Global robust power-rate stability of delayed genetic regulatory networks with noise perturbations

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Abstract In this paper, by using the Lyapunov method, Itô's differential formula and linear matrix inequality (LMI) approach, the global robust power-rate stability in mean square is discussed for genetic regulatory networks with unbounded time-varying delay, noise perturbations and parameter uncertainties. Sufficient conditions are given to ensure the robust power-rate stability (in mean square) of the genetic regulatory networks. Meanwhile, the criteria ensuring global power-rate stability in mean square are a byproduct of the criteria guaranteeing global robust power-rate stability in mean square. The obtained conditions are derived in terms of linear matrix inequalities (LMIs) which are easy to be verified via the LMI toolbox. An illustrative example is given to show the effectiveness of the obtained result.

Keywords Global robust power-rate stability in mean square · Global power-rate stability in mean square · Genetic regulatory networks · Stochastic perturbation · Unbounded time-varying delay · Linear matrix inequality (LMI)

Introduction

Nowadays, With the fast development of cognitive science, it is current to study cognitive science with the viewpoint of neurodynamics. In Wang and Zhang (2006), the authors showed that principle of the energy coding can reveal mechanism of brain information processing in physical nature, which make it is possible to indicate a new road from the point of view of globe of brain information processing for comprehending role of information coding of neural-network system of brain. And the analytic results in Wang et al. (2009b) is beneficial to the study of the important scientific problem as how the brain performs coding at the level of locla neural networks. And some conclusions about robust stability, and so on have been obtained for cognitive neurodynamics, we refer to Wang et al. (2009a), Liu et al. (2009) and the references cited therein. Meanwhile, with the development of genome sequencing and gene recognition, the study for the dynamics of gene networks has some contributions to the development of cognitive neurodynamics. Genetic regulatory networks have become an important new area of research in the biological and biomedical sciences and attracted great attention over past few years (Gardner et al. 2000; Elowitz and Leibler 2000; Austin et al. 2006; McAdams and Shapiro 1995; Becskei and Serrano 2000; Monk 2003; Xu and Tao 2006). Nowadays, one of the main challenges in systems biology is to understand the genetic regulatory networks, such as, how genes and proteins interact to form a complex network that performs complicated biological functions. Because genetic networks are biochemically dynamical systems, mathematical modelling of genetic networks as dynamical system models can be viewed as a powerful tool for studying gene regulation processes in living organisms. Mathematical models of genetic networks in literature can be roughly classified into two types, i.e., the

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Boolean model and the differential equation model (De Jong 2002; Smolen et al. 2000; Bolouri and Davidson 2002).

Gene regulation is an intrinsically noisy process, which is subject to intracellular and extracellular noise perturbations and environment fluctuations (Elowitz 2002; Paulsson 2004; Paulsson and Ehrenberg 2001; Raser and OShea 2005; Chen et al. 2005; Li et al. 2006a, b, 2007; Wang et al. 2007). Such noises will undoubtedly affect the dynamics of the gene networks. In Li et al. (2006b), based on Lyapunov method and the Lur's system approach, the asymptotical stability in mean square is studied for genetic networks with SUM regulatory logic. In Li et al. (2007), the authors studied the stability of the delayed genetic networks with non-vanishing perturbation term.

Besides, time delays are also unavoidable in genetic regulatory networks particularly owing to the slow processes of transcription, translation, and translocation (Wang et al. 2009a; Monk 2003; Smolen et al. 2002; Bratsun et al. 2005; Chen and Aihara 2002; He and Cao 2008; Yu et al. 2009). Such time delays may make the dynamics in the genetic networks be more complicated. About the study of delayed genetic regulatory networks, we refer to Monk (2003), Li et al. (2006b), (2007), Wang et al. (2007), Smolen et al. (2002), Bratsun et al. (2005), Chen and Aihara (2002), Ren and Cao (2008), Cao and Ren (2008) and the references cited therein. Also, there are often some unavoidable uncertainties in modelling networks due to modelling errors and parameter fluctuations. The uncertainties can cause the network to be unstable and perform poorly. Hence, the study of the robust stability for genetic regulatory networks with parameter uncertainties has attracted some author's interest (Wang et al. 2007; Ren and Cao 2008). In Wang et al. (2007), a robust variance-constrained filtering problem is investigated for a gene expression model with stochastic disturbances and norm-bounded parameter uncertainties, where the stochastic perturbation is in the form of a scalar Gaussian white noise with constant variance and the parameter uncertainties enter both the system matrix and the output matrix. In Ren and Cao (2008), based on the Lyapunov stability theory and linear matrix inequality (LMI), the authors analyzed the robust asymptotical stability issues of the genetic regulatory networks with time-varying delays and norm bounded uncertainties.

However, it should be noted that most authors took account of bounded delays or assumed that time-varying delays are differentiable and their derivatives are with a upper boundedness which is less than 1 when they studied the dynamic behaviors of networks (Monk 2003; Li et al. 2006b, 2007; Wang et al. 2007; Smolen et al. 2002; Bratsun et al. 2005; Chen and Aihara 2002; Ren and Cao 2008; Cao and Ren 2008). But, if the delays are unbounded, what will happen? It's an interesting issue. To the best of our knowledge, neural networks with unbounded time-varying delays are seldom considered (Zeng et al. 2005; Chen and Wang

2007). In Zeng et al. (2005), the global asymptotic stability and the global exponential stability are discussed for neural networks with unbounded time-varying delays and with bounded and Lipschitz continuous activation functions. In Chen and Wang (2007), the authors considered global power-rate stability which is firstly proposed for dynamical systems with unbounded time-varying delays.

In the applications and designs of networks, the faster the convergence rate of the networks is, the higher the cost is. It is well-known that the asymptotic convergence rate is slower than that of exponential convergence. But, in order to reduce the cost and keep fast convergence rate, the power-rate stability, the convergence rate of which is between asymptotic convergence rate and exponential convergence rate, can be considered.

