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Analytic Derivation and Monte Carlo Validation of a Sensitivity Formula for Slit-Slit Collimation With Penetration

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Abstract

A slit-slit collimator consists of two orthogonal slits and can be conceptualized as a generalized pinhole. Since the two slits are independent of each other, there can be independent axial and transaxial acceptance angles. A small axial acceptance angle may help mitigate axial blurring with circular orbits, allowing multiple copies axially. In addition, since the two slit planes can be placed at different distances with respect to the source, a better detector usage can be achieved, especially in the case of detectors and imaged objects with different aspect ratios. In this paper an analytical expression is derived for the sensitivity of slit-slit collimation including effective slit widths for photon penetration. An analytical expression for sensitivity is necessary in order to accurately model the system response. This expression could also be useful for comparing the slit-slit's sensitivity performance with others. When the effective slit width is used instead of the geometric slit width, the derived analytical expression accurately accounts for photon penetration of the aperture. The derived expression for the sensitivity was validated by Monte Carlo simulation for both geometric and penetrative cases.

Index Terms

slit-slit; skew-slit; SPECT; sensitivity; penetration

I. INTRODUCTION

A slit-slit, also known as skew-slit, collimator is formed by two orthogonal slits, usually located at different distances from the detector (Fig. 1). In a pinhole collimator, the magnification is determined by the distance h from the source to the aperture plane and the distance f from the aperture plane to the detector. In the case of a slit-slit, since the two slits forming the collimator are in different planes, there are two independent magnifications: axial and transaxial. One slit (slit 1 in Fig. 1) is parallel to the Axis of Rotation (AoR) and its distance from the source determines the magnification in the transaxial direction. The other slit is perpendicular to the AoR (slit 2 in Fig. 1) and its distance from the source determines the axial magnification. When the two slits are placed on the same plane, they degenerate into a rectangular pinhole whose sensitivity is

$$g_{PH} = \frac{w_1 w_2}{4\pi h^2} \sin^3 \theta, \quad (1)$$

analogous to sensitivity for circular pinholes [1].

A pinhole collimator imaging along a circular orbit results in projections that do not fully fill the Radon space. This leads to axial artifacts in the reconstructed image [2] – [4]. Slit-slit collimation has been proposed as an alternative to pinhole collimation because of its favorable sampling properties [5]. Since slit-slit collimators can have two independent, axial and transaxial cone angles, the severity of the axial artifacts may be reduced by allowing a smaller cone angle in the axial direction.

Since its proposal as an alternative to a pinhole collimator [5], the slit-slit collimator's characteristics have not been evaluated yet. In [5], an analytical image reconstruction algorithm based on tilted fan-beam inversion with non-uniform attenuation was developed; the authors also showed that the reconstructed images had less severe axial artifacts compared to pinhole collimation. Later, in [6], the reconstruction algorithm presented in [5] was extended to include a uniform attenuation correction.

However in order to be able to assess the usefulness of the slit-slit collimator in different imaging scenarios, its characteristics such as sensitivity and resolution need to be evaluated. An analytic expression for the sensitivity and resolution for a slit-slit collimator are not yet available. An analytic expression for sensitivity, for example, is necessary to be able to make the comparison of sensitivity performance with other collimation schemes, such as a pinhole and slit-slat [7] – [13].

In this paper, we will first derive an analytical expression for the sensitivity of a slit-slit collimator. It will be shown that the effect of photon penetration on the sensitivity can be handled by the use of an effective slit width [14], which is defined as the width of an ideal slit (i.e., one that does not allow penetration) that passes the same number of photons as the real slit, which allows penetration. The effective slit width is always at least as large as the geometric width of the slit. In the second part of the paper, the sensitivity expression will be validated with Monte Carlo simulations for both geometric and penetrative cases.

II. THEORETICAL DERIVATION

A. Definition of the coordinate system and parameters

Both slits, numbered as 1 and 2, are on x - z planes at distances of h_1 and h_2 from the photon source, respectively. The slit widths are w_1 and w_2 . Slit 1 was chosen to run parallel to the z -axis, which is parallel to the AoR, and collimates transaxially. Slit 2 was chosen to lie along the x -axis and collimates axially. The origin of the coordinate system was chosen so that slit 1 is centered on the plane $x = 0$ and slit 2 is centered on the line $y = z = 0$.

