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Biomechanics of Fundamental Frequency Regulation:

Constitutive Modeling of the Vocal Fold Lamina Propria*

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Abstract

Accurate characterization of biomechanical characteristics of the vocal fold is critical for understanding the regulation of vocal fundamental frequency (F_0), which depends on the active control of the intrinsic laryngeal muscles as well as the passive biomechanical response of the vocal fold lamina propria. Specifically, the tissue stress-strain response and viscoelastic properties under cyclic tensile deformation are relevant, when the vocal folds are subjected to length and tension changes due to posturing. This paper describes a constitutive modeling approach quantifying the relationship between vocal fold stress and strain (or stretch), and establishes predictions of F_0 with the string model of phonation based on the constitutive parameters. Results indicated that transient and time-dependent changes in F_0 , including global declinations in declarative sentences, as well as local F_0 overshoots and undershoots, can be partially attributed to the time-dependent viscoplastic response of the vocal fold cover.

Keywords

Larynx; Vocal fold; Phonation; Fundamental frequency; Tissue mechanics; Viscoelasticity; Viscoplasticity

Introduction

In human phonation changes in vocal fold length and tension are directly linked to changes in vocal fundamental frequency (F_0) (1,2). For an understanding of the regulation of F_0 from a continuum mechanics perspective, it is necessary to describe the elastic and viscoelastic properties of the vocal fold, particularly those of the lamina propria, the major vibratory portion of the vocal fold. For a quantitative description of vocal fold tissue response under tensile deformation or longitudinal tension, one has to develop constitutive models that can reliably relate vocal fold tensile stress (force per unit area) to vocal fold tensile strain (ratio of change in length to original length) or stretch (fractional change in length, given by strain plus 1.0).

The most basic description of the biomechanical response of soft tissues like vocal fold tissues is linear elasticity. Linear tissue response has been assumed in, for example, studies of the

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linear viscoelastic properties of vocal fold tissues under small-strain shear deformation or small-amplitude oscillation (3,4). Nonetheless, there is evidence that vocal fold tissues are strongly nonlinear when subjected to large-strain deformation. Min et al. (5) introduced a nonlinear elastic model based on fitting stress-strain data to exponential functions, and extracted linearized Young's modulus of the vocal ligament for low and high strains. Chan et al. (6) applied a similar approach to correlate the tangent Young's moduli of the vocal fold cover and the vocal ligament to their relative densities of collagen and elastin. A more comprehensive approach is to substitute such nonlinear elastic descriptions of the stress-strain response by hyperelastic (large-strain) constitutive models (7,8). This allows the large-strain response to be more accurately captured, for the advancement of continuum models of phonation (9). In particular, our previous work has focused on Ogden's hyperelastic model for describing the vocal fold lamina propria (7,8,10).

Experimental studies have also provided evidence on the time-dependent nature of vocal fold tissue response, as shown in the observations of rate-dependent and hysteretic stress-strain response (6). As is the case with most biological tissues, a stabilized mechanical response is only achieved after a number of load cycles have been applied. Such behavior is the result of viscoplastic deformation, creep and relaxation phenomena, commonly referred to as the "preconditioning" response (11). This transient time-dependent tissue response has to be quantified for phonatory processes to be described as time functions, including the regulation of F_0 . In order to capture the time-dependent response of vocal fold tissues under small-strain conditions, various viscoelastic models have been proposed (3,4,12-14). However, no previous attempts have been made to develop constitutive models to characterize vocal fold tissues with respect to the transient large-strain response, such as cyclic stress relaxation and increase in specimen reference length through creep, as observed in other soft tissues (15-19). Such models need to be developed in combination with the tissue hyperelastic response. We have recently developed such a constitutive framework based on the formulation of a large-strain viscoplastic model, following the approach of Bergström and Boyce (20,21), including the representation of both a short-term and a long-term time dependence (10).

This paper represents an effort to further develop this large-strain viscoplastic constitutive characterization of the vocal fold lamina propria. We hypothesized that such a formulation would not only allow us to describe experimental data, but also to predict stress-strain response under loading conditions not accessible experimentally. To test this hypothesis, vocal fold cover specimens were dissected from the left and the right vocal folds of an excised human larynx, assumed to be mechanically symmetric (3). One specimen was subjected to cyclic tensile deformation with a simple constant-load, constant-frequency load history, and constitutive model parameters were determined from the data. The second specimen was subjected to a complex load history, including stress relaxation periods and changing amplitudes. Simulations were performed to predict the tissue response to this complex load history based on the constitutive model parameters extracted from the first specimen. Modeling results were then compared to the experimentally measured response of the second specimen.