This paper aims to investigate the global robust power-rate stability (in mean square) for genetic regulatory networks with unbounded time-varying delay, noise perturbations and parameter uncertainties by using the Lyapunov method, Itô's differential formula and linear matrix inequality (LMI) technique which is used more frequently for its easily being verified. Novel criterion is derived to guarantee the global robust power stability (in mean square) of the considered model. Here, the time-varying delay is assumed to be unbounded and less than or equal to μt with $0 < \mu < 1$. In addition, as a byproduct, the global power stability in mean square is also considered for given model.

The rest of the paper is organized as follows. In Sect. 2, model description and preliminaries are presented. The main results are stated in Sect. 3. In Sect. 4, an example is given to show the validity of the obtained result. Finally, in Sect. 5, the conclusions are drawn.

Notations Throughout the paper, A^T denotes the transpose of square matrix A . For real symmetric matrices X and Y , the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, positive semi-definite). I denotes the identity matrix with compatible dimensions. $\lambda_{\max}(A)$, $\lambda_{\min}(A)$ are used as the maximum eigenvalue and the minimum eigenvalue of matrix A , respectively. The notation “*” is used as an ellipsis for terms that are induced by symmetry.

Model description and preliminaries

Genetic regulatory networks without noise perturbations are expressed in the following (Ren and Cao 2008):

$$\begin{cases} \frac{dM_i(t)}{dt} = -a_i M_i(t) + \sum_{j=1}^n G_{ij}(P_j(t - \epsilon(t))), \\ \frac{dP_i(t)}{dt} = -c_i P_i(t) + d_i M_i(t - \tau(t)) \quad i = 1, 2, \dots, n, \end{cases} \quad (1)$$

where $M_i(t)$ and $P_i(t)$ are the concentrations of messenger RNA and protein of the i th node, respectively; a_i and c_i are

the decay rates of messenger RNA and protein, respectively; d_i is the translation rate; $\epsilon(t)$, $\tau(t)$ are time-varying delays. The function G_{ij} denotes the feedback regulation of the j th protein on the transcription, which is a monotonic function of the Hill form (Li et al. 2006c): if transcription factor j is an activator of gene i , then

$$G_{ij}(P_j(t)) = b_{ij} \frac{(P_j(t)/\beta_j)^{H_j}}{1 + (P_j(t)/\beta_j)^{H_j}},$$

if transcription factor j is a repressor of gene i , then

$$\begin{aligned} G_{ij}(P_j(t)) &= b_{ij} \frac{1}{1 + (P_j(t)/\beta_j)^{H_j}} \\ &= b_{ij} \left(1 - \frac{(P_j(t)/\beta_j)^{H_j}}{1 + (P_j(t)/\beta_j)^{H_j}} \right), \end{aligned}$$

where H_j is the Hill coefficient, β_j is positive constant, b_{ij} is a bounded constant, which is the dimensionless transcriptional rate of transcription factor j to i .

Hence, model (1) can be rewritten as follows:

$$\begin{cases} \frac{dM_i(t)}{dt} = -a_i M_i(t) + \sum_{j=1}^n w_{ij} f_j(P_j(t - \epsilon(t))) + B_i, \\ \frac{dP_i(t)}{dt} = -c_i P_i(t) + d_i M_i(t - \tau(t)) \quad i = 1, 2, \dots, n, \end{cases} \quad (2)$$

where $f_j(x) = \frac{(x/\beta_j)^{H_j}}{1 + (x/\beta_j)^{H_j}}$, $B_i = \sum_{j \in I_i} b_{ij}$ and I_i is the set of all the j which is a repressor of gene i , $W = (w_{ij})_{n \times n}$ and w_{ij} is defined as follows:

$$w_{ij} = \begin{cases} b_{ij}, & \text{if transcription factor } j \text{ is an activator of gene } i, \\ 0, & \text{if there is no link from node } j \text{ to } i, \\ -b_{ij}, & \text{if transcription factor } j \text{ is a repressor of gene } i. \end{cases}$$

The compact form of model (2) is expressed as follows:

$$\begin{cases} \frac{dM(t)}{dt} = -AM(t) + Wf(P(t - \epsilon(t))) + B, \\ \frac{dP(t)}{dt} = -CP(t) + DM(t - \tau(t)), \end{cases} \quad (3)$$

where $M(t) = (M_1(t), \dots, M_n(t))^T$, $P(t) = (P_1(t), \dots, P_n(t))^T$, $f(P(t - \epsilon(t))) = (f_1(P_1(t - \epsilon(t))), \dots, f_n(P_n(t - \epsilon(t))))^T$, $A = \text{diag}(a_1, a_2, \dots, a_n)$, $C = \text{diag}(c_1, c_2, \dots, c_n)$, $D = \text{diag}(d_1, d_2, \dots, d_n)$, $B = (B_1, \dots, B_n)^T$.

From the definition of f_j , one can obtain that f_j is a monotonically increasing function with saturation, and it satisfies

$$0 \leq \frac{f_j(x) - f_j(y)}{x - y} \leq k_j, \quad (4)$$

for all $x, y \in \mathbb{R}$ with $x \neq y$. Let $(M^{*T}, P^{*T})^T$ be an equilibrium point of model (3), using the following transformation, we shift it to the origin:

$$m(t) = M(t) - M^*, \quad p(t) = P(t) - P^*,$$

and the model (3) can be rewritten as follows:

$$\begin{cases} \frac{dm(t)}{dt} = -Am(t) + Wg(p(t - \epsilon(t))), \\ \frac{dp(t)}{dt} = -Cp(t) + Dm(t - \tau(t)), \end{cases} \quad (5)$$

where $m(t) = (m_1(t), m_2(t), \dots, m_n(t))^T$, $p(t) = (p_1(t), p_2(t), \dots, p_n(t))^T$, $g(p(t)) = (g_1(p_1(t)), g_2(p_2(t)), \dots, g_n(p_n(t)))^T$ with $g_i(p_i(t)) = f_i(P_i(t)) - f_i(P_i^*)$.