A point source P is positioned in space at a location $\vec{r}_P$ on the side of slit 1 opposite to slit 2. The y direction given by $\hat{n} = (0, 1, 0)$ is positive on the side of slit 2 opposite to P . The polar angle θ is defined as the angle between $\vec{r}_P$ and its projection on the plane of slit 2. The azimuthal angle ϕ is defined as the angle between the projection of $\vec{r}_P$ on the plane of slit 2 and the x -axis: it is measured in the clock-wise direction as seen from the point source. With these definitions, $\vec{r}_P$ can be expressed as

$$\vec{r}_P \equiv (h_2 \cot \theta \cos \phi, -h_2, -h_2 \cot \theta \sin \phi). \quad (2)$$

A photon going through the center of slits 1 and 2 intersects the plane of slit 2 at point Q (see Fig. 1b) whose position is given by

$$\vec{r}_Q \equiv \left(-h_2 \frac{h_2 - h_1}{h_1} \cot \theta \cos \phi, 0, 0 \right). \quad (3)$$

The angle γ measures the angle that the photon path from P to Q forms with the planes of slits 1 and 2 ($\widehat{PQT}$ in Fig. 1), and can be expressed in terms of θ and ϕ as

$$\sin \gamma = \frac{|\vec{r}_Q - \vec{r}_P|}{|\vec{r}_Q - \vec{r}_P|} \cdot \hat{n} = \frac{\sin \theta}{\left[1 + \frac{h_2^2 - h_1^2}{h_1^2} \cos^2 \theta \cos^2 \phi \right]^{1/2}}. \quad (4)$$

B. Derivation of analytical expression

Sensitivity is herein defined as the fraction of emitted photons from P that reach the detector unscattered. The derivation starts by observing that the sensitivity g can be calculated from the flux through the plane of slit 2:

$$g \equiv \frac{1}{N} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\mu \Delta L} \vec{\Phi} \cdot \hat{n} \, dx dz, \quad (5)$$

where $\vec{\Phi}$ is the photon flux originating from the point source, which emits N photons per second, ΔL is the path of the photon through the material forming the slits and μ is the linear attenuation coefficient of the slit material.

It is now assumed that the incident photon flux is constant over the portion of the slit that passes the photons at the value of the center of slit. This is usually very accurate when P is at least a few slit widths away from the aperture and is equivalent to assuming that all photons in the beam are traveling along paths parallel to

$$\vec{r} = \vec{r}_Q - \vec{r}_P = \left(-\frac{h_2^2}{h_1} \cot \theta \cos \phi, h_2, h_2 \cot \theta \sin \phi \right). \quad (6)$$

With this parallel-beam approximation,

$$\vec{\Phi} = N \vec{r} / (4\pi |\vec{r}|^3). \quad (7)$$

Therefore (5) becomes

$$g = \frac{\vec{r} \cdot \hat{n}}{4\pi |\vec{r}|^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\mu \Delta L} \, dx dz. \quad (8)$$

In principle, integration over the whole slit is not consistent with the approximation of constant flux. However, in most practical cases μ is sufficiently large that the exponential term limits meaningful contributions to an area over which the flux can be considered constant.

The photons can reach the detector in four different ways (see Fig. 2): by not penetrating any material or by penetrating either slit or both. Hence (8) can be written as

$$g = \frac{\vec{r} \cdot \vec{n}}{4\pi|\vec{r}|^3} \left[\int_{|x| < \frac{w_1'}{2}} \int_{|z| < \frac{w_2}{2}} dx dz + \int_{|z| > \frac{w_2}{2}} \int_{|x| > \frac{w_1'}{2}} e^{-\mu \Delta L} dx dz \right. \\ \left. + \int_{|x| < \frac{w_1'}{2}} \int_{|z| > \frac{w_2}{2}} e^{-\mu \Delta L} dx dz + \int_{|z| < \frac{w_2}{2}} \int_{|x| > \frac{w_1'}{2}} e^{-\mu \Delta L} dx dz \right] \quad (9)$$

where

$$w_1' \equiv w_1 h_2 / h_1 \quad (10)$$

is the projection width of slit 1 on the plane of slit 2.

We define ΔL_1 and ΔL_2 as the photon path length through slits 1 and 2, respectively, so that the total photon path length through the material is $\Delta L = \Delta L_1 + \Delta L_2$. In the hypothesis of parallel beam, Φ is constant over the area contributing counts and ΔL_1 is a function of x but not z since a change in z is parallel to the slit axis and does not change path length; ΔL_2 is a function of z but not x . Then, the integrals in (9) become separable and integration yields:

$$g = \frac{\vec{r} \cdot \vec{n}}{4\pi|\vec{r}|^3} \left[w_2 w_1' + \int_{|z| > \frac{w_2}{2}} e^{-\mu \Delta L_2(z)} dz \int_{|x| > \frac{w_1'}{2}} e^{-\mu \Delta L_1(x h_1 / h_2)} dx \right. \\ \left. + w_1' \int_{|z| > \frac{w_2}{2}} e^{-\mu \Delta L_2(z)} dz + w_2 \int_{|x| > \frac{w_1'}{2}} e^{-\mu \Delta L_1(x h_1 / h_2)} dx \right]. \quad (11)$$

