molenaar, 1992). What is especially noteworthy is that if a nonlinear system approaches a bifurcation, then the variability of its output (i.e., $\mathbf{y}(t)$) increases substantially.

In sum, the dynamic systems model scheme (1^a) - (1^d) is characterized by time-dependent parametric changes occurring at different time scales, where each time scale defines a distinct level within a hierarchy of time scales. This model involves an integration of the three subthemes which figure in all six papers of this special issue: dynamic laws, different time scales and hierarchies of multiple levels. Moreover, the model has important additional virtues in that it yields lawful and testable connections between microscopic and more macroscopic developmental processes. Last but not least, it can explain the waxing and waning of latent factor processes as well as the emergence of qualitative new attractor landscapes (i.e., new configurations of equilibria) during development.

On the Necessity to Study Intraindividual Variation

The standard approach in psychology is analysis of interindividual variation. Its defining features are the following: a) a random sample of subjects is drawn from a given population; b) it is assumed that the population of subjects is homogeneous, i.e., subjects are exchangeable and all obey the same statistical model; c) statistics of interest are estimated by pooling across the sampled subjects; d) the results of statistical analysis are generalized to the population of subjects from which the random sample was drawn. This definition of analysis of

interindividual variation can be straightforwardly refined for mixture analysis. In contrast, the defining features of analysis of intraindividual variation are: a) a given subject is repeatedly measured during an interval of time, where this interval of time is considered to be a block of time points randomly drawn from the entire time axis; b) statistics of interest are estimated by pooling across time points; c) the results of statistical analysis are generalized along the entire time axis (retrodiction, interpolation, prediction).

It is an implicit assumption of methods for the analysis of interindividual variation that results thus obtained also apply at the level of intraindividual variation. However, this assumption is invalid. The reason is that many psychological processes violate a fundamental mathematical theorem stipulating the conditions which have to be obeyed in order that results obtained in analyses of interindividual variation can be validly generalized to intraindividual variation, and vice versa. This theorem is called the individual ergodic theorem and its profound consequences for the analysis of psychological processes are explained in Molenaar (2004). In particular, all learning and developmental processes violate the conditions stipulated by the individual ergodic theorem and therefore their analysis should be based on intraindividual variation, not interindividual variation. Hence for all learning and developmental processes, in fact all processes having time-varying statistical characteristics, it is not an option but a necessity to base their analysis on intraindividual variation.

Full-blown analysis of intraindividual variation uses time series analysis techniques. Modern statistical tools for multivariate time series analysis can be applied not only to the data of a single subject, but also to data obtained with multiple subjects in a replicated time series design. If time series of multiple subjects are available, then in the first step a subject-specific dynamic systems model is fitted to the data of each subject. In the next step it is tested whether parameters of these subject-specific dynamic systems models are invariant across subjects. This opens up the possibility that detailed conclusions can be drawn, based on dedicated statistical tests, about the respects in which subjects differ from each other and other respects in which they are homogeneous. Such a finding, namely that the dynamic systems models fitted to the time series of different subjects have parameters which are invariant across subjects, constitutes an important step along the path to the derivation of nomothetic laws for the structures of intraindividual variation. Several papers in this special issue do not fit dynamic systems models to the data, but instead reduce the information inherent in the time series of each individual subject to two statistics: the level of the time series and the standard deviation of the time series. This, of course, constitutes a severe reduction of the information inherent in time series. Then, in the second step, it is tested whether these statistics bear lawful relationships with other variables. The first step in which statistics are derived from individual time series should be replaced by the fit of dynamic systems models in order to make optimal use of the intraindividual information in the data.

The Idiographic Filter

Along with an emphasis on the individual and modeling intraindividual variability, there has been a renewed interest in measurement issues. Because theory is built on concepts (latent variables) and their interrelationships and the concepts must have empirical representation, either directly (in manifest variables) or through other concepts, if its hypotheses are to be falsifiable, measurement operations play a critical role in empirical research. In behavioral science, after approximately a century of investment of energy, time, and talent, *measurement invariance* has surfaced as a *sine qua non* of “good” measurement. Heavy reliance is placed on the factor analysis model and its parts: factor loadings, uniquenesses, intercepts, and factor covariances to provide a basis for distinguishing among weak, strong, and strict forms of measurement invariance (Meredith, 1993; Millsap & Meredith, 2007).

Under standard measurement practice, however, there is virtually no useful way to deal with idiosyncrasy in the manifest and latent variables, or in their relations. Yet, when the unit of analysis is the person, idiosyncrasy is a natural, substantial phenomenon with which to deal. Concepts such as expertise, intelligence, and creativity, for example, imply idiosyncrasy and concepts such as *syndrome* recognize it explicitly. A syndrome presumes a common core of meaning across individuals but allows different but overlapping subsets of indicators of that core from one individual to another. Cancer victims, for example, have unique manifestations but also enough common ones to be labeled as cancer cases. Idiosyncrasy is also present in subgroups. Older adults, for example, do not necessarily construe (and respond to) a given set of items in the same way as younger adults, rendering inter-group comparisons of item responses problematic.

Results from individually-oriented modeling procedures (e.g., P-technique factor analysis see e.g., Lebo & Nesselroade, 1978; Molenaar & Nesselroade, 2009) suggested a new measurement approach that can accommodate necessary idiosyncrasy with mathematical rigor and that holds some promise for revolutionizing the way we conduct behavioral measurement (Nesselroade, Gerstorf, Hardy, & Ram, 2007). Termed “the idiographic filter,” the proposal explicitly recognizes that individuals and subgroups differ in important ways in how they construe items, in their learning/conditioning histories, etc., and frankly accepts the possibility that a rigidly standardized measurement framework *at the observable level* may not be the most appropriate way to proceed with the assessment of abstract constructs. Rather, the idiographic filter rests on individual or subgroup level factor analyses, using theory-guided conceptions to the extent possible that do not impose strictly invariant factor loadings across comparisons but, instead, allows for necessary *individuality* in the manifest expression of underlying constructs (the factor loadings). Rigor is imposed by defining invariance on the relationships among the constructs (the factor intercorrelations). Thus, the pattern of relations between constructs and observable indicators (the factor loadings) can exhibit idiosyncrasies but the interrelations of the constructs—the factor intercorrelations—are invariant across individuals (or subgroups, if that is the basis of the analyses). The singularly important feature of this resolution is that if one now factor analyzes the intercorrelations of the factors i.e., conducts a second-order factor analysis (Loehlin, 1998), those factor loadings will be metrically invariant across analyses, thus reinstituting the classical notion of factorial invariance but at a more abstract level. Moreover, transformations can be performed to obtain the loadings of the manifest variables directly on the second- or higher-order factors. This amounts to individually “tailored” measurement schemes for the same underlying higher-order constructs and hence the term *idiographic filter*.

Use of the *idiographic filter* is tantamount to asserting that the underlying mechanisms are the same (nomothetic relations) but the observable manifestations of those mechanisms reflect individuality (idiographic relations). As has been indicated elsewhere (Nesselroade & Molenaar, in press), this seems to be a productive way to approach the measurement of behavioral and psychological processes.

Conclusions

The contents of this special issue are strong evidence that methodology for the study of intraindividual variability has begun to mature. However, there is a long road ahead before it will be time to sit in a rocking chair on the front porch and fondly recall the good old days of innovation in this field. The field is rapidly evolving even as you read these words. It is an extremely exciting time to be involved in psychological inquiry. We have our work cut out for us, but the rewards are great. The individual has once again regained a rightful place as the primary unit of analysis in psychology.

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