From a functional biomechanical perspective it is necessary to correlate the tissue mechanical response to the phonatory response, in particular the F_0 response. It is hypothesized that the transient and time-dependent changes in F_0 , including short-term and long-term events, can be partially attributed to the time-dependent response of the lamina propria. The long-term tissue time-dependence could affect F_0 by gradually lowering the tensile stress, accounting for a significant portion of F_0 declination (22-25), whereas the short-term tissue time-dependence could capture local changes in F_0 , such as overshoots and undershoots (26). To test this hypothesis, relevant stretch histories characterizing vocal fold length changes during phonation were implemented in the simulations, with the stretch histories being hypothetical but

reasonable as they were based on previous empirical studies on vocal fold length changes (27-30).

Empirical measurements of vocal fold tissue response

As described in Chan et al. (6), vocal fold cover and ligament specimens were dissected from human larynges obtained from autopsy within 24 hours postmortem, with instruments for phonomicrosurgery. Subjects were non-smokers, had no history of head and neck disease, laryngeal pathologies, or trauma. Physical examination of the larynges by a laryngologist revealed no abnormalities. Before dissection, the *in situ* vocal fold length was measured with an accuracy of 0.01 mm. The tissue procurement protocol and the experimental protocol were approved by the Institutional Review Board of UT Southwestern Medical Center.

All specimens were kept in Krebs-Ringer solution buffered at a physiological pH of 7.4 and at a temperature of 37°C until testing. The mechanical response of each specimen was measured with the use of a dual-model servo-control lever system (Aurora Scientific Model 300B-LR, Aurora, ON, Canada). The system allowed for precise real-time measurements of the displacement and force of the lever arm, with a displacement accuracy of 1.0 µm and a force resolution of 0.3 mN, and a displacement range of up to 8-9 mm in the frequency range of 1-10 Hz (Figure 1). Displacement and force output of the lever system were digitized at a sampling rate of 1000 samples per second per channel with a 14-bit signal amplitude resolution (Windaq Model DI-722, DATAQ Instruments, Akron, OH). Specimen to be tested were mounted vertically (Fig. 1) to the lever arm through 3-0 sutures (Ethicon, Somerville, NJ) in Krebs-Ringer solution at pH 7.4 in an environmental chamber at 37°C. Slackness in the sutures was prevented and specimens were mounted at an initial length as close to its *in situ* length as possible. The *in situ* length was used as the reference length for calculation of stretch values. The passive uniaxial tensile response was measured with the servo-control lever system under displacement control. Displacement and tensile force of the specimen under cyclic stretch-release were measured. From the vocal fold *in situ* length (mounting length) L and the displacement Δl applied by the system, the uniaxial stretch (λ_u) of a specimen was given by

$$\lambda_u = 1 + \frac{\Delta l}{L}.$$

This paper reports the results of vocal fold cover specimens dissected from the left and the right vocal folds of an 85-year-old white male. The *in situ* length of both specimens was 18.07 mm. The mass of the left vocal fold cover specimen was 0.0540 g, and that of the right cover specimen was 0.0399 g. The right vocal fold cover specimen was loaded with 600 cycles of uniaxial stretch with a constant maximum stretch of 1.27 at 1 Hz. The left vocal fold cover specimen was subjected to a more complex stretch (or load) history, namely different levels of stretch (1.1, 1.2, 1.3, 1.35) for 300 cycles each, at 1 Hz. The specimen was rested for 2 minutes (120 seconds) before loading to the next higher level of stretch. These levels of stretch and frequency were chosen as they are typical of vocal fold length changes in speech (27-30).