From (4), one can have that the function g_i satisfies the sector condition:

$$0 \leq \frac{g_i(x)}{x} \leq k_i, \quad (6)$$

for all $x \in \mathbb{R}$ with $x \neq 0$.

In practice, parameter uncertainties exist during electronic implementation of neural networks, and can lead to deviations and perturbations. So, in this paper, we consider above genetic regulatory networks with unbounded time-varying delays, noise perturbations and parameter uncertainties, the model is expressed as follows:

$$\begin{cases} dm(t) = [-(A + \Delta A(t))m(t) + (W + \Delta W(t))g(p(t - \tau(t)))]dt + \sigma(p(t), p(t - \tau(t)))d\omega(t), \\ dp(t) = [-(C + \Delta C(t))p(t) + (D + \Delta D(t))m(t - \tau(t))]dt, \end{cases} \quad (7)$$

where $\sigma(p(t), p(t - \tau(t))) \in \mathbb{R}^{n \times n}$ is called the noise intensity matrix, $\omega(t)$ is an n -dimensional Wiener process (Arnold 1974) with $E\{d\omega(t)\} = 0$, $E\{[d\omega(t)]^2\} = dt$; $\tau(t)$ is unbounded time-varying delay and satisfies $0 \leq \tau(t) \leq \mu t$ with $0 < \mu < 1$; $A + \Delta A(t) = \text{diag}(a_i + \Delta a_i(t))$, $C + \Delta C(t) = \text{diag}(c_j + \Delta c_j(t))$, $D + \Delta D(t) = \text{diag}(d_j + \Delta d_j(t))$, $W + \Delta W(t) = (w_{ij} + \Delta w_{ij}(t))_{n \times n}$.

The initial conditions are given by

$$\begin{aligned} m_i(s) &= \phi_i(s), p_i(s) = \psi_i(s), s \in (-\infty, 0], \\ i &= 1, 2, \dots, n, \end{aligned}$$

where $\phi_i(s), \psi_i(s), i = 1, 2, \dots, n$, are bounded and continuous on $(-\infty, 0]$.

The parametric uncertainties $\Delta A(t), \Delta C(t), \Delta D(t), \Delta W(t)$ are time-varying matrices which have the following form:

$$\begin{aligned} \Delta A(t) &= HF(t)E_1, \Delta W(t) = HF(t)E_2, \Delta C(t) \\ &= H^*F^*(t)E_1^*, \Delta D(t) = H^*F^*(t)E_2^*, \end{aligned} \quad (8)$$

where $H, H^*, E_1, E_2, E_1^*, E_2^*$ are known real constant matrices with appropriate dimensions, and $F(t), F^*(t)$ are unknown real time-varying matrices satisfying

$$F(t)^T F(t) \leq I, \quad F^{*T}(t) F^*(t) \leq I. \quad (9)$$

As in many stochastic system studies (Li et al. 2006b, 2007), we make the following assumption: (A1):

$$\begin{aligned} & \text{trace}[\sigma(p(t), p(t - \tau(t))) \sigma^T(p(t), p(t - \tau(t)))] \\ & \leq p^T(t) L_1 p(t) + p^T(t - \tau(t)) L_2 p(t - \tau(t)), \end{aligned}$$

where $L_1 \geq 0, L_2 \geq 0$.

Definition 1 The genetic regulatory network model (7) with $\Delta A(t) = \Delta W(t) = \Delta C(t) = \Delta D(t) = 0$ is said to be globally power-rate stable in mean square, if there exist scalars $\alpha > 0$ and $\gamma > 0$ such that

$$E \| (m^T(t), p^T(t))^T \|^2 \leq \alpha t^{-\gamma}.$$

Here, γ is power convergence rate.

Definition 2 The genetic regulatory network model (7) is said to be globally robust power-rate stable in mean square, if the equilibrium point of the model is globally power-rate stable in mean square in the presence of the parameters uncertainties satisfying (8) and (9).

holes for all F satisfying $F^T F \leq I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \varepsilon H H^T + \varepsilon^{-1} E^T E < 0.$$

Lemma 3 (Schur Complement) Boyd et al. (1994) The linear matrix inequality (LMI)

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix} > 0$$

is equivalent to the following condition:

$$S_{22} > 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^T > 0,$$

where $S_{11}^T = S_{11}, S_{22}^T = S_{22}$.

Main results

In this section, we analyze the global robust power-rate stability in mean square of model (7) by using appropriate Lyapunov functional.

Theorem 1 If there exist two symmetric positive definite matrices Q_1, Q_2 and five positive scalar $\rho > 0, \varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_1^* > 0, \varepsilon_2^* > 0$ such that

$$\begin{pmatrix} Q_1 A + A^T Q_1 - Q_1 & -Q_1 W & Q_1 H & -\varepsilon_1 E_1^T & -Q_1 H & -\varepsilon_2 E_2^T \\ * & \frac{Q_2 - \rho L_2}{k^2} & 0 & 0 & 0 & 0 \\ * & * & \varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & \varepsilon_1 I & 0 & 0 \\ * & * & * & * & \varepsilon_2 I & 0 \\ * & * & * & * & * & \varepsilon_2 I \end{pmatrix} > 0, \quad (10)$$

$$\begin{pmatrix} Q_2 C + C^T Q_2 - Q_2 - \rho L_1 & -Q_2 D & Q_2 H^* & -\varepsilon_1^* E_1^{*T} & -Q_2 H^* & -\varepsilon_2^* E_2^{*T} \\ * & Q_1 & 0 & 0 & 0 & 0 \\ * & * & \varepsilon_1^* I & 0 & 0 & 0 \\ * & * & * & \varepsilon_1^* I & 0 & 0 \\ * & * & * & * & \varepsilon_2^* I & 0 \\ * & * & * & * & * & \varepsilon_2^* I \end{pmatrix} > 0, \quad (11)$$