The scaling factor h_1/h_2 in the argument of ΔL_1 is needed because x is in the plane of slit 2. The two different 1-D integrals in (11) are both in the same form and account for the photons penetrating the slits. If the slits have a double-knife-edge profile, the result of these integrals can be found in [14], which also assumed a parallel beam, and written in terms of the sensitivity-effective slit widths w_{se} :

$$\begin{aligned}
w'_{se,1} - w'_1 &= \int_{|x| > \frac{w'_1}{2}} e^{-\mu \Delta L_1(xh_1/h_2)} dx = \frac{h_2}{h_1 \mu} \left[1 - \cot^2 \frac{\alpha_1}{2} \left(\frac{h_2}{h_1} \right)^2 \cot^2 \theta \cos^2 \phi \right] \sin \gamma \tan \frac{\alpha_1}{2} \\
w_{se,2} - w_2 &= \int_{|z| > \frac{w_2}{2}} e^{-\mu \Delta L_2(z)} dz = \frac{1}{\mu} \left[1 - \cot^2 \frac{\alpha_2}{2} \cot^2 \theta \sin^2 \phi \right] \sin \gamma \tan \frac{\alpha_2}{2},
\end{aligned} \tag{12}$$

where w_{se} incorporates both the geometric (w) and the penetrative (w_p) contributions to the sensitivity, and $\alpha_{1(2)}$ is the acceptance angle of slit 1(2). Note that for no penetration $\mu \rightarrow \infty$, so that $w_{se} = w$.

The sensitivity-effective width depends exclusively on the shape of the Point Spread Function (PSF). It can be rescaled back to the plane of the coordinate system as:

$$w'_{se,1} \equiv w_{se,1} h_2 / h_1. \tag{13}$$

With (12), equation (11) can be written as

$$\begin{aligned}
g &= \frac{\vec{r} \cdot \hat{n}}{4\pi |\vec{r}|^3} \left[w_2 w'_1 + (w'_{se,1} - w'_1)(w_{se,2} - w_2) + w'_1(w_{se,2} - w_2) + w_2(w'_{se,1} - w'_1) \right] \\
&= \frac{w_{se,1} w_{se,2}}{4\pi |\vec{r}|^3} \vec{r} \cdot \hat{n}.
\end{aligned} \tag{14}$$

Next we evaluate, with the help of (6)

$$\begin{aligned}
\frac{\vec{r} \cdot \hat{n}}{|\vec{r}|^3} &= \frac{(\vec{r}_Q - \vec{r}_P) \cdot \hat{n}}{|\vec{r}_Q - \vec{r}_P|^3} = - \frac{\vec{r}_P \cdot \hat{n}}{|\vec{r}_Q - \vec{r}_P|^3} \\
&= \frac{-\frac{h_2}{h_1} \cot^2 \theta \cos^2 \phi + h_2^2 \cot^2 \theta \sin^2 \phi}{\left[\left(\frac{h_2}{h_1} \right)^2 \cot^2 \theta \cos^2 \phi + h_2^2 \cot^2 \theta \sin^2 \phi \right]^{\frac{3}{2}}} \\
&= h_2^{-2} \left[\frac{h_2^2 - h_1^2}{h_1^2} \cot^2 \theta \cos^2 \phi + 1 + \cot^2 \theta \right]^{-\frac{3}{2}}.
\end{aligned} \tag{15}$$

Finally substituting (13), and (15) into (14), the general analytical expression for the sensitivity of a slit-slit collimator is reached:

$$g_{ss} = \frac{w_{se,1} w_{se,2}}{4\pi h_1 h_2} \sin^3 \gamma = \frac{w_{se,1} w_{se,2}}{4\pi h_1 h_2} \frac{\sin^3 \theta}{\left[1 + \left[\left(\frac{h_2}{h_1} \right)^2 - 1 \right] \cos^2 \theta \cos^2 \phi \right]^{3/2}}. \tag{16}$$

This expression determines the point source sensitivity of a slit-slit collimator. Sensitivity-effective slit widths account for the effects of photon penetration through the slits. In Fig. 3, the derived sensitivity expression (16) was plotted for a slit-slit collimator with $h_2/h_1 = 3$ as a function of polar angle θ and azimuthal angle ϕ . The sensitivity was normalized with respect to its maximum value, which occurs at $\theta = 90^\circ$.

C. Special cases

The derived expression for the slit-slit sensitivity, (16), is the generalization of the rectangular pinhole sensitivity formula (1). The denominator of the angular part of (16) becomes unity and (16) reduces to (1) when $h_2 = h_1$, $\theta = \pi/2$ or $\phi = \pi/2$. Here we note three special cases:

1. $h_1 = h_2$: . When $h_1 = h_2 = h$, the two slits are placed on the same plane and form a square pinhole. In this case, (16) correctly reduces to the rectangular pinhole sensitivity formula (1).
2. $\theta = \pi/2$: . When $\theta = \pi/2$, the source is at the apex of the slit-slit collimator. This configuration is equivalent to a rectangular pinhole with sides $(w_{se,1}h_2/h_1)$ and $w_{se,2}$ at a distance h_2 . In this case, $\cos \theta = 0$ and (16) correctly reduces to

$$g_{ss} = \frac{w_{se,1}w_{se,2}}{4\pi h_1 h_2} = \frac{w_{se,1}(h_2/h_1)w_{se,2}}{4\pi h_2^2} \quad (17)$$

which is equivalent to the sensitivity of a pinhole with sides $(w_{se,1}h_2/h_1)$ and $w_{se,2}$.

