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Comparison Study of Three Different Image Reconstruction Algorithms for MAT-MI

Rongmin Xia,

Department of Biomedical Engineering, University of Minnesota, Minneapolis, MN 55455 USA. He is now with Methodist Hospital, Houston Medical Center, Houston, TX 77030 USA

Xu Li [Student Member, IEEE], and

Department of Biomedical Engineering, University of Minnesota, Minneapolis, MN 55455 USA

Bin He [Fellow, IEEE]

Department of Biomedical Engineering, University of Minnesota, Minneapolis, MN 55455 USA
binhe@umn.edu

Abstract

We report a theoretical study on magnetoacoustic tomography with magnetic induction (MAT-MI). According to the description of signal generation mechanism using Green's function, the acoustic dipole model was proposed to describe acoustic source excited by the Lorentz force. Using Green's function, three kinds of reconstruction algorithms based on different models of acoustic source (potential energy, vectored acoustic pressure, and divergence of Lorentz force) are deduced and compared, and corresponding numerical simulations were conducted to compare these three kinds of reconstruction algorithms. The computer simulation results indicate that the potential energy method and vectored pressure method can directly reconstruct the Lorentz force distribution and give a more accurate reconstruction of electrical conductivity.

Keywords

Green's function; time-domain reconstruction; magnetoacoustic tomography; magnetoacoustic tomography with magnetic induction; MAT-MI

I. INTRODUCTION

Hybrid imaging methods that can combine the strength of more than two imaging techniques have attracted more and more attentions. For example, hybrid imaging modalities that combine ultrasound technique with optical imaging, microwave imaging or electromagnetic source and impedance imaging techniques were developed and demonstrated in photoacoustic imaging [1], thermoacoustic imaging [2], [3], magnetoacoustic imaging [4]–[11] and acoustic electrical imaging [12]. By combining with ultrasound technique, photoacoustic imaging can get larger penetration depth than pure optical imaging methods and at the same time obtain the large optical contrast in biological tissues. Similarly thermoacoustic imaging can provide microwave absorption contrast with better spatial resolution than pure microwave imaging technique. In the magnetoacoustic imaging methods such as magnetoacoustic tomography (MAT) [4], [5], Hall Effect imaging (HEI) [6] and magnetoacoustic tomography with magnetic induction

(MAT-MI) [7]–[11], ultrasound technique is involved to increase the sensitivity for detecting bioelectrical current or to get better spatial resolution for electrical impedance imaging.

On the other hand, noninvasive electrical impedance imaging has been an active research field for decades. Besides those hybrid magnetoacoustic imaging approaches, several other methods have been developed including electrical impedance tomography (EIT) [13], [14], magnetic resonance electrical impedance tomography (MREIT) [15], [16] and magnetic induction tomography (MIT) [17], [18]. The major limitations of both EIT and MIT are their limited spatial resolution and the need to solve ill-posed inverse problems. MREIT can provide high spatial resolution conductivity images with the aid of MRI measurements, but currently it is limited by its need of relatively high level of current injection to reach good signal to noise ratio (SNR).

In principle, those hybrid magnetoacoustic imaging methods including MAT, HEI and MAT-MI all incorporate the electrical-acoustic coupling mechanism achieved through the Lorentz force acting on the injected or magnetically induced current in the conductive sample. In MAT-MI, for example, a sample is placed in a static magnetic field and a stimulating coil is used to generate a time varying magnetic field and induce eddy current in the sample. This eddy current under static magnetic field is subject to the Lorentz force and induces vibrations that can be detected by ultrasound transducers.

In this paper we report a theoretical study on the acoustic wave generation mechanism of MAT-MI using Green's function and propose a new reconstruction algorithm. In previous forward model of MAT-MI, the divergence of the Lorentz force is taken as an acoustic source. Correspondingly, the reconstructed image would indicate only this divergence [7]–[10]. Using Green's function, three kinds of reconstruction algorithms based on different models of acoustic source (potential energy, vectored acoustic pressure, and divergence of Lorentz force) are deduced and compared. The corresponding numerical simulation results were given.

II. THEORY

A. Forward problem

In MAT-MI, the wave equation for the induced pressure $p(\mathbf{r}, t)$ can be written as in (1) [7],

$$\nabla^2 p(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} p(\mathbf{r}, t) = \nabla \cdot (\mathbf{J} \times \mathbf{B}_0) \delta(t) \quad (1)$$

where $\mathbf{B}_0$ is the static magnetic field flux density, $\mathbf{J}$ is the induced eddy current density vector. Taking Fourier transform on variable $t' = ct$, here c is the acoustic speed, the above equation can be written as

$$(\nabla^2 + k^2) \tilde{p}(\mathbf{r}, k) = \nabla \cdot (\mathbf{J} \times \mathbf{B}_0) c \quad (2)$$

The magnetoacoustic signal can be measured on a surface S_0 where transducer is located and encloses the sample. According to Green's function, the signal measured at position $\mathbf{r}_0$ on the surface S_0 can be expressed as [19]

$$\tilde{p}(\mathbf{r}_0, k) = -c \iiint_V d\mathbf{v} \cdot \nabla \cdot (\mathbf{J} \times \mathbf{B}_0) \cdot \tilde{G}_k(\mathbf{r}, \mathbf{r}_0) \quad (3)$$

where $\tilde{G}_k(\mathbf{r}, \mathbf{r}_0)$ is Green's function, $\mathbf{r}$ is the object coordinate vector, $\mathbf{r}_0$ is the observing point and V is the volume of the to-be-imaged object. This formula, taking the divergence of the Lorentz force as an acoustic source was proposed by Xu & He [7]. According to (3) the acoustic source is generated by the heterogeneity of the Lorentz force distribution. Normally the acoustic signal generated at the conductivity boundary is much larger than that generated from a homogeneous conductivity volume [10].

Denoting $\mathbf{F} = \mathbf{J} \times \mathbf{B}_0$ and taking the factor that the Lorentz force is zero outside the source region, according to the vector identity $\tilde{G}_k \Delta \cdot \mathbf{F} = -\mathbf{F} \cdot \Delta \tilde{G}_k + \nabla \cdot (\mathbf{F} \tilde{G}_k)$ and the divergence theorem, (3) can be rewritten as in (4)

$$\tilde{p}(\mathbf{r}_0, k) = c \iiint_V d\mathbf{v} \cdot \mathbf{F} \cdot \nabla \tilde{G}_k(\mathbf{r}, \mathbf{r}_0) \quad (4)$$

As we all know, a dipole source is composed of two monopole sources close to each other, opposite in sign and equal in magnitude. If the dipole vector $\mathbf{D}_w$ (vector dipole strength) is centered at position $\mathbf{r} = (x, y, z)$, the pressure wave at point $\mathbf{r}_0$ can be written in terms of the radius vector $\mathbf{R} = \mathbf{r}_0 - \mathbf{r}$ and the spherical angles of vector $\mathbf{R}$ [20] as in (5)

$$\tilde{p} = -ik\rho c \mathbf{D}_w \cdot \nabla G_k(\mathbf{r}, \mathbf{r}_0) \quad (5)$$

where ρ is the density of the medium. Comparing Eq. (4) and Eq. (5), the Lorentz force item in Eq. (4) can be considered as an acoustic dipole source similar to the dipole vector $\mathbf{D}_w$ in (5).

From the identity $\nabla \tilde{G}_k(\mathbf{r}, \mathbf{r}_0) = -(jk - \frac{1}{|\mathbf{r} - \mathbf{r}_0|}) \tilde{G}_k \mathbf{n}(\mathbf{r}, \mathbf{r}_0)$ with $\mathbf{n}(\mathbf{r}, \mathbf{r}_0)$ denoting the direction of vector $\mathbf{r}_0 - \mathbf{r}$, we get the following from Eq. (4)

$$\tilde{p}(\mathbf{r}_0, k) = -c \iiint_V d\mathbf{v} \cdot (jk - \frac{1}{|\mathbf{r}_0 - \mathbf{r}|}) (\mathbf{F} \cdot \mathbf{n}(\mathbf{r}, \mathbf{r}_0)) \nabla \tilde{G}_k(\mathbf{r}, \mathbf{r}_0) \quad (6)$$

To better understand the dipole source here, consider the movement of particles under the Lorentz force as a swing vibration, with compression at one side and expansion at the opposite side, thus forming a dipole like source. Here we give a one-dimensional illustration shown as in Fig. 1. Point A and E represent the outside of object. The force at point B, C, and D are $\mathbf{F}_b$, $\mathbf{F}_c$, and $\mathbf{F}_d$ respectively. According to the point source theory, the acoustic pressure at observing point G can be written as $(\nabla \cdot \mathbf{F}_a + \nabla \cdot \mathbf{F}_b + \nabla \cdot \mathbf{F}_c + \nabla \cdot \mathbf{F}_d + \nabla \cdot \mathbf{F}_e) G_k$, assuming the distance between two point sources next to each other within the sample is equal and much smaller than the distance between the sample and the observing point. The top line in Fig. 1 is the values of force divergence at each point. A dipole source consists of two monopole sources of equal strength but opposite phase and separated by a small distance compared with the acoustic wavelength. At point B, C and D in Fig. 1, the sources that satisfy the condition of dipole sources, such as the positive $\mathbf{F}_b$ on the right side of point B and negative $\mathbf{F}_b$ on the left side of point B, form a dipole source at point B. Fig. 1 and Eq. (6) suggest that the acoustic sources of MAT-MI are dipole like sources, and the force at each point represents the part of dipole vector.

B. Inverse problem

The inverse problem of MAT-MI can be generally divided into three steps: acoustic source reconstruction, the eddy current reconstruction and the conductivity reconstruction. According to Green's function, three different reconstruction algorithms were deduced here for conductivity image reconstruction.

Taking $\nabla \cdot (\mathbf{J} \times \mathbf{B}_0)$ as acoustic source, using Green's function and Eq. (3) in three-dimensional cylinder coordinate system (ρ, ϕ, z) , the reconstruction formula [7] can be written as

$$\nabla \cdot (\mathbf{J} \times \mathbf{B}_0) = -\frac{1}{2\pi c^3} \iint_{S_0} dS_0 \sqrt{1 - \frac{(z_0 - z)^2}{|\mathbf{r} - \mathbf{r}_0|^2}} \frac{1}{t} \frac{\partial^2 p(\mathbf{r}_0, t)}{\partial^2 t} \Big|_{t=|\mathbf{r} - \mathbf{r}_0|/c} \quad (7)$$

According to Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}_1}{\partial t} \quad (8)$$

Assuming the time varying magnetic field has step function in time i.e. $\mathbf{B}_1(\mathbf{r}, t) = \mathbf{B}_1(\mathbf{r})u(t)$, we have $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})\delta(t)$, where $\mathbf{E}$ is the spatial component of the electrical field and obeys

$$\nabla \times \mathbf{E} = -\mathbf{B}_1 \quad (9)$$

Combing equation (9) with Ohm's Law $\mathbf{J} = \sigma \mathbf{E}$, we have

$$\nabla \times (\mathbf{J}/\sigma) = -\mathbf{B}_1 \quad (10)$$

where σ is the electrical conductivity. After taking a dot product with $\mathbf{B}_0$ on both sides of equation (10) [7], we have

$$\nabla \cdot (\mathbf{F}/\sigma) = -\mathbf{B}_1 \cdot \mathbf{B}_0 \quad (11)$$

where we replace $(\nabla \times \mathbf{J}) \cdot \mathbf{B}_0$ with $\nabla \cdot (\mathbf{J} \times \mathbf{B}_0)$, due to the fact that the magnetic field is generated by the sources outside the sample, i.e. $\nabla \times \mathbf{B}_0 = 0$ inside of object.

After expanding the dot product, we have

$$\frac{1}{\sigma(\mathbf{r})} (\nabla \cdot \mathbf{F} - \frac{\sigma(\mathbf{r})'}{\sigma(\mathbf{r})} \mathbf{n}_r \cdot \mathbf{F}) = -\mathbf{B}_1 \cdot \mathbf{B}_0 \quad (12)$$

where $\mathbf{n}_r$ is the unit vector of $\mathbf{r}$, $\sigma(\mathbf{r})$ is the electrical conductivity at point $\mathbf{r}$ and $\sigma(\mathbf{r})'$ is the derivative of $\sigma(\mathbf{r})$. From Eq. (12), we know that the sound source of MAT-MI relates to the conductivity distribution $\sigma(\mathbf{r})$ and projection of Lorentz force along the $\mathbf{r}$ direction. The Lorentz force $\mathbf{F}$ could not be obtained from Eq. (7), so we could not deduce an analysis result of conductivity from Eq. (12). However, if we take the approximation $\sigma \gg \sigma'$, the above equation can be simplified as in [7]

$$\sigma(\mathbf{r}) = - \frac{\nabla \cdot (\mathbf{J} \times \mathbf{B}_0)}{\mathbf{B}_1 \cdot \mathbf{B}_0} \quad (13)$$

Eq. (13) is only applicable to piece-wise homogeneous medium and in practice a band-pass filter [7] has been used to improve the imaging quality in areas close to the boundary between regions with different conductivity. All conductive volumes with small size and sharp boundaries may disappear in the reconstructed image, if the size information is out of the band-pass frequency band of the filter.

In order to reconstruct more accurate conductivity distribution, the Lorentz force $\mathbf{F}$ must be solved. By constructing a vector acoustic pressure in Eq. (6), the Lorentz force $\mathbf{F}$ can be reconstructed directly. The details have been reported in another paper [21], here we only give the result as in (14)

$$|\mathbf{F}(\rho, \varphi, z)| \exp(i\theta_F) = - \frac{1}{2\pi c^3} \times \iint_{S_0} dS_0 \sqrt{1 - \frac{(z_0 - z)^2}{|\mathbf{r} - \mathbf{r}_0|^2}} \frac{1}{t} \left. \frac{\partial p(\mathbf{r}_0, t) \exp(i\varphi_0)}{\partial t} \right|_{t=|\mathbf{r} - \mathbf{r}_0|/c} \quad (14)$$

where φ_F is the azimuth of Lorentz force $\mathbf{F}$ vector, and φ_0 is the azimuth of the location vector of transducer. This formula has a condition that the detecting distance is much larger than the radius of the imaging object. From $\mathbf{J} = \sigma \mathbf{E}$, taking a cross product with $\mathbf{B}_0$ on both sides of it, we can get

$$\sigma = \frac{|\mathbf{J} \times \mathbf{B}_0|}{|\mathbf{E} \times \mathbf{B}_0|} = \frac{|\mathbf{F}|}{|\mathbf{E} \times \mathbf{B}_0|} \quad (15)$$

If Lorentz force and electric field are reconstructed, the conductivity distribution can be calculated easily using Eq. (15). However, the induced electric field $\mathbf{E}$ is not easily obtained, so we go back to Eq. (12), and notice that if we have the value of $\mathbf{F}$ and divergence of $\mathbf{F}$, the electric conductivity can be calculated by applying an iterative algorithm as in Eq. (16)

$$\sigma(\mathbf{r}) = \frac{-\nabla \cdot \mathbf{F} - \sqrt{(\nabla \cdot \mathbf{F})^2 + 4 \mathbf{B}_1 \cdot \mathbf{B}_0 (\nabla \sigma(\mathbf{r}) \cdot \mathbf{F})}}{2 \mathbf{B}_1 \cdot \mathbf{B}_0} \quad (16)$$

Ignoring the term of $\nabla \sigma(\mathbf{r}) \cdot \mathbf{F}$, the above formula becomes Eq. (13)

In the second method, we notified that the force is a vector, so we constructed a vector acoustic pressure to reserve the vector information during the reconstruction process. Using potential function is another way to keep vector information, just like acoustic pressure vs. velocity potential function. Here we introduce an item V called potential energy, and the corresponding force $\mathbf{F} = -\nabla V$. Substitute these into Eq. (2), we get

$$(\nabla^2 + k^2) \tilde{p}(\mathbf{r}, k)/c = -\nabla \cdot (\nabla V) \quad (17)$$

Moving $\nabla^2 V$ to the left hand side of Eq. (17), and subtracting $k^2 V$ from both hand sides, the above equation can be rewritten as

$$(\nabla^2 + k^2)(\tilde{p}(\mathbf{r}, k)/c - V) = k^2 V \quad (18)$$

As V is not a quantity of wave, the wave equation of V should be zero, that is $(\nabla^2 + k^2)V = 0$, so we can obtain

$$\tilde{p}(\mathbf{r}_0, k) = -c \iiint_V d\mathbf{v} \cdot k^2 V \cdot \tilde{\mathbf{G}}_k(\mathbf{r}, \mathbf{r}_0) \quad (19)$$

Applying the similar deduction process with Eq. (3), the corresponding reconstruct formula can be written as

$$V(\rho, \varphi, z) = -\frac{1}{2\pi c^3} \iint_{S_0} dS_0 \sqrt{1 - \frac{(z_0 - z)^2}{|\mathbf{r} - \mathbf{r}_0|^2}} \frac{1}{t} p(\mathbf{r}_0, t) \Big|_{t=|\mathbf{r} - \mathbf{r}_0|/c} \quad (20)$$

The corresponding Lorentz force $\mathbf{F}$ can be calculated by taking the negative gradient of V , and the conductivity distribution can be obtained using Eq. (16).