Lemma 1 Liao et al. (2002) Given any real matrices $\Sigma_1, \Sigma_2, \Sigma_3$ of appropriate dimensions and a scalar $\lambda > 0$ such that $0 < \Sigma_3 = \Sigma_3^T$. Then, the following inequality holds:

$$\Sigma_1^T \Sigma_2 + \Sigma_2^T \Sigma_1 \leq \lambda \Sigma_1^T \Sigma_3 \Sigma_1 + \lambda^{-1} \Sigma_2^T \Sigma_3^{-1} \Sigma_2.$$

Lemma 2 Zou and Wang (2007) Given matrices $Q = Q^T, H, E$ of appropriate dimensions, the matrix inequality

$$Q + H F E + E^T F^T H^T < 0$$

and

$$Q_1 \leq \rho I, \quad Q_2 > \rho L_2,$$

where $k = \max\{k_1, k_2, \dots, k_n\}$, then, the model (7) is globally robust power-rate stable in mean square.

Proof If the LMI (10) and (11) hold, we can find two sufficient small constants $\beta > 0$ and $\gamma > 0$, such that

$$\begin{pmatrix} Q_1 A + A^T Q_1 - \beta Q_1 - (1 - \mu)^{-\gamma} Q_1 & -Q_1 W & Q_1 H & -\varepsilon_1 E_1^T & -Q_1 H & -\varepsilon_2 E_2^T \\ * & \frac{Q_2 - \rho L_2}{k^2} & 0 & 0 & 0 & 0 \\ * & * & \varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & \varepsilon_1 I & 0 & 0 \\ * & * & * & * & \varepsilon_2 I & 0 \\ * & * & * & * & * & \varepsilon_2 I \end{pmatrix} > 0, \quad (12)$$

$$\begin{pmatrix} Q_2 C + C^T Q_2 - \beta Q_2 - (1 - \mu)^{-\gamma} Q_2 - \rho L_1 & -Q_2 D & Q_2 H^* & -\varepsilon_1^* E_1^{*T} & -Q_2 H^* & -\varepsilon_2^* E_2^{*T} \\ * & Q_1 & 0 & 0 & 0 & 0 \\ * & * & \varepsilon_1^* I & 0 & 0 & 0 \\ * & * & * & \varepsilon_1^* I & 0 & 0 \\ * & * & * & * & \varepsilon_2^* I & 0 \\ * & * & * & * & * & \varepsilon_2^* I \end{pmatrix} > 0 \quad (13)$$

Furthermore, there exists a sufficiently large T , such that for all $t > T$,

$$\frac{\gamma}{t} \leq \beta. \quad (14)$$

In the following, we always assume $t > T$.

Consider the following Lyapunov function:

$$V(t, m(t), p(t)) = t^\gamma m^T(t) Q_1 m(t) + t^\gamma p^T(t) Q_2 p(t). \quad (15)$$

Denote $\alpha(t) = \sup_{s \leq t} E(V(s, m(s), p(s)))$, then, $\alpha(t)$ is nondecreasing and $E(V(t, m(t), p(t))) \leq \alpha(t)$.

In the following, we will prove that for all $t \geq T$, $\alpha(t) = \alpha(T)$.

For any $t_0 > T$, if $E(V(t_0, m(t_0), p(t_0))) < \alpha(t_0)$, then $\alpha(t)$ is non-increasing at t_0 . Now, we consider the case that $E(V(t_0, m(t_0), p(t_0))) = \alpha(t_0)$. By Itô's formula (Li et al. 2006b), we obtain the following stochastic differential:

$$\begin{aligned} dV(t_0, m(t_0), p(t_0)) &= \mathcal{L}V(t_0, m(t_0), p(t_0)) dt_0 \\ &\quad + 2t_0^\gamma m^T(t_0) Q_1 \sigma(p(t_0)) \\ &\quad \times p(t_0 - \tau(t_0)) d\omega(t_0), \end{aligned} \quad (16)$$

where $\mathcal{L}$ is the diffusion operation, and

$$\begin{aligned} \mathcal{L}V(t_0, m(t_0), p(t_0)) &= \gamma t_0^{\gamma-1} (m^T(t_0) Q_1 m(t_0) + p^T(t_0) Q_2 p(t_0)) \\ &\quad + t_0^\gamma [2m^T(t_0) Q_1 (-A + \Delta A(t)) m(t_0) \\ &\quad + (W + \Delta W(t)) g(p(t_0 - \tau(t_0))) \\ &\quad + 2p^T(t_0) Q_2 (-C + \Delta C(t)) p(t_0) \\ &\quad + (D + \Delta D(t)) m(t_0 - \tau(t_0)) \\ &\quad + \text{trace}(\sigma(p(t_0), p(t_0 - \tau(t_0))) \\ &\quad \times Q_1 \sigma^T(p(t_0), p(t_0 - \tau(t_0)))]. \end{aligned} \quad (17)$$

From Lemma 1, one can obtain

$$\begin{aligned} &2m^T(t_0) Q_1 (W + \Delta W(t)) g(p(t_0 - \tau(t_0))) \\ &\leq m^T(t_0) Q_1 (W + \Delta W(t)) \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} \\ &\quad \times (Q_1 (W + \Delta W(t)))^T m(t_0) + g^T(p(t_0 - \tau(t_0))) \\ &\quad \times \frac{Q_2 - \rho L_2}{k^2} g(p(t_0 - \tau(t_0))), \end{aligned} \quad (18)$$

and

$$\begin{aligned} &2p^T(t_0) Q_2 (D + \Delta D(t)) m(t_0 - \tau(t_0)) \leq p^T(t_0) Q_2 \\ &\quad \times (D + \Delta D(t)) Q_1^{-1} (Q_2 (D + \Delta D(t)))^T p(t_0) \\ &\quad + m^T(t_0 - \tau(t_0)) Q_1 m(t_0 - \tau(t_0)). \end{aligned} \quad (19)$$