3. $\phi = \pi/2$: . This configuration corresponds to moving P along slit 1. This configuration is equivalent to a rectangular pinhole with sides $w_{se,1}h_2/h_1$ and $w_{se,2}$ placed at h_2 since the projection onto the plane of slit 2 along the x -axis does not shift (see (3)). Correctly (16) reduces again to (1).

III. VALIDATION WITH MONTE CARLO SIMULATION

The derived analytic expression for the sensitivity was validated using Monte Carlo (MC) simulations. This validation was necessary especially for the penetrative sensitivity expression which used sensitivity effective slit widths to account for the photon penetration through the slit apertures. In addition we used the MC simulations to evaluate the effect of the collimator scatter in slit-slit compared to a pinhole.

The MC code was developed in the framework of the GEANT4 simulation package [15]. The code simulates two double-knife-edge slits made of tungsten with 2.5 mm slit width. The tungsten material ($\mu = 3.6 \text{ mm}^{-1}$) was defined by using GEANT4's standard material definition database. Each slit had an acceptance angle $\alpha = 90^\circ$. For each simulated slit-slit configuration, 2×10^8 photons with 140 keV energy (^{99m}Tc) originating from a point source were emitted isotropically. The pure geometric and penetrative sensitivity cases were simulated by allowing and disallowing photon penetration through the slits. The detector was assumed to be 100% efficient and have perfect energy and spatial resolution.

A. Sensitivity validation

These validation studies did not include Compton scatter in the simulation. In order to test the validity of (16) three sets of simulations, for configurations in which all but one parameter were kept constant, were performed.

In the first set (cases 1 through 8 in Table I), θ was varied for constant values of ϕ , h_1 , and h_2 . Cases with $\phi = 0^\circ$ and varying θ correspond to the angular movement of the point source parallel to slit 2 (perpendicular to AoR and slit 1). When $\phi = 90^\circ$, as θ changes the source moves along slit 1, i.e, parallel to AoR and slit 1 and perpendicular to slit 2.

In the second set (cases 9 through 12 in Table I), ϕ was varied for fixed θ , h_1 , and h_2 . The source was moved away from the geometric center of slit 2 and the corresponding θ was calculated. The radial distance, and hence angle θ , was chosen arbitrarily provided that the

source is in the field of view of slit 1. For each angle θ , ϕ was changed from 0° to 180° in 30° steps.

In case 13 (Table I), $\theta = 90^\circ$, $\phi = 0^\circ$ and $h_2 - h_1 = 30$ mm were constant for varying values of h_1 . Finally in case 14, $\theta = 90^\circ$, $\phi = 0^\circ$, and $h_1 = 30$ mm were constant for varying h_2 .

The parameters h_1 and h_2 were chosen such that a cylindrical object of varying diameter and lengths can be imaged without any truncation in typical small-animal imaging scenarios. For example, $h_1=20$ mm and $h_2=60$ mm are necessary distances from the source to the slit 1 and 2 respectively so that a mouse with an approximate length:diameter aspect ratio of 3:1 is not truncated with an aperture angle $\alpha=90^\circ$.

The agreement between the analytically calculated sensitivity and the MC simulation was quantified by calculating a χ^2 statistic as

$$\chi^2 = \sum_{i=1}^I \left[\frac{(g_i^{\text{analytical}} - g_i^{\text{MC}})}{\sigma_i^{\text{MC}}} \right]^2, \quad (18)$$

where $g^{\text{analytical}}$ is the analytically calculated sensitivity (16), g^{MC} is the MC sensitivity, σ_i^{MC} is the uncertainty on MC sensitivity, and I is the number of data points in each plot. The MC sensitivity and its uncertainty are calculated as $g^{\text{MC}} = N_{\text{detected}}/N_{\text{emitted}}$ and $\sigma_i^{\text{MC}} = \sqrt{N_{\text{detected}}/N_{\text{emitted}}}$ respectively.

B. Collimator scatter fraction

The slit-slit collimator is a generalized pinhole collimator. However, since the slits are located at different planes, the collimator scatter fraction may be different. For this study two different pinhole sizes, representing the likely range of use, were considered: $w=0.1$ mm and $w=2.5$ mm. For the slit-slit collimator, both slit widths were the same as the pinhole size. The collimator scatter fractions for slit-slit and pinhole collimators were studied with dedicated simulations. These simulations included Compton scattering. The simulation recorded the detected photon's energy. The simulated pinhole collimator had a double-knife-edge square pinhole and was made of tungsten. The collimator scatter fraction was calculated as the ratio of the number of detected scattered photons to the total number of detected photons (scattered + unscattered) photons in a given energy window. Different detector energy resolutions were considered in the range from 1% to 20%. Scatter fractions as a function of energy resolution were calculated for four different energy windows (5%, 10%, 15%, and 20%).

