Fitts' law is not continuous in reciprocal aiming

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It takes longer to accomplish difficult tasks than easy ones. In the context of motor behaviour, Fitts' famous law states that the time needed to successfully execute an aiming movement increases linearly with task difficulty. While Fitts' explicit formulation has met criticism, the relation between task difficulty and movement time is invariably portrayed as continuous. Here, we demonstrate that Fitts' law is discontinuous in reciprocal aiming owing to a transition in operative motor control mechanisms with increasing task difficulty. In particular, rhythmic movements are implemented in easy tasks and discrete movements in difficult ones. How movement time increases with task difficulty differs in both movement types. It appears, therefore, that the human nervous system abruptly engages a different control mechanism when task difficulty increases.

Keywords: speed–accuracy trade-off; motor control; transition

1. INTRODUCTION

Fitts' law (Fitts 1954; Fitts & Peterson 1964) unusual robustness forcefully suggests that it captures a fundamental property of human motor performance. Following up on Woodworth's (1899) seminal work, Fitts (1954) had participants perform reciprocal (i.e. back and forth) movements between two targets whose width (W) and distance (D) were systematically varied across trials. He discovered that the time required to perform an aiming movement—movement time (MT)—relates to D and W according to $MT = a + b \times \log_2(2D/W)$, where a and b are constants. The index of difficulty ($ID = \log_2(2D/W)$) quantifies task difficulty via the amount of information (in bits) that is required to specify target width W relative to distance D (Shannon & Weaver 1949; Fitts 1954; Fitts & Peterson 1964). Fitts' task thus reduces the inherently complex relation between environmental constraints and motor control to a one-dimensional problem relating two scalar variables. As such, Fitts' aiming paradigm may be considered to capture the essence of goal-directedness in its simplest and most elegant form.

Although over the decades alternative formulations for the trade-off between movement speed and task difficulty have been formulated (Schmidt *et al.* 1979; Meyer *et al.* 1988; MacKenzie 1992), the smooth continuous character of the ID–MT relation has never been questioned. The finding that during reciprocal aiming the patterning of the movements' kinematics yielding Fitts' law changes gradually with task difficulty (Mottet & Bootsma 1999; Mottet *et al.* 2001; Bootsma *et al.* 2002) has further reinforced the idea of a continuous relationship. This

gradual adjustment may, however, evoke a transition in control mechanism (electronic supplementary material 1). Here, we demonstrate the use of distinct control mechanisms during reciprocal aiming movements and its consequences for Fitts' law.

2. MATERIAL AND METHODS

Ten participants were instructed to reciprocally move a line cursor between two targets for 30 cycles (i.e. 60 aiming movements) per trial under instructions stressing speed as well as accuracy. Trials with more than five target misses in the last 25 cycles were performed anew. Left–right stylus movements over a graphics tablet (WACOM UltraPad A3) produced time-locked left–right motion of a vertical red line cursor on a cathode ray tube screen (Dell M991, 1024×768 pixels resolution) positioned at eye level and at a horizontal distance of about 60 cm. The stylus, weighting 11 g, was 145 mm long with a diameter of 12 mm. Measurement accuracy was 0.15 mm. Two vertically elongated white bars on the screen represented the targets against a blue background. The stylus' position was sampled at a frequency of 150 Hz. Twelve different target widths were implemented under a constant 20 cm distance between target midpoints, resulting in 12 IDs (from 2.5 to 6.9, with steps of 0.4). Each ID condition was repeated 10 times. Trials were presented in blocks of 12 in which each condition occurred once (randomly ordered). The experiment was performed in two five-block sessions held on different days. Six habituation trials (covering the entire range of IDs) preceded the experiment proper.

Each trial's first five cycles were removed. For the remaining 25 cycles, we determined the effective amplitude (A) as the average distance between consecutive position extrema and effective target width (W_e) as 1.96 times the mean standard deviation averaged at both extrema (cf. Welford 1968). Effective ID was calculated as $ID_e = \log_2(2A/W_e)$. For each

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aiming movement, we determined the total duration of movement (movement time MT, defined as the time between successive position extrema) and the durations of the corresponding acceleration and deceleration phases. Acceleration time (AT) and deceleration time (DT) were defined as the duration prior to and following peak velocity, respectively. Although for higher IDs ($ID_e > 5.5$) fluctuations in velocity, indicative of corrective submovements, were observed towards the end of the movement, such fluctuations were generally small and did not affect the detection of the much larger peak velocity. As these submovements are not of primary interest for our present purposes, they will not further be addressed. Averaging over the 50 aiming movements (25 cycles) of a trial, we then computed the MT, AT and DT trial averages.