By Assumption (A1), we have

$$\begin{aligned} &\text{trace}(\sigma(p(t_0), p(t_0 - \tau(t_0))) Q_1 \sigma^T(p(t_0), p(t_0 - \tau(t_0)))) \\ &\leq \lambda_{\max}(Q_1) \text{trace}(\sigma^T(p(t_0), p(t_0 - \tau(t_0))) \sigma(p(t_0), \\ &\quad \times p(t_0 - \tau(t_0)))) \leq \rho [p^T(t_0) L_1 p(t_0) + p^T(t_0 - \tau(t_0)) \\ &\quad \times L_2 p(t_0 - \tau(t_0))]. \end{aligned} \quad (20)$$

From (17), (18), (19) and (20), we obtain

$$\begin{aligned} \mathcal{L}V(t_0, m(t_0), p(t_0)) &\leq t_0^\gamma \left[m^T(t_0) \left(\frac{\gamma}{t_0} Q_1 - 2Q_1 (A + \Delta A(t)) \right. \right. \\ &\quad + Q_1 (W + \Delta W(t)) \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} (Q_1 (W + \Delta W(t)))^T) m(t_0) \\ &\quad + p^T(t_0) \left(\frac{\gamma}{t_0} Q_2 - 2Q_2 (C + \Delta C(t)) + Q_2 (D + \Delta D(t)) \right. \\ &\quad \times Q_1^{-1} (Q_2 (D + \Delta D(t)))^T + \rho L_1) p(t_0) \\ &\quad + g^T(p(t_0 - \tau(t_0))) \frac{Q_2 - \rho L_2}{k^2} g(p(t_0 - \tau(t_0))) \\ &\quad + m^T(t_0 - \tau(t_0)) Q_1 m(t_0 - \tau(t_0)) \\ &\quad \left. \left. + \rho p^T(t_0 - \tau(t_0)) L_2 p(t_0 - \tau(t_0)) \right] \right]. \end{aligned} \quad (21)$$

From (6), the following inequality holds:

$$\begin{aligned} & t_0^\gamma \left[g^T(p(t_0 - \tau(t_0))) \frac{Q_2 - \rho L_2}{k^2} g(p(t_0 - \tau(t_0))) \right. \\ & \quad \left. + \rho p^T(t_0 - \tau(t_0)) L_2 p(t_0 - \tau(t_0)) \right] \\ & \leq t_0^\gamma \left[p^T(t_0 - \tau(t_0)) k^2 \frac{Q_2 - \rho L_2}{k^2} p(t_0 - \tau(t_0)) \right. \\ & \quad \left. + \rho p^T(t_0 - \tau(t_0)) L_2 p(t_0 - \tau(t_0)) \right] \\ & = t_0^\gamma p^T(t_0 - \tau(t_0)) Q_2 p(t_0 - \tau(t_0)). \end{aligned} \quad (22)$$

Also, we have

$$\begin{aligned} & t_0^\gamma [p^T(t_0 - \tau(t_0)) Q_2 p(t_0 - \tau(t_0)) + m^T(t_0 - \tau(t_0)) \\ & \quad \times Q_1 m(t_0 - \tau(t_0))] = \left(\frac{t_0}{t_0 - \tau(t_0)} \right)^\gamma (t_0 - \tau(t_0))^\gamma \\ & \quad \times [m^T(t_0 - \tau(t_0)) Q_1 m(t_0 - \tau(t_0)) + p^T(t_0 - \tau(t_0)) \\ & \quad \times Q_2 p(t_0 - \tau(t_0))] \leq (1 - \mu)^{-\gamma} V(t_0 - \tau(t_0)), \\ & \quad \times m(t_0 - \tau(t_0)), p(t_0 - \tau(t_0)). \end{aligned} \quad (23)$$

So, we obtain

$$\begin{aligned} & dV(t_0, m(t_0), p(t_0)) = \mathcal{L}V(t_0, m(t_0), p(t_0)) dt_0 \\ & + 2t_0^\gamma m^T(t_0) Q_1 \sigma(p(t_0), p(t_0 - \tau(t_0))) d\omega(t_0) \\ & \leq t_0^\gamma \left[m^T(t_0) \left(\beta Q_1 - 2Q_1(A + \Delta A(t)) + Q_1(W + \Delta W(t)) \right. \right. \\ & \quad \times \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} (Q_1(W + \Delta W(t)))^T m(t_0) \\ & \quad + p^T(t_0) (\beta Q_2 - 2Q_2(C + \Delta C(t)) \\ & \quad + Q_2(D + \Delta D(t)) Q_1^{-1} (Q_2(D + \Delta D(t)))^T + \rho L_1) p(t_0) \\ & \quad \left. \left. + (1 - \mu)^{-\gamma} V(t_0 - \tau(t_0), m(t_0 - \tau(t_0)), p(t_0 - \tau(t_0))) \right) \right. \\ & \quad \left. + 2t_0^\gamma m^T(t_0) Q_1 \sigma(p(t_0), p(t_0 - \tau(t_0))) d\omega(t_0) \right]. \end{aligned} \quad (24)$$