IV. RESULTS

A. Comparison of theoretical sensitivity with simulations

The sensitivities for each source position and slit-slit collimator configuration are plotted for both the MC simulation results and the calculated results from (16). The expressions of $w_{se,1}$ and $w_{se,2}$ are found in [14] and can be calculated from (12).

In Fig. 4, sensitivity as a function of θ is shown for case 1 (top left), and case 2 (top right) (see Table I). Filled marker points denote the photon penetration MC while open marker points show the geometric only MC. Penetration (geometric) theory is shown with solid (dashed) line and penetration (geometric) MC is shown with filled (open) marker points. The solid and dashed lines show the sensitivity calculated from (16) for the penetrative and geometric only

cases respectively. The dash-dotted line which shows the $\sin^3 \theta$ angular dependence of a pinhole collimator, was also drawn for comparison.

Similarly, simulation results and the theoretical predictions for cases 3 and 4 of Table I are shown in Fig. 4 at the bottom left and right, respectively.

In Fig. 5, the sensitivities corresponding to cases 5 (top left), 6 (top right), 7 (bottom left), and 8 (bottom right) are plotted (see Table I).

Cases 9, 10, 11, and 12 in Table I are shown in Fig. 6. In Fig. 7(a), we show the results for case 13 and in Fig. 7(b) the results for case 14 are presented.

The agreement was quantified by calculating χ^2 and χ^2/ndf , where ndf is the number of degrees of freedom, and tabulated in Table II for both the geometric and penetrative cases. For all of the cases investigated, the agreement between simulation and the derived expression is within statistical uncertainty better than $< 1\%$.

B. Collimator scatter fraction

The collimator scatter fractions measured by MC simulations for four energy windows of 5%, 10%, 15%, and 20% are shown in Fig. 8 with filled circles, filled squares, open circles and open squares respectively. In Fig. 8 (a) and (c), the scatter fractions as a function of energy resolution is plotted for slit-slit for $w=0.1$ mm and $w=2.5$ mm respectively. The slit-slit collimator was configured with $h_1=20$ mm and $h_2=80$ mm. The scatter fractions for pinhole collimators of sizes $w=0.1$ mm and $w=2.5$ mm are shown in Fig. 8 (b) and (d) respectively.

V. DISCUSSION

The derived analytic expression describes the sensitivity for slit-slit collimation. Two slit planes are located at different distances from the source and the sensitivity was shown to have angular dependence on both polar (θ) and azimuthal angles (ϕ) as well as the slit distances from the source. This angular dependence of slit-slit sensitivity is different from that of the pinhole in that pinhole sensitivity has only a polar angle (θ) dependence.

The derived expression for the sensitivity of the slit-slit collimation reduces to that of a pinhole collimator in three cases: $h_1 = h_2$, $\theta = \pi/2$, and $\phi = \pi/2$. If the two slits are on the same plane, $h_1 = h_2$, the geometry becomes a rectangular pinhole and the derived expression correctly describes the case as shown earlier in Section II-C. In the case where $\theta = \pi/2$, the source is at the apex of the slit-slit collimator, and the geometry behaves as a rectangular pinhole collimator with sides $w_1 h_2 / h_1$ and w_2 at a distance h_2 . Similarly, in the case $\phi = \pi/2$, the geometry again acts as a rectangular pinhole collimator with sides $w_1 h_2 / h_1$ and w_2 , as shown in Fig. 5, because the projection on the slit 2 plane does not shift. In these three aforementioned cases, the derived formula reduces to a pinhole sensitivity formula since γ and θ are the same (See Fig. 1). In the case $\phi = 0$, (16) does not reduce to (1); MC simulations in Fig. 4 confirm that the angular dependence of sensitivity for the slit-slit collimator is different than that of the pinhole collimator. In this case, the angle of incidence of the photon, γ , does not coincide with the spatial coordinate θ . This occurs when the source projects to a position where the x coordinate is other than zero (Fig. 1).

The choice of the coordinate system is arbitrary. We chose the laboratory coordinate system in which the origin of the coordinate system is the center of the slit 2 plane, independent of the source position. If one chooses a coordinate system that would move with the source, the pinhole sensitivity formula with a polar angle dependence is obtained (See (4)). However this coordinate system is less useful for predicting the response in the laboratory frame and less

direct in the forward and backward steps of reconstruction. Still, the relationship between the coordinate systems given by (4) allows for direct conversion.