Biomechanical modeling

Constitutive characterization of tissue mechanical response

A one-dimensional rheological representation of the three-dimensional constitutive model is considered (Figure 2). Three networks (A), (B) and (C) are in parallel with one another such that the applied deformation gradient $\mathbf{F}$ is identical to the deformation gradients $\mathbf{F}_i$ in the networks with the subscript i denoting network (A), (B) or (C), $\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$. Following this parallel arrangement of networks, the total Cauchy stress $\mathbf{T}$ is the sum of the Cauchy stresses in networks (A), (B) and (C), $\mathbf{T}_A$, $\mathbf{T}_B$, $\mathbf{T}_C$, $\mathbf{T} = \mathbf{T}_A + \mathbf{T}_B + \mathbf{T}_C$. The deformation gradient

of the time-dependent components (B) and (C) is decomposed multiplicatively into an elastic component $\mathbf{F}^e$ and a viscoplastic component $\mathbf{F}^p$:

$$\mathbf{F} = \mathbf{F}_B = \mathbf{F}_B^e \cdot \mathbf{F}_B^p = \mathbf{F}_C = \mathbf{F}_C^e \cdot \mathbf{F}_C^p \quad (1)$$

A one-dimensional form of Eq. (1) in the longitudinal direction of the vocal fold is:

$$\lambda_u = \lambda_B^e \lambda_B^p = \lambda_C^e \lambda_C^p \quad (2)$$

where λ_u is the applied stretch, and λ_i^e and λ_i^p are the elastic and viscoplastic stretch components, respectively, with the subscript i denoting network (B) or (C). The elastic response of all network components is described by the Ogden model, with a shear modulus μ and a power α describing the degree of nonlinearity of the elastic response (31). In this model it is assumed that the three networks (A), (B) and (C) have the same power $\alpha = \alpha_A = \alpha_B = \alpha_C$ in the hyperelastic response, but different values of the initial network shear moduli μ_A , μ_B and μ_C . These assumptions are based upon studies that fibrous proteins (collagen and elastin) and interstitial proteins (proteoglycans and glycoproteins) in the lamina propria contribute differentially to tissue viscoelasticity (6,32,33).

The first-order Ogden model is described by a strain energy density function w

$$w_i = \frac{2\mu_i}{\alpha^2} \left[\left(\lambda_{i,1}^e \right)^\alpha + \left(\lambda_{i,2}^e \right)^\alpha + \left(\lambda_{i,3}^e \right)^\alpha - 3 \right] \quad (i=A, B, C) \quad (3)$$

where $\lambda_{i,1}^e$ is the principal stretch of the elastic component of network (A), (B) or (C) in the longitudinal direction $\lambda_{i,1}^e = \lambda_i^e$ and $\lambda_{i,2}^e$ and $\lambda_{i,3}^e$ are the principal stretches in the transverse directions, which satisfy $\lambda_{i,1}^e \lambda_{i,2}^e \lambda_{i,3}^e = 1$. For uniaxial loading the principal stretches in the transverse directions are $\lambda_{i,2}^e = \lambda_{i,3}^e = 1 / \sqrt{\lambda_i^e}$. The nominal stress is obtained as the derivative of the strain energy density with respect to the principal stretch in the longitudinal direction λ_i^e :

$$\sigma_i = \frac{\partial w_i}{\partial \lambda_i^e} = \frac{2\mu_i}{\alpha} \left[\left(\lambda_i^e \right)^{\alpha-1} - \left(\lambda_i^e \right)^{-\frac{\alpha}{2}-1} \right] \quad (4)$$

The formulation of the inelastic rate of the shape change of the time-dependent components (B) and (C) follows Bergström and Boyce (20,21),

$$\begin{aligned} \dot{\mathbf{D}}_i^p &= \lambda_i^p \frac{\dot{\mathbf{S}}_i}{\bar{\sigma}_i} \\ &= \mathbf{F}_i^e \mathbf{F}_i^p (\mathbf{F}_i^p)^{-1} (\mathbf{F}_i^e)^{-1} \quad (i=B, C) \end{aligned} \quad (5)$$

where $\mathbf{S}_i$ is the deviator of the Cauchy stress tensor in network (B) or (C), and $\bar{\sigma}_i$ the corresponding effective stress in network (B) or (C), $\bar{\sigma}_i = \sqrt{\frac{3}{2} \mathbf{S}_i : \mathbf{S}_i}$. The effective inelastic 2 stretch rate $\dot{\lambda}_i^p$ depends on the effective inelastic stretch λ_i^p and the effective stress $\bar{\sigma}_i$ through

$$\dot{\lambda}_i^p = Z_i \left(\lambda_i^p - 1 + \delta \right)^{c_i} \left(\dot{\sigma}_i \right)^{m_i} \quad (6)$$