Exponential regression was performed on individual trial values of AT (i.e. 120 data points per participant) according to $a - b \times \exp(-\lambda \times ID_e)$. To anticipate, visual inspection of the AT suggested that its evolution as a function of ID_e involved a discontinuity. In order to localize it, per participant two linear regressions were performed over data windows of varying size but minimally including 30 data points; one regression over the data window from 1 to n , and one regression over the (remaining) data window from $N - n$ to N , with n increasing from 31 to 89, and N being 120. The discontinuity, or breakpoint, was identified as the ID_e at which the total (i.e. of both linear regressions) of the residuals' squared sum was smallest.

For the concepts and technical details of the vector field reconstruction, we refer the reader to Friedrich & Peinke (1997a,b), Van Mourik *et al.* (2006) and Huys *et al.* (2008). Summarizing briefly, stochastic behaviour, such as biological movement, can be viewed as the time evolution of a deterministic process under the impact of random (i.e. stochastic) fluctuations, referred to as the drift and diffusion components, respectively. Both components can be disentangled from a time series via computation of its conditional probability matrix $P(x, y, t | x_0, y_0, t_0)$, which indicates the probability to find the system at state (x, y) at a time t , given its state (x_0, y_0) at an earlier time step t_0 . From the position $x(t)$ and velocity $y(t)$ time series, we computed each trial's conditional probability matrix defined over a 41×41 equally bin-sized grid with the aim to focus on the dynamics' deterministic (i.e. the drift) component. Importantly, the drift coefficients numerically express the system's vector field in phase space, that is, they allow recovery of the flow that allows for the unambiguous qualification of dynamical systems (cf. Strogatz 1994; Huys *et al.* 2008). Drift coefficients were computed for each bin according to

$$D_x(x, y) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} \iint (x' - x) P(x', y', t + \tau | x, y, t) dx' dy'$$

and

$$D_y(x, y) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} \iint (y' - y) P(x', y', t + \tau | x, y, t) dx' dy',$$

and averaged across blocks of 10 trials with adjacent ID_e 's. From the first two drift coefficients (representing the velocity vector's x and y components), we determined for each bin the angles θ between its neighbouring velocity vectors and extracted its maximum value ($\theta_{\max}$). Note that the presence of locally opposing vectors (i.e. with an angle of approximately 180°) indicates the existence of a fixed point, while its absence

is indicative of a limit cycle. For each participant and each condition, the transition point between both dynamical regimes was determined through the location of the inflection point in the sigmoidal function $\theta_{\max} = 1/(1 + e^{a(b + ID)})$.

For the principal component analysis (PCA), the phase space (x, y) was divided into eight equal and around-the-origin-point symmetrical areas from 0 to $\frac{1}{4}\pi$, $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$, ..., and $\frac{3}{4}\pi$ to 2π . For each trial, the movement trajectories within each area were selected, resampled to equal length ($N = 40$) and organized into a state vector $\mathbf{q}(t)$ that was subjected to PCA (Haken 1996; Daffertshofer *et al.* 2004). The eigenvalue corresponding to the first mode, interpreted as the amount of variance captured by it, was determined and the values corresponding to the point symmetric areas were averaged (figure 3). The so-obtained areas P_1 , P_2 , P_3 and P_4 correspond to the first, second, third and fourth quarter of aiming movements, respectively.

ID block averages of MT, AT, DT and $\theta_{\max}$ were subjected to repeated-measures ANOVA with ID (12) as the within-participant factor while systematically applying a Huynh–Feldt correction. Significant main effects were followed up by Bonferroni-adjusted post hoc tests (reported in the electronic supplementary material 2).

3. RESULTS

Twelve equidistant IDs were repeated 10 times, resulting in 120 trials per participant. Each trial consisted of 60 reciprocal aiming movements between two targets under instructions stressing speed and accuracy (see §2 for details). The ID_e averaged across 10-trial blocks increased monotonously from 4.47 to 8.23 (the mean $ID_e \pm$ the coefficient of variation was 4.47 ± 0.03 , 4.80 ± 0.03 , 5.02 ± 0.02 , 5.24 ± 0.02 , 5.51 ± 0.02 , 5.83 ± 0.02 , 6.24 ± 0.02 , 6.65 ± 0.03 , 7.06 ± 0.02 , 7.37 ± 0.02 , 7.73 ± 0.02 and 8.23 ± 0.02 for blocks 1–12, respectively).