Hence, one can obtain

$$\begin{aligned} & d(EV(t_0, m(t_0), p(t_0))) \leq \\ & E \left(t_0^\gamma \left[m^T(t_0) \left(\beta Q_1 - 2Q_1(A + \Delta A(t)) + Q_1(W + \Delta W(t)) \right. \right. \right. \\ & \quad \times \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} (Q_1(W + \Delta W(t)))^T m(t_0) \\ & \quad + p^T(t_0) (\beta Q_2 - 2Q_2(C + \Delta C(t)) + Q_2(D + \Delta D(t)) \\ & \quad \times Q_1^{-1} (Q_2(D + \Delta D(t)))^T + \rho L_1) p(t_0) \left. \left. \right] + (1 - \mu)^{-\gamma} \right. \right. \\ & \quad \times E(V(t_0 - \tau(t_0), m(t_0 - \tau(t_0)), p(t_0 - \tau(t_0)))) \\ & \quad \left. + E(2t_0^\gamma m^T(t_0) Q_1 \sigma(p(t_0), p(t_0 - \tau(t_0))) d\omega(t_0)) \right) \\ & = E \left(t_0^\gamma \left[m^T(t_0) \left(\beta Q_1 - 2Q_1(A + \Delta A(t)) + Q_1(W + \Delta W(t)) \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned} & \times \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} (Q_1(W + \Delta W(t)))^T m(t_0) \\ & + p^T(t_0) (\beta Q_2 - 2Q_2(C + \Delta C(t)) + Q_2(D + \Delta D(t)) \\ & \times Q_1^{-1} (Q_2(D + \Delta D(t)))^T + \rho L_1) p(t_0) \left. \left. \right] + (1 - \mu)^{-\gamma} \right. \\ & \times E(V(t_0 - \tau(t_0), m(t_0 - \tau(t_0)), p(t_0 - \tau(t_0))))). \end{aligned} \quad (25)$$

Because $E(V(t_0 - \tau(t_0), m(t_0 - \tau(t_0)), p(t_0 - \tau(t_0)))) \leq \alpha(t_0) = E(V(t_0, m(t_0), p(t_0)))$, we have

$$\begin{aligned} & d(EV(t_0, m(t_0), p(t_0))) \leq \\ & E \left(t_0^\gamma \left[m^T(t_0) \left(\beta Q_1 - 2Q_1(A + \Delta A(t)) \right. \right. \right. \\ & \quad + Q_1(W + \Delta W(t)) \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} (Q_1(W + \Delta W(t)))^T \\ & \quad \times m(t_0) + p^T(t_0) (\beta Q_2 - 2Q_2(C + \Delta C(t)) \\ & \quad + Q_2(D + \Delta D(t)) Q_1^{-1} (Q_2(D + \Delta D(t)))^T + \rho L_1) p(t_0) \left. \left. \right] \right. \\ & \quad \left. + (1 - \mu)^{-\gamma} E(V(t_0, m(t_0), p(t_0))) \right) = E \left(t_0^\gamma \left[m^T(t_0) \right. \right. \\ & \quad \times \left(\beta Q_1 - 2Q_1(A + \Delta A(t)) + Q_1(W + \Delta W(t)) \right. \\ & \quad \times \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} (Q_1(W + \Delta W(t)))^T + (1 - \mu)^{-\gamma} Q_1 m(t_0) \\ & \quad + p^T(t_0) (\beta Q_2 - 2Q_2(C + \Delta C(t)) + Q_2(D + \Delta D(t)) \\ & \quad \times Q_1^{-1} (Q_2(D + \Delta D(t)))^T + \rho L_1 + (1 - \mu)^{-\gamma} Q_2) p(t_0) \left. \left. \right] \right). \end{aligned} \quad (26)$$

From (8) and (9), we obtain

$$\begin{aligned} & \beta Q_1 - 2Q_1(A + \Delta A(t)) + Q_1(W + \Delta W(t)) \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} \\ & \times (Q_1(W + \Delta W(t)))^T + (1 - \mu)^{-\gamma} Q_1 = \beta Q_1 - 2Q_1 \\ & \times (A + HF(t)E_1) + Q_1(W + HF(t)E_2) \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} \\ & \times (Q_1(W + HF(t)E_2))^T + (1 - \mu)^{-\gamma} Q_1, \end{aligned} \quad (27)$$

$$\begin{aligned} & \beta Q_2 - 2Q_2(C + \Delta C(t)) + Q_2(D + \Delta D(t)) \\ & \times Q_1^{-1} (Q_2(D + \Delta D(t)))^T + \rho L_1 + (1 - \mu)^{-\gamma} Q_2 \\ & = \beta Q_2 - 2Q_2(C + H^*F^*(t)E_1^*) + Q_2(D + H^*F^*(t)E_2^*) \\ & \times Q_1^{-1} (Q_2(D + H^*F^*(t)E_2^*))^T + \rho L_1 + (1 - \mu)^{-\gamma} Q_2. \end{aligned} \quad (28)$$