with the effective inelastic stretch λ_i^p calculated as $\lambda_i^p = \sqrt{\frac{1}{3} \mathbf{I} : \mathbf{C}_i^p}$ in which $\mathbf{C}_i^p = (\mathbf{F}_i^p)^T \mathbf{F}_i^p$. The 3 stress exponent m_i characterizes the dependence of the inelastic deformation on the stress level in network (B) or network (C), the stretch exponent c_i ($-1 \leq c_i \leq 0$) characterizes the dependence of the rate of inelastic deformation on the current magnitude of inelastic deformation, the viscosity scaling constant Z_i defines the absolute magnitude of the inelastic deformation, and δ is a small positive number ($\delta < 0.001$) introduced to avoid singularities in the inelastic stretch rate when the inelastic stretch is close to unity (21). The inelastic component of the deformation gradient in network (B) or (C) is obtained by inverting the relationships in Eq. (5) and then substituting into Eq. (6),

$$\dot{\lambda}_i^p = \frac{2}{3} \lambda_i^p \cdot Z_i \cdot \operatorname{sgn}(\lambda_i^e - 1) \cdot \left\{ \sqrt{\frac{1}{3} \left[\left(\lambda_i^p \right)^2 + \frac{2}{\lambda_i^p} \right]} - 1 + \delta \right\}^{c_i} \cdot \left| \frac{2\mu_i}{\alpha} \left[\left(\lambda_i^e \right)^\alpha - \left(\lambda_i^e \right)^{-\frac{\alpha}{2}} \right] \right|^{m_i} \quad (i=B, C) \quad (7)$$

The magnitude of stress is calculated from the elastic components of stretch λ_A^e , λ_B^e and λ_C^e , all of which depend on the applied stretch λ_u and the applied stretch rate $\dot{\lambda}_u$. For network (A) the elastic stretch rate is $\dot{\lambda}_A^e = \dot{\lambda}_u$ and the elastic stretch is $\lambda_A^e = \lambda_u$. In networks (B) and (C) the elastic stretch rate $\dot{\lambda}_B^e$ and $\dot{\lambda}_C^e$ can be expressed by substituting Eq. (2) into Eq. (7),

$$\dot{\lambda}_i^e = \dot{\lambda}_u \frac{\lambda_i^e}{\lambda_u} - \frac{2}{3} \lambda_i^e \cdot Z_i \cdot \operatorname{sgn}(\lambda_i^e - 1) \cdot \left\{ \sqrt{\frac{1}{3} \left[\left(\frac{\lambda_u}{\lambda_i^e} \right)^2 + \frac{2\lambda_i^e}{\lambda_u} \right]} - 1 + \delta \right\}^{c_i} \cdot \left| \frac{2\mu_i}{\alpha} \left[\left(\lambda_i^e \right)^\alpha - \left(\lambda_i^e \right)^{-\frac{\alpha}{2}} \right] \right|^{m_i} \quad (i=B, C) \quad (8)$$

During loading and the beginning of unloading, the applied stretch rate is prescribed by the lever arm as constant ($\dot{\lambda}_u = \text{constant}$), the elastic stretch component λ_B^e and λ_C^e can then be found through numerical integration of Eq. (8) using MATLAB 7.0 (The MathWorks, Natick, MA). With these formulations, the total nominal stress σ can be obtained from Eq. (4):

$$\sigma = \frac{2\mu_A}{\alpha} \left[\lambda_u^{\alpha-1} - \lambda_u^{-\frac{\alpha}{2}-1} \right] + \frac{2\mu_B}{\alpha} \left[\left(\lambda_B^e \right)^{\alpha-1} - \left(\lambda_B^e \right)^{-\frac{\alpha}{2}-1} \right] + \frac{2\mu_C}{\alpha} \left[\left(\lambda_C^e \right)^{\alpha-1} - \left(\lambda_C^e \right)^{-\frac{\alpha}{2}-1} \right] \quad (9)$$

Considering cyclic loading on tissue specimens mounted with sutures (Fig. 1), special boundary conditions apply if during loading length of the specimen increased permanently. Recall that the lever system was displacement (or stretch) controlled. Toward the end of unloading of each cycle the sutures attached to both ends of the tissue specimen would be slack due to permanent specimen deformation, with no tension applied on the specimen. During this special

“relaxation” period, the total stress σ remains zero and the overall stretch rate $\dot{\lambda}_u$ is unknown. The governing equation is that the time derivative of the stress [Eq. (9)] remains zero,