The derived analytic expression was verified by MC simulations for 14 different cases. In cases 1–12, the angular dependence of (16) was tested with MC simulations. While the source to slit plane distances, h_1 and h_2 , were constant in each case, either θ or ϕ was varied. Cases 1–4 (Fig. 4) tested the angular dependence when $\phi = 0^\circ$ at different h_2/h_1 configurations. Both pinhole and the slit-slit collimator sensitivities reach their maximum values when $\theta = 90^\circ$. Equation (16) predicts that as the angle θ deviates from 90° , the sensitivity of slit-slit collimator decreases faster than that of pinhole collimator, which has a $\sin^3 \theta$ dependence. When $\phi = 90^\circ$ (cases 5–8, Fig. 5), the slit-slit angular dependence reduces to $\sin^3 \theta$ which is the same as the pinhole angular dependence. Cases 9–12 (Fig. 6) show that when the polar angle θ along with h_1 and h_2 are kept constant, the sensitivity of the slit-slit collimator varies as a function of the azimuthal angle ϕ . For comparison, the pinhole sensitivity predicts a constant sensitivity at a fixed polar angle. For example, cases 9 and 10 show a much increased dependence on ϕ as θ changes by only $\sim 10^\circ$. Further, cases 11 and 12 show that this effect decreases with decreasing h_2/h_1 . The agreement between the MC simulation results and theoretical predictions shows that (16) describe the angular dependence of sensitivity of a slit-slit collimator accurately. Finally, in cases 13 and 14, the h_1 and h_2 dependence of the sensitivity was verified with simulations. In both cases MC simulations agreed well with the derived analytical expression.

In the derivation of the sensitivity, the effect of the photon penetration was accounted for by the use of the sensitivity-effective slit widths. Equation (16) describes the penetrative sensitivity by replacing the geometric slit widths with the sensitivity-effective slit widths as described by (12). MC simulations presented in Fig. 4–7 tested the penetrative sensitivity expression and yielded good agreement with the predictions by (16).

The collimator scatter fraction of a pinhole and a slit-slit collimator of the same pinhole size and slit width was shown in Fig. 8. When $w = 0.1$ mm, the scatter fraction was $\sim 2\%$ for both pinhole and slit-slit collimators with a 20% energy window and for detector energy resolutions of range 1–20%. When the energy window was decreased to 5%, the collimator scatter fractions decreased as the detector energy resolutions decreased and plateau at $\sim 1\%$ at 4% energy resolution. When $w = 2.5$ mm, the collimator scatter fraction was less than 0.5% for both slit-slit and pinhole collimators for all energy windows and resolutions considered. The existence of two slits at different planes in a slit-slit collimator does not change the collimator scatter fraction compared to a pinhole collimator of the same pinhole size as the slit width.

In the derivation of the sensitivity expression, two arbitrary assumptions were made: the center of the coordinate system was positioned arbitrarily at the center of the slit 2 and slit 1 was assumed to be in the direction of z -axis and closer to point P than slit 2. The first assumption leads to the variables θ and ϕ being measured from slit 2 where $h_2 > h_1$. If the coordinate system were located at the center of slit 1, the polar and azimuthal angle would be measured with respect to slit 1. In this case, one could arrive to a similar sensitivity expression with the variables change $h_1 \leftrightarrow h_2$. Instead of the second assumption, if slit 1 is moved closer to the detector, the position of point Q would be located on slit 1 and given by

$$\vec{r}_Q \equiv (0, h_1 - h_2, \frac{h_1 - h_2}{h_2} h_2 \cot \theta \sin \phi). \quad (19)$$

Also the projection width of slit 2 on slit 1 would be

$$w'_{se,2} = \frac{h_1}{h_2} w_{se,2}. \quad (20)$$

With these expressions, one could repeat the derivation to show that the same form of the sensitivity expression is calculated.

Slit-slit collimation can be considered as a generalization of pinhole collimation. Consequently, it is natural to compare the two. Pinhole collimation yields the same magnification in both directions. In contrast, a slit-slit collimator can offer different magnifications, which can be used to give reduced magnification in one direction. This reduction allows making multiple copies of the image or to image different portions of the object with different portions of the detector. If multiple images are made axially, the acceptance angle in that direction can be reduced to mitigate axial blur from oblique photons. However, a consequence of reducing magnification is the potential reduction in spatial resolution. A more thorough investigation of this comparison will be the subject of future research.

VI. CONCLUSION

An analytical expression for the sensitivity of a slit-slit collimator was derived and validated with MC simulations. The agreement between the derived slit-slit sensitivity expression and MC better than $< 1\%$ for both the penetrative and geometric sensitivities shows that the derived analytical expression describes the sensitivity of the slit-slit collimator very well. We have also shown with MC simulations that the penetrative sensitivity expression can be obtained from the geometric sensitivity expression by replacing the geometric slit width by the sensitivity effective slit width. The collimator scatter fraction characteristics of the slit-slit were studied with MC simulations and showed that the collimator scatter fraction of a slit-slit is the same as that of a pinhole collimator. An advantage of a slit-slit compared to a pinhole collimator could be the capability to position two slit planes independent of each other with respect to the source which might lead to better detector usage. The derived expression along with expressions for field of view and resolution may be important for evaluating the scenarios in which slit-slit collimation may be useful when compared with other collimation schemes. We will investigate these scenarios in our future work.