By Lemma 2, Lemma 3 and LMI (12) and (13), we have

$$\begin{aligned}
& \begin{pmatrix} 2Q_1(A + HF(t)E_1) - \beta Q_1 - (1 - \mu)^{-\gamma} Q_1 & -Q_1(W + HF(t)E_2) \\ -(W + HF(t)E_2)^T Q_1 & \frac{Q_2 - \rho L_2}{k^2} \end{pmatrix} \\
&= \begin{pmatrix} 2Q_1 A - \beta Q_1 - (1 - \mu)^{-\gamma} Q_1 & -Q_1 W \\ -W^T Q_1 & \frac{Q_2 - \rho L_2}{k^2} \end{pmatrix} + \begin{pmatrix} Q_1 H \\ 0 \end{pmatrix} F(t) \begin{pmatrix} E_1^T \\ 0 \end{pmatrix}^T \\
&+ \begin{pmatrix} E_1^T \\ 0 \end{pmatrix} F^T(t) \begin{pmatrix} Q_1 H \\ 0 \end{pmatrix}^T + \begin{pmatrix} -Q_1 H \\ 0 \end{pmatrix} F(t) \begin{pmatrix} 0 \\ E_2^T \end{pmatrix}^T \\
&+ \begin{pmatrix} 0 \\ E_2^T \end{pmatrix} F^T(t) \begin{pmatrix} -Q_1 H \\ 0 \end{pmatrix}^T > 0,
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
& \begin{pmatrix} 2Q_2(C + H^* F^*(t)E_1^*) - \beta Q_2 - (1 - \mu)^{-\gamma} Q_2 - \rho L_1 & -Q_2(D + H^* F^*(t)E_2^*) \\ -(D + H^* F^*(t)E_2^*)^T Q_2 & Q_1 \end{pmatrix} \\
&= \begin{pmatrix} 2Q_2 C - \beta Q_2 - (1 - \mu)^{-\gamma} Q_2 - \rho L_1 & -Q_2 D \\ -D^T Q_2 & Q_1 \end{pmatrix} + \begin{pmatrix} Q_2 H^* \\ 0 \end{pmatrix} F^*(t) \begin{pmatrix} E_1^{*T} \\ 0 \end{pmatrix}^T \\
&+ \begin{pmatrix} E_1^{*T} \\ 0 \end{pmatrix} F^{*T}(t) \begin{pmatrix} Q_2 H^* \\ 0 \end{pmatrix}^T + \begin{pmatrix} -Q_2 H^* \\ 0 \end{pmatrix} F^*(t) \begin{pmatrix} 0 \\ E_2^{*T} \end{pmatrix}^T \\
&+ \begin{pmatrix} 0 \\ E_2^{*T} \end{pmatrix} F^{*T}(t) \begin{pmatrix} -Q_2 H^* \\ 0 \end{pmatrix}^T > 0.
\end{aligned} \tag{30}$$

Also, by Lemma 3, the inequalities (29) and (30) are equivalent to the following inequalities:

$$\begin{aligned}
& \beta Q_1 - 2Q_1(A + HF(t)E_1) \\
&+ Q_1(W + HF(t)E_2) \left(\frac{Q_2 - \rho L_2}{k^2} \right)^{-1} \\
&\times (Q_1(W + HF(t)E_2))^T + (1 - \mu)^{-\gamma} Q_1 < 0,
\end{aligned} \tag{31}$$

$$\begin{aligned}
& \beta Q_2 - 2Q_2(C + H^* F^*(t)E_1^*) \\
&+ Q_2(D + H^* F^*(t)E_2^*) Q_1^{-1} (Q_2(D + H^* F^*(t)E_2^*))^T \\
&+ \rho L_1 + (1 - \mu)^{-\gamma} Q_2 < 0.
\end{aligned} \tag{32}$$

Therefore, $d(EV(t, m(t), p(t)))|_{t=t_0} \leq 0$, and $\alpha(t)$ is also non-increasing at t_0 in the case that $E(V(t_0, m(t_0), p(t_0))) = \alpha(t_0)$.

In summary, for all $t \geq T$, $\alpha(t) = \alpha(T)$. From the definition of $\alpha(t)$, we obtain that $E(V(t, m(t), p(t))) \leq \alpha(t) \leq \alpha(T)$ for all t . Meanwhile, we have

$$\begin{aligned}
& EV(t, m(t), p(t)) = t^\gamma E(m^T(t)Q_1 m(t) + p^T(t)Q_2 p(t)) \\
&\geq \min\{\lambda_{\min}(Q_1), \lambda_{\min}(Q_2)\} t^\gamma E \| (m^T(t), p^T(t))^T \|^2.
\end{aligned} \tag{33}$$

Hence, one can obtain

$$E \| (m^T(t), p^T(t))^T \|^2 \leq \frac{\alpha(T)}{\min\{\lambda_{\min}(Q_1), \lambda_{\min}(Q_2)\}} t^{-\gamma}, \tag{34}$$

that is, the model (7) is globally robust power-rate stable in mean square with the parameters uncertainties satisfying (8) and (9), and power-rate convergence rate is γ . The proof of Theorem 1 is completed.

Remark 1 From the proof of Theorem 1, the power-rate convergence rate γ can be estimated. From (12) and (13), we have

$$Q_1 A + A^T Q_1 - \beta Q_1 - (1 - \mu)^{-\gamma} Q_1 > 0, \tag{35}$$

$$Q_2 C + C^T Q_2 - \beta Q_2 - (1 - \mu)^{-\gamma} Q_2 - \rho L_1 > 0, \tag{36}$$

Hence, we have that

$$\begin{aligned}
& 0 < \gamma < \min \\
& \left\{ \log_{\frac{1}{1-\mu}} \frac{\lambda_{\max}(Q_1 A + A^T Q_1)}{\lambda_{\min}(Q_1)}, \log_{\frac{1}{1-\mu}} \frac{\lambda_{\max}(Q_2 C + C^T Q_2 - \rho L_1)}{\lambda_{\min}(Q_2)} \right\}.
\end{aligned} \tag{37}$$

Remark 2 For simplicity, in model (7), we consider the delays are both $\tau(t)$. Also, when considering $\tau(t) \neq \epsilon(t)$, we can take consider of μ such that $0 \leq \tau(t), \epsilon(t) \leq \mu t$. Under

this condition, similar result can be obtained for model (5) with noise perturbations.

In addition, from the proof of Theorem 1, if the parameter uncertainties vanish, global power-rate stability in mean square can be derived. It can be seen in the following corollary.

Corollary 1 *If there exist two symmetric positive definite matrices Q_1 , Q_2 and a positive scalar $\rho > 0$ such that*

$$\begin{pmatrix} Q_1 A + A^T Q_1 - Q_1 & -Q_1 W \\ -W^T Q_1 & \frac{Q_2 - \rho L_2}{k^2} \end{pmatrix} > 0, \quad (38)$$

$$\begin{pmatrix} Q_2 C + C^T Q_2 - Q_2 - \rho L_1 & -Q_2 D \\ -D^T Q_2 & Q_1 \end{pmatrix} > 0, \quad (39)$$

and

$$Q_1 \leq \rho I, \quad Q_2 > \rho L_2,$$

where $k = \max\{k_1, k_2, \dots, k_n\}$, then, the model (7) with $\Delta A(t) = \Delta W(t) = \Delta C(t) = \Delta D(t) = 0$ is globally power-rate stable in mean square.

Illustrative example

In this section, we present an example to show the effectiveness of our theoretical results.

We consider the genetic network in Li et al. (2006b), [Fig. 2] with unbounded time-varying delay and stochastic perturbation. Let

$$A = C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$W = 0.5 \times \begin{pmatrix} 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad B = 0.5 \times \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$H = H^* = \begin{pmatrix} 0.02 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0 & 0.02 \end{pmatrix},$$

$$E_1 = E_1^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, E_2 = E_2^* = \begin{pmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{pmatrix}, F(t) = F^*(t) = \begin{pmatrix} \sin t & 0 & 0 & 0 & 0 \\ 0 & \cos t & 0 & 0 & 0 \\ 0 & 0 & \sin t & 0 & 0 \\ 0 & 0 & 0 & \cos t & 0 \\ 0 & 0 & 0 & 0 & \sin t \end{pmatrix}.$$

and $g_i(x) = \frac{x^2}{1+x^2}$, $i = 1, 2, \dots, 5$, i.e., the Hill coefficient is 2. It is obvious that $k = 0.65$.

Assuming that the delay $\tau(t) = \frac{t}{5}$, we have $\mu = \frac{1}{5}$. The noise intensity is assumed as follows:

$$\sigma(p(t), p(t - \tau(t))) = (\sigma_1(p(t), p(t - \tau(t))), \dots, \sigma_5(p(t), p(t - \tau(t))))^T$$

with $\sigma_i(p(t), p(t - \tau(t))) = 10 \sum_{j=1}^5 p_j(t)$, $i = 1, \dots, 5$.

We can easily obtain that

$$L_1 = \begin{pmatrix} 2500 & 0 & 0 & 0 & 0 \\ 0 & 2500 & 0 & 0 & 0 \\ 0 & 0 & 2500 & 0 & 0 \\ 0 & 0 & 0 & 2500 & 0 \\ 0 & 0 & 0 & 0 & 2500 \end{pmatrix}$$

and $L_2 = 0$.

By using the MATLAB LMI Toolbox, the LMIs (10) and (11) hold with $\rho = 0.0395$, $\varepsilon_1 = 0.0152$, $\varepsilon_1^* = 0.0031$, $\varepsilon_2 = 5.5035$, $\varepsilon_2^* = 0.0164$,

$$Q_1 = \begin{pmatrix} 0.0352 & 0 & 0 & 0 & 0 \\ 0 & 0.0351 & 0 & 0 & 0 \\ 0 & 0 & 0.0352 & 0 & 0 \\ 0 & 0 & 0 & 0.0352 & 0 \\ 0 & 0 & 0 & 0 & 0.0352 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 0.0212 & 0 & 0 & 0 & 0 \\ 0 & 0.0212 & 0 & 0 & 0 \\ 0 & 0 & 0.0212 & 0 & 0 \\ 0 & 0 & 0 & 0.0212 & 0 \\ 0 & 0 & 0 & 0 & 0.0212 \end{pmatrix}.$$

Hence, the considered model with the parameter uncertainties is globally robust power-rate stable in mean square.

When the parameter uncertainties vanish, the unique equilibrium point of this network without stochastic perturbation is $M^* = (0.3955, 0.5208, 0.1164, 0.4493,$

Fig. 1 Transient behavior of $m_1, m_2, m_3, m_4, m_5, p_1, p_2, p_3, p_4, p_5$ with different parameters

$0.0803)^T, P^* = (0.3905, 0.5296, 0.1295, 0.4408, 0.0850)^T$. We shift the equilibrium point to the origin to get the equations of m and p . By using the MATLAB LMI Toolbox, the LMIs (38) and (39) hold with $\rho = 0.0163$,

$$Q_1 = \begin{pmatrix} 0.0128 & 0 & -0.0001 & 0 & 0 \\ 0 & 0.0129 & 0 & -0.0001 & 0 \\ -0.0001 & 0 & 0.0127 & 0 & 0 \\ 0 & -0.0001 & 0 & 0.0128 & 0 \\ 0 & 0 & 0 & 0 & 0.0127 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 0.0088 & -0.0002 & -0.0004 & 0.0001 & -0.0003 \\ -0.0002 & 0.0093 & -0.0003 & -0.0003 & 0.0002 \\ -0.0004 & -0.0003 & 0.0083 & -0.0003 & 0 \\ 0.0001 & -0.0003 & -0.0003 & 0.0089 & 0.0003 \\ -0.0003 & 0.0002 & 0 & 0.0003 & 0.0083 \end{pmatrix}.$$

Hence, the equilibrium point of the considered model without the parameter uncertainties is globally power-rate stable in mean square, and the power convergence rate α is satisfied $0 < \alpha < \log_{1.25} 2.0476$. Simulation result is depicted in Fig. 1.

Conclusions

In this paper, the global robust power-rate stability in mean square is studied for genetic regulatory networks with unbounded time-varying delay, noise perturbations and parameter uncertainties. The method combining Lyapunov stability theory, Itô's differential formula and linear matrix

inequality (LMI) approach was adopted to study this issue. Sufficient criterion ensuring global robust power-rate stability (in mean square) of the genetic regulatory networks is given, which is expressed in terms of linear matrix inequalities. Meanwhile, when the parameter uncertainties vanish, sufficient criterion ensuring global power-rate stability (in mean square) of the genetic regulatory networks is also provided. The boundedness and differentiability of the time delay function is removed in this paper. Also, the conditions which we obtain are easy to be verified and checked in practice.

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