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{\partial \sigma}{\partial \lambda_u} \dot{\lambda}_u + \frac{\partial \sigma}{\partial \lambda_B^e} \dot{\lambda}_B^e + \frac{\partial \sigma}{\partial \lambda_C^e} \dot{\lambda}_C^e \\ &= \sum_{i=A,B,C} \frac{2\mu_i}{\alpha} \left[(\alpha-1) (\lambda_i^e)^{\alpha-2} + \left(\frac{\alpha}{2}+1\right) (\lambda_i^e)^{\frac{\alpha}{2}-2} \right] \cdot \dot{\lambda}_i^e = 0 \end{aligned} \quad (10)$$

in which $\lambda_A^e = \lambda_u$ is the overall stretch, λ_B^e and λ_C^e are the elastic stretch components of networks (B) and (C), respectively. From Eqs. (8) and (10), all the stretch components can be calculated numerically. During unloading, when the total stress σ vanishes, it serves as an indicator to replace the condition $\dot{\lambda}_u = \text{constant}$ with Eq. (10) in the numerical integration process; during the relaxation period, when the lever arm displacement catches up with the real stretch of the tissue specimen, the numerical integration process will be switched back into Eq. (8) together with the constant applied stretch rate condition.

The optimization procedure to determine the constitutive parameter values was based on Bergström and Boyce (21), as described in Zhang et al. (10). Briefly, the first stress-stretch cycle of the right vocal fold cover specimen was used to derive the parameters of the equilibrium network (A) and the short-term viscoplastic network (B), whereas the data on transient effects, i.e., peak stress decay (stress relaxation) and increase in specimen length (creep) over the cycles were used to derive parameters for the long-term viscoplastic network (C).

Fundamental frequency regulation

The investigation of vocal F_0 begins with the string model of phonation (1,2). The partial differential equation governing the vibration of a flexible string with two fixed ends is (34):

$$T \frac{\partial^2 v}{\partial x^2} = \rho A \frac{\partial^2 v}{\partial t^2} \quad (11)$$

where ρ is density, T is tensile force applied to the composite string, A is the current total cross-section area of the string, and v , the displacement from the equilibrium position (in the medial-lateral direction) of an infinitesimal part of the string with a distance x (anterior-posterior) from a string endpoint, is a function of x and time t , $v = v(x, t)$. Equation (11) can be written as:

$$\sigma_{\text{cauchy}} \frac{\partial^2 v}{\partial x^2} = \rho \frac{\partial^2 v}{\partial t^2} \quad (12)$$

where $\bar{\sigma}_{\text{cauchy}}$ is the longitudinal Cauchy stress (or true stress), $\sigma_{\text{cauchy}} = T/A$. The current cross-section area of the cover A is related to the initial cross-section area A_0 through $A = A_0/\lambda_u$. Then, the Cauchy stress σ_{cauchy} in Eq. (12) is related to the nominal stress, as used in the constitutive characterization, and the longitudinal stretch λ_u as:

$$\sigma_{\text{cauchy}} = \lambda_u \sigma \quad (13)$$

The lowest allowed frequency for the string model, i.e. the fundamental frequency F_0^{string} , can be expressed as a function of the current vocal fold length $l = L\lambda_u$, the tissue density ρ , and the tissue longitudinal Cauchy stress σ_{cauchy} :

$$\begin{aligned}
 F_0^{\text{string}} &= \frac{1}{2l} \sqrt{\frac{\sigma_{\text{cauchy}}}{\rho}} \\
 &= \frac{1}{2L\lambda_u} \sqrt{\frac{\sigma_{\text{cauchy}}}{\rho}}
 \end{aligned}
 \tag{14}$$

F_0 is thus dependent on the magnitude of deformation applied as a result of vocal fold length changes due to posturing. Predictions of F_0 can be obtained from Eq. (14) with use of the constitutive model once the model parameters are determined. It should be noted that Eq. (14) was developed by the use of Cauchy stress (true stress) instead of nominal stress, a factor not considered in previous work (1,7,35).

Results and Discussion

Empirical tissue mechanical response and simulations

The investigation began with the constitutive characterization of the empirical tissue mechanical response of the right vocal fold cover specimen. Tensile stress and stretch data were quantified under cyclic deformation at a constant-amplitude stretch at 1 Hz for 600 cycles. The stress-stretch curves obtained are shown in Figure 3 for the first cycle ($N = 1$) and the 600th cycle ($N = 600$). Considering the data for $N = 1$ [Fig. 3(a)], the tissue specimen exhibited a nonlinear elastic and hysteretic response. Next, when considering the mechanical response for $N = 600$ [Fig. 3(b)], the specimen exhibited a time dependence in the stress level such that the peak stress reached during a load cycle decayed as cyclic loading progressed. Furthermore, a permanent (residual) stretch of about 1.05 can be seen to have developed. Details of the time dependence of the peak stress from cycle to cycle are shown in Fig. 3(c). It can be seen that the vocal fold cover did not quite reach a stable mechanical response even after a significant number of load cycles (600 cycles). Hence, no truly “preconditioned” tissue response can be achieved.