MT as well as its constituents, AT and DT, increased with increasing ID_e (all p 's < 0.0001 ; figure 1). While MT and DT always differed for adjacent ID_e pairs, for AT this was the case up to $ID_e = 5.46$ but never for $ID_e > 5.81$, where it hardly increased at all (see table S1 and figure S1 in electronic supplementary material 2 for individual participants' data). Regression analysis confirmed the visual impression that AT's evolution as a function of ID_e was discontinuous rather than exponential (figure 1). For each participant, the sum of the residuals squared was lower for the (double) linear regression than for the exponential regression, and across participants the difference was significant (two-tailed t -test, $p < 0.01$). The average breakpoint was located at $ID_e = 5.88$ (± 0.43 s.d.).

To examine whether distinct control mechanisms operate in the regimes separated by the discontinuity, we reconstructed the phase spaces' vector fields (§2). At low ID_e , the maximal angle between adjacent vectors was always small (less than 45°), while at high ID_e it approached 180° (figure 2). In mathematical terms, these observations imply the existence of limit cycle and fixed-point dynamics, respectively; dynamical structures that are, respectively, associated with rhythmic and discrete movements (Huys *et al.* 2008). On average, the transition between both control mechanisms was located at $ID_e = 5.41$ (± 0.37 s.d.).

Figure 1. Movement time and its partitioning for a single participant. The horizontal axis represents ID_e ; the vertical axis time (s). Each dot represents a single trial ($N = 120$). Black, blue and red dots represent MT, acceleration, and deceleration phase (see main text). Black lines represent linear regressions. For the acceleration phase below the discontinuity, the slope was 0.13 ± 0.02 (mean $\pm$ s.d., $R^2 = 0.73 \pm 0.12$; all p 's < 0.01); above it 0.04 ± 0.02 ($R^2 = 0.42 \pm 0.22$; all p 's < 0.01 , except one $p > 0.05$). For the deceleration phase, 0.19 ± 0.04 ($R^2 = 0.74 \pm 0.12$; all p 's < 0.01) and 0.28 ± 0.07 ($R^2 = 0.87 \pm 0.07$; all p 's < 0.01), respectively.

Figure 2. Reconstructed angle diagrams as a function of ID_e averaged across participants. The horizontal axes represent normalized position (x); the vertical axes normalized velocity (y). ID_e increases from the upper left panel ($ID_e = 4.47$) from left to right and up to down to the lower right panel ($ID_e = 8.23$). The colour coding (right side of the panel) represents the maximum angle in degrees between adjacent vectors (§2). Red areas indicate the existence of locally opposing angles and imply the presence of a fixed point. Its absence implies the existence of a limit cycle.

In the fixed-point regime, MT increased with ID_e predominantly owing to DT's lengthening (figure 1). Dynamical systems theory states that, the more attractive a fixed point, the faster the system converges towards it

(Strogatz 1994). The lengthening of DT with higher task difficulty thus predicts (counterintuitively) that the fixed point's attractiveness decreases with diminishing target width. In the presence of noise, this evokes

Figure 3. PCA. (a) Partitioning of trajectories in phase space for (i) $ID_e = 8.25$ and (ii) $ID_e = 4.48$ for one participant. The horizontal and vertical axes represent position (x) and velocity (y). P_1 , P_2 , P_3 and P_4 represent the first, second, third and fourth part of the movement (here shown for the lower half of the phase space only; §2). (b) Variance captured by the first PCA mode as a function of ID_e for P_1 , P_2 , P_3 and P_4 .

increasing trajectory variability during target approach even though the variability at the target is smaller (as dictated by the task). This prediction was confirmed via PCA (see figure 3 and §2). While ID_e increments did not affect trajectory variability in the second and third quarter of the aiming movements (both p 's > 0.05 ; figure 3), it led to increased variability in the first and particularly last quarter of the movements ($p < 0.0001$ and $p = 0.0001$, respectively).

4. DISCUSSION

We show that when task difficulty is high ($ID_e \gtrsim 5.5$), reciprocal aiming is accomplished via (slower) discrete movements while (faster) rhythmic movements are used for low levels of task difficulty ($ID_e \lesssim 5.5$). In frequency-paced movement tasks without accuracy constraints, a switch from discrete to rhythmic movements (at about 2 Hz) has been observed (Huys *et al.* 2008). To examine whether our results are (trivially) owing to movement frequency instead of accuracy constraints, we performed an experiment in which the same participants performed reciprocal back-and-forth movements at the same tempos but without targets. We found no evidence for fixed-point dynamics and the

achieved ID_e 's did not exceed approximately 5 (electronic supplementary material 3). The scaling of task difficulty thus truly underlies the here-observed transition in control mechanism and our results suggest that high precision can only be achieved by the discrete mechanism. We thus show that a gradual scaling of task difficulty—and the dynamics that lead to the transition (electronic supplementary material 1)—evokes an instantaneous (binary) transition between qualitatively distinct control mechanisms (Huys *et al.* 2008).

The qualitative nature of the distinction between discrete and rhythmic movements (i.e. one cannot be reduced to the other) is based on the notion of topological equivalence of structures in phase space that follows from theorems in the dynamical system theory (cf. Huys *et al.* 2008 and references therein). While this distinction does not necessitate distinct spatio-temporal (kinematic) properties of the corresponding movements, it is no surprise that distinct control mechanisms entail dissimilar movement characteristics as evidenced by the here observed transition (in AT). The latter, in fact, appears inconsistent with alternative contemporary approaches to (motor) trajectory formation. For instance, movement timing, as it was recently proposed, results from different geometries (Euclidian, equi-affine

and full affine) that are implemented in a task-dependent mixture, while movement segments are distinguished by more or less fixed combinations of geometries (Bennequin *et al.* 2009). The combined geometries approach appears quite successful in reconstructing rather complex movement trajectories, and may to some extent be complementary to the present one in that multiple geometries may, by hypothesis, play a role in the phase flow formation. However, no explicit argument based on first principles for the delineation of movement segments via mixtures of geometries can be offered by a geometrical composition, and it is in this sense that our presented approach is complementary.

In the present study, task difficulty was manipulated via target width but not target distance. One may wonder therefore whether our results can be generalized to target distance-induced changes in task difficulty. While both ingredients of task difficulty are known to affect movement time (cf. Mackenzie 1992; Meyer *et al.* 1988) and the movement dynamics (Mottet & Bootsma 1999) distinctly, the difference is moderate (cf. Mackenzie 1992; Mottet & Bootsma 1999) and quantitative only. In other words, our qualitative pattern of results is, in all likelihood, independent of how changes in task difficulty are brought about even though the exact location (in terms of index of difficulty) at which the transition occurs may vary slightly as a function thereof.

Precision aiming movements are often applied to directly manipulate the environment, as in reaching movements (cf. Zaal *et al.* 1999), as well as indirectly via particular tools (cf. MacKenzie 1992; Baird *et al.* 2002). Not surprisingly, the impact of tool use on motor performance depends largely on the tool characteristics; it being smaller the smaller the tool used, the impact vanishes in the limit (Baird *et al.* 2002; Bongers *et al.* 2003). It thus seems reasonable to suggest that our findings generalize for manual pointing and reaching movements as well as those involving many tools of daily usage (pens, needles, screwdrivers, etc.).

We documented that how speed was traded off against accuracy depended on the motor mechanism involved. When moving rhythmically the entire movement slowed down with increasing task difficulty; for discrete movements, this was mainly owing to the lengthening of the deceleration phase. Both control mechanisms apparently comply differentially with increasing task demands. As a result, AT was not a smooth continuous function of ID_e . Since MT is a linear addition of AT and DT, it follows that in reciprocal aiming Fitts' law is discontinuous (figure 1), owing to a change in the control mechanism. The reason that this discontinuity has remained unobserved is most likely owing to the strong focus on discrete movement tasks that has prevailed in the field of motor control since the start of information-processing theories. While the rhythmic regime cannot be implemented in a discrete movement task, it might however be implied in serial chaining tasks involving a reversal of movement direction (Adam *et al.* 2000).

It appears that evolution has endowed us with different functional 'modules' (control mechanisms), each with its own merits and limitations, that we can

implement to optimize motor performance under concurrent speed and accuracy constraints. As movement tempo increases, different motor control mechanisms are needed for simple and difficult spatio-temporal tasks.

The protocol was approved by the ethics committee of the University of the Mediterranean and was in agreement with the Declaration of Helsinki. Informed consent was obtained from all participants.

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