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REFERENCES

1. Mallard JR, Myers MJ. The Performance of a Gamma Camera for the Visualization of Radioactive Isotopes in vivo. *Phys. Med. Biol* 1963;vol. 18:165–182. [PubMed: 13932556]
2. Jaszczak RJ, Li J, Wang H, Zalutsky MR, Coleman RE. Pinhole collimation for Ultra-High-Resolution, Small-Field-of-View SPECT. *Phys. Med. Biol* 1994;vol. 39:425–437. [PubMed: 15551591]
3. Tuy HK. An inversion formula for cone-beam reconstruction. *J. Apply. Math* 1983;vol. 43(no. 3):546–552.
4. Smith BD. Image reconstruction from cone-beam projections: Necessary and sufficient conditions and reconstruction methods. *IEEE. Trans. Med. Imag* 1985;vol. MI-4:14–25.
5. Huang Q, Zeng GL. An analytical algorithm for skew-slit imaging geometry with nonuniform attenuation correction. *Med. Phys* 2006;vol. 33(no 4):997–1004. [PubMed: 16696476]
6. Tang Q, Zeng GL, Huang Q. An analytical algorithm for skew-slit collimator SPECT with uniform attenuation correction. *Phys. Med. Biol* 2006;vol. 51:6199–6211. [PubMed: 17110780]

7. Metzler SD, Accorsi R, Novak JR, Ayan AS, Jaszczak RJ. On-axis sensitivity and resolution of a Slit-Slat collimator. *J. Nucl. Med* 2006;vol. 47:1884–1890. [PubMed: 17079823]
8. Rogers WL, et al. SPRINT: A stationary detector single photon ring tomography for brain imaging. *IEEE Trans. Med. Imaging* 1982;vol. MI-1:63–68. [PubMed: 18238259]
9. Rogers WL, et al. Performance evaluation of SPRINT: A single photon ring tomograph for brain imaging. *J. Nucl. Med* 1984;vol. 25:1013–1018. [PubMed: 6332182]
10. Rogers WL, et al. SPRINT II: A second generation single photon ring tomograph. *IEEE Trans. Med. Imaging* 1988;vol. 7:291–297. [PubMed: 18230481]
11. Chang W, Liang H, Liu J. Design concepts and potential performance of MarC-SPECT: A high-performance cardiac SPECT system. *J. Nucl. Med* 2006;vol. 47:190P–191P.
12. Walrand S, Jamar F, de Jong M, Pauwels S. Evaluation of novel whole-body high-resolution rodent SPECT (linoview) based on direct acquisition of linogram projections. *J. Nucl. Med* 2005;vol. 46:1872–1880. [PubMed: 16269602]
13. Brasse, D.; Piqueras, I.; Guyonnet, J-L. Development of a high resolution SPECT system dedicated to small-animal imaging. *Proc. Nuclear Science Symposium Conf. Record; Rome*. 2004. p. 3868-3871.
14. Accorsi R, Novak JR, Ayan AS, Metzler SD. Derivation and validation of a sensitivity formula for Slit-Slat collimation. *IEEE Trans. Med. Imaging* 2008;vol. 27:709–722. [PubMed: 18450543]
15. Allison J, et al. Geant4 developments and applications. *IEEE Trans. Nucl. Sci* 2006;vol. 53(no.1): 270–278.

Fig. 1.

(a) Schematic drawing showing the slit-slit geometry and the symbols used. $w_{1(2)}$ is the slit 1 (2) width, and $h_{1(2)}$ is the perpendicular distance from the source P to the plane of slit 1(2). The polar angle is θ and the azimuthal angle ϕ is measured with the right hand rule about the y -axis with respect to the direction of slit 2. (b) The projection of slit 1 on the plane of slit 2 has width w_1' . Q is the center of the projection, and γ is the angle between the photon path and the slit 2 plane.

Fig. 2.

Top view of the two slits. Photons can reach the detector through four different regions: without penetrating any slit (region 1), penetrating both slits (region 2), penetrating slit 2 but not penetrating slit 1 (region 3), penetrating slit 1 but not penetrating slit 2 (region 4). Q represents the center of the projection on the plane of slit 2. Although photon penetration extends beyond the area shown, (11)–(14) show that the effective widths of the two slits are independent, despite overlap in region 2.

Fig. 3. Normalized sensitivity as a function of the azimuthal angle ϕ and the polar angle θ for a slit-slit collimator with $h_2/h_1 = 3$. The dotted lines correspond to the constant value of θ used in MC cases 9 and 10 (see Table I).

Fig. 4.

Sensitivity of a point source (^{99m}Tc 140 keV) as a function of angle θ while $\phi = 0^\circ$ for cases: 1 (top left, $h_1=30$ mm and $h_2=60$ mm), 2 (top right, $h_1=40$ mm and $h_2=70$ mm), 3 (bottom left, $h_1=30$ mm and $h_2=70$ mm), and 4 (bottom right, $h_1=40$ mm and $h_2=80$ mm). Penetration (geometric) theory is shown with solid (dashed) line and penetration (geometric) MC is shown with filled (open) marker points. The dash-dotted line represents the $\sin^3 \theta$ dependence of a pinhole collimator. Error bars ($\sigma_i^{MC} = \sqrt{N_{\text{detected}}/N_{\text{emitted}}}$) are smaller than the size of the data point marker. See Table I for explanation of cases.

Fig. 5.

Sensitivity of a point source (^{99m}Tc 140 keV) as a function of angle θ while $\phi = 90^\circ$ for cases: 5 (top left, $h_1=30$ mm and $h_2=60$ mm), 6 (top right, $h_1=40$ mm and $h_2=70$ mm), 7 (bottom left, $h_1=30$ mm and $h_2=70$ mm), and 8 (bottom right, $h_1=40$ mm and $h_2=80$ mm). Penetration (geometric) theory is shown with solid (dashed) line and penetration (geometric) MC is shown with filled (open) marker points. In this case, the $\sin^3 \theta$ angular dependence of a pinhole collimator is the same as the angular dependence of a slit-slit collimator. Error bars

($\sigma_i^{MC} = \sqrt{N_{\text{detected}}/N_{\text{emitted}}}$) are smaller than the size of the data point marker. See Table I for explanation of cases.

Fig. 6.

Sensitivity of a point source (140 keV) as a function of angle ϕ while θ is constant for cases: 9 (top left, $\theta = 85.2^\circ$, $h_1 = 20$ mm and $h_2 = 60$ mm), 10 (top right $\theta = 75.9^\circ$, $h_1 = 20$ mm and $h_2 = 60$ mm), 11 (bottom left $\theta = 85.9^\circ$, $h_1 = 30$ mm and $h_2 = 70$ mm), and 12 (bottom right $\theta = 77.9^\circ$, $h_1 = 30$ mm and $h_2 = 70$ mm). Penetration (geometric) theory is shown with solid (dashed) line and penetration (geometric) MC is shown with filled (open) marker points. The dash-dotted line represents the $\sin^3 \theta$ dependence of a pinhole collimator. Error bars

($\sigma_i^{MC} = \sqrt{N_{detected}/N_{emitted}}$) are smaller than the size of the data point marker. See Table I for explanation of cases.

Fig. 7.

(a) Penetrative and geometric sensitivities as a function of h_1 when $\phi = 0^\circ$, $\theta = 90^\circ$, and $h_2 - h_1 = 30$ mm (case 13: See Table I). (b) Penetrative and geometric sensitivities as a function of h_2 when $\phi = 0^\circ$, $\theta = 90^\circ$, and $h_1 = 30$ mm (case 14: See Table I). Penetration (geometric) theory is shown with solid (dashed) line and penetration (geometric) MC is shown with filled (open) marker points. Error bars ($\sigma_i^{MC} = \sqrt{N_{detected}/N_{emitted}}$) are smaller than the size of the data point marker.

(a)

(b)

(c)

(d)

Fig. 8.

Collimator scatter fractions as a function of energy resolution are shown for a slit-slit collimator with $h_1=20$ mm, $h_2=80$ mm with (a) $w=0.1$ mm (c) $w=2.5$ mm, and square double-knife-edge pinhole collimator with pinhole size (b) $w=0.1$ mm (d) $w=2.5$ mm.

TABLE I

Summary of Monte Carlo experimental parameters.

MC case#	ϕ [°]	θ [°]	h_1 [mm]	h_2 [mm]
1	0	67–90	30	60
2	0	63–90	40	70
3	0	70–90	30	70
4	0	66–90	40	80
5	90	56–90	30	60
6	90	60–90	40	70
7	90	60–90	30	70
8	90	55–90	40	80
9	0–180	85.2	20	60
10	0–180	75.9	20	60
11	0–180	85.9	30	70
12	0–180	77.9	30	70
13	0	90	20–70	h_1+30
14	0	90	30	50–105

TABLE II

χ^2 and χ^2/ndf for each MC configuration of Table I.

MC case #	geometric	penetrative
	χ^2/ndf	χ^2/ndf
1	6.6 / 6 = 1.1	1.3/6 = 0.2
2	4.8 / 8 = 0.6	9.9/8 = 1.2
3	17.4 / 6 = 2.9	8.1/6 = 1.4
4	1.9 / 8 = 0.2	6.9/8 = 0.9
5	9.3 / 9 = 1.0	15.6/9 = 1.7
6	11.1 / 9 = 1.2	16.3/9 = 1.8
7	4.2 / 9 = 0.5	16.8/9 = 1.9
8	11.2 / 9 = 1.2	7.9/9 = 0.9
9	5.8 / 7 = 0.8	14.2/7 = 2.0
10	15.0 / 7 = 2.1	8.2/7 = 1.2
11	10.3 / 7 = 1.5	11.5/7 = 1.6
12	8.8 / 7 = 1.3	9.2/7 = 1.3
13	11.6 / 6 = 1.9	3.9/6 = 0.7
14	5.8 / 7 = 0.8	5.5/7 = 0.8