Based on the stress-stretch data, the constitutive model parameters characterizing the tissue specimen were obtained. A total of ten parameters were extracted from the data, including the shear moduli $\mu_A = 47.7$ [kPa], $\mu_A = 800$ [kPa], $\mu_C = 45.2$ [kPa]; the power of the Ogden model characterizing the degree of nonlinearity $\alpha = 15.0$; and the viscoplastic parameters $m_B = 1.1$, $m_C = 1.2$, $c_B = 0.0$, $c_C = 1.0$, $Z_B = 3 \cdot 10^{-5}$ [$\text{s}^{-1} (\text{kPa})^{-m_B}$], $Z_C = 3 \cdot 10^{-4}$ [$\text{s}^{-1} (\text{kPa})^{-m_C}$]. Once these constitutive parameters were determined, the capability of the model to simulate the response of the right vocal fold cover was verified. Figure 3 shows the comparisons between the empirical data and the simulations for cycles $N = 1$ and $N = 600$, with Fig. 3(c) depicting the development of the peak stress in dependence of the number of applied stretch cycles. Overall, a very good agreement between the model and the experimental data was obtained.

To be able to assess the model for its predictive capabilities, a comparison of data and simulations of the right vocal fold cover was not sufficient, since this was only a curve-fitting procedure. To investigate our hypothesis that the model could predict stress-strain response under other loading conditions, we predicted the tissue response of the left vocal fold cover under a more complex stretch history. Figure 4(a) shows the empirical response of the left vocal fold cover for the first 600 load cycles (i.e., 300 load cycles at a stretch of 1.1 at 1 Hz, followed by a rest period of 120 s, and subsequently 300 load cycles at a stretch of 1.2 at 1 Hz). As stretch levels varied from a load block to the next the strain rate was constant only in a given load block, and would always differ from that applied on the right vocal fold cover specimen. Results showed that the tissue response was nonlinear elastic, hysteretic, and time-dependent. Stresses in the second load block ($N = 301$ to 600) were lower than those in the

first load block ($N = 1$ to 300), and in each load block the peak stress gradually decayed, with a residual stretch. Figure 4(b) shows the predicted response based on constitutive parameters extracted from the right vocal fold cover. A comparison of Fig. 4(a) and Fig. 4(b) shows that the constitutive model was reasonably capable of predicting the tissue response of the left vocal fold cover, at least for the first 300 cycles. The predictions became less accurate as the number of load cycles increased, although the overall trend remained well predicted. Figure 4(c) provides a comparison of the predicted and the measured peak stresses for all four load blocks and the inter-block rest periods. Again, it is clear that the model could simulate the tissue response very well up to around 600 cycles, but the accuracy of the predictions deteriorated as the number of load cycles increased.

Simulation of fundamental frequency changes in an utterance

To examine the capability of the biomechanical model (Eq. 14) for predicting fundamental frequency changes in phonation, a stretch history typical of vocal fold length changes in speech (27-30) was prescribed as the active driving parameter for the model, with an initial stretch of $\lambda_u = 1.32$, and subsequent cycles of stretch varying between short periods of stretch held at $\lambda_u = 1.32$ and $\lambda_u = 1.28$, i.e., a square wave of input stretch. Figure 5 shows the fundamental frequency contours predicted by the biomechanical model under the stretch history prescribed, normalized to the maximum value of F_0 reached during the initial phase of stretch.