Eqs. (7), (14) and (20) are similar with the major difference in the order of time derivative of the pressure. The reconstruction algorithm in Eq. (20) is called potential energy reconstruction, whose characteristic is to use the acoustic pressure directly in integral equation. The vectored acoustic pressure method is to reconstruct the distribution of vector force field, as shown in Eq. (14), and the key point is to vectorize an acoustic pressure scalar with a first-order time derivative before the integral procedure. If the integral is based on the second-order time derivative of acoustic pressure, as in Eq. (7), the algorithm is based on reconstruction of the divergence of the Lorentz force.

III. SIMULATION AND DISCUSSION

In order to verify the potential energy reconstruction algorithm (Eq. (20)) and compare its performance with the other two algorithms, we conducted a two-dimensional numerical simulation. As shown in Fig. 2 (a), the original force field has three circular regions. The largest circle locates at (8, 0) mm with 6mm radius. The medium circular region has its center at (-10, 8) mm and has its radius to be 4mm. The smallest one centers at (-10, -10) mm with 2mm radius. Each region has a force distribution that is along the radial directions from center to periphery. The force amplitude at each center is zero, and increases linearly along the radius. The corresponding conductivity distribution and divergence of force are shown in Figs. 2(b) and (c) respectively. This force distribution simulates the Lorentz force distribution in MAT-MI with several homogeneous conductive objects while in each object the eddy current induced by time varying magnetic field is zero at object center and reaches its maximum value at the boundary. In addition, the direction of the Lorentz force is always perpendicular with eddy current. The imaging area of simulation was $40\text{mm} \times 40\text{mm}$ with calculation grids of 200×200 . The acoustic signals driven by force field are collected at 360 detection positions that are evenly distributed on a circumference or a circle with radius of 500mm.

According to the numerical simulation and experiment results in Ref [8], the acoustic signal generated from the object boundary is much larger than that from inner area of the object. The acoustic signal from object boundary has the same frequency as the magnetic stimulating

source, but the signal from inner area of the object has much lower frequency that is related to the conductivity distribution. The simulation results are shown in Fig. 3. The left column shows the results of acoustic source reconstruction and the right column shows the conductivity reconstruction results. The first reconstruction algorithm takes a second-order time derivative of acoustic pressure as back projection source. As the time derivative operation has similar effect of a ramp filter [3], the signal at the conductivity boundaries is magnified and signal from the inner area is depressed. Fig. 3(a) shows the reconstructed divergence of the force field. A clear object, boundary can be reconstructed, but the internal distribution of the object is missing. Using Eq. (13) and band-pass filter, the objects appear, but the image, Fig. 3(b), is blurred due to the use of filter. Fig. 3(c) shows the distribution of the Lorentz force reconstructed using Eq. (14). Compared with the original force distribution, we can see that the edge of the object was smoothed out and a ring appeared outside the object. These effects are mainly caused by the first-order time derivative. Fig. 3(e) demonstrates the reconstructed potential energy distribution using Eq. (20). It is shown that the boundary is not quite distinct. From all the results, we can see that when reconstructing the force divergence as in Eq. (7) the high frequency signal from the object boundary is dominant. On the contrary, in the reconstruction of potential energy, we use lower frequency signals for reconstruction. Figs. 3 (d) and (f) show the corresponding reconstructed conductivity distributions using the results shown in (c) and (e) respectively. In Fig. 3 (d) and (f), a dark point at the center of each object is shown. This effect is caused by singularity due to the induced electrical field is zero at the center of each conductive object [22]. The difference between them can be seen from Fig. 4, which is the force amplitude profile along the horizontal central line across the center of biggest circular object. The blue dash line is obtained from potential energy method, and the red solid line is calculated using vector acoustic method. The black solid line represents the original force amplitudes. For the reconstruction of the electric conductivity, until now no algorithm could solve it perfectly. The divergence method can only give a good boundary reconstruction, and others also need to overcome the singularity in the center. How to obtain the electric conductivity from the Lorentz force needs further investigation.

In