Fig. 5(a) focuses on the global changes in F_0 in the simulated utterance. The predicted F_0 contours showed that with the three-network constitutive model in combination with the string model, there was indeed a gradual decrease of the predicted peak values of F_0 , reflecting the transient mechanical response of cyclic stress relaxation of the vocal fold cover. The percentage of peak F_0 decay was calculated to be on the order of 20%. Under a constant mean value of applied stretch, the decay percentage of the predicted peak F_0 was lower compared to the actual decay percentage (about 25%) observed in F_0 declinations in declarative utterances (25, 36). This finding implied that such magnitudes of decay of the peak F_0 could be achieved if the stretch history was modified to include a continuous decay in the mean value of applied stretch, with a stronger decay in stretch resulting in a more significant decay in F_0 . This appeared to indicate that the viscoplastic (or viscous) response of the vocal fold lamina propria can only contribute partially to the F_0 decay observed in declarative utterances, consistent with the fact that active neuromuscular control of the vocal fold plays a significant role in global F_0 changes.

Fig. 5(b) shows the short-term time response, or local changes in F_0 in the simulated utterance. Phenomena such as F_0 overshoots, undershoots, and preparations were predicted. Overshoots occurred at the instances of step increases in stretch, where F_0 quickly increased and subsequently decayed slowly. Undershoots occurred at the instances of step decreases in stretch, where F_0 quickly decreased and then steadily increased. Both stretch increases and decreases also led to instantaneous changes in F_0 at the onset of the step changes in stretch. Such variations were of only very short durations (on the order of milliseconds) and were associated with reversals in the direction of stretch. These findings were in good qualitative agreement with experimental data on local changes in F_0 (26). It can be seen from Fig. 5(b) that overshoots, undershoots and preparations of F_0 (i.e., local fluctuations of F_0) can be characterized by our model corresponding to the step activations of stretch followed by hold periods in the stretch histories. The present model predicted that the F_0 response was directly related to the short-term viscoplastic tissue response. These results suggested that when the speaker or singer lengthens or shortens the vocal folds, and then keeping them at a constant length for even a short period of time (in milliseconds), the tensile stress in the vocal fold cover would relax, causing the F_0 to drop or rise depending on whether the vocal folds are being elongated or shortened. Reversals of stretch direction would lead to additional variations in F_0 that are equally short in durations.

The regulation of vocal fundamental frequency obviously depends on both the passive biomechanical response of the vocal fold lamina propria as described, as well as the active neurologic control of the intrinsic laryngeal muscles, particularly the cricothyroid (CT) and the thyroarytenoid (TA) muscles, which was not addressed in the present study. Future models should incorporate the contributions of the CT and TA muscles to the regulation of vocal fold stretch, stress, and stiffness. The present findings should also be corroborated by further studies involving more specimens.

Conclusion

The results of this study showed that the large-strain three-network constitutive model formulated was capable of the description of stress-strain and viscoplastic response of the vocal fold cover, as well as reasonably accurate prediction of such tissue mechanical response up to about 600 cycles of cyclic tensile deformation. Through a biomechanical model developed by combining the constitutive model of vocal fold tissue response with the string model of phonation, this study also demonstrated that transient and time-dependent changes in fundamental frequency in sentence production, including both global and local changes, can be partially attributed to the time-dependent transient mechanical response of the lamina propria.

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Figure 1.

(a) Schematic of the experimental setup for uniaxial cyclic tensile deformation of vocal fold tissues with a dual-mode servo-control lever system [after Chan et al. (6)]; (b) A photograph of a vocal ligament specimen mounted in the lever system with suture attachment through dissected cartilage sections. The *in situ* length (reference length) of the specimen (L) is shown.

Figure 2.

A one-dimensional rheological representation of the three-network constitutive model, with a hyperelastic equilibrium network (A) in parallel with a short-term viscoplastic network (B) and a long-term viscoplastic network (C) [after Zhang et al. (10)].

Figure 3.

Experimental data and corresponding simulation results: (a) first cycle of the right vocal fold cover specimen, (b) 600th cycle of the right vocal fold cover specimen, (c) peak stress as a function of the number of loading cycles.

Figure 4.

(a) Experimental response of the left vocal fold cover specimen (cycles 1, 300, rest, 301, 600); (b) Simulated response of the left vocal fold cover based on constitutive parameters derived from the right vocal fold cover (cycles 1, 300, rest, 301, 600); (c) Peak stress as a function of the loading cycles comparing the experimental data and the model predictions.

Figure 5.

(a) Time history of normalized fundamental frequency predicted by the biomechanical model (Eq. 14) driven by an input stretch history simulating an utterance; (b) Magnified view of part (a) in the time interval of 2.0-3.0 s. Stretch history input to the model: uniaxial stretch from 0 to $\lambda_u = 1.32$, followed by cyclic step changes between $\lambda_u = 1.32$ and $\lambda_u = 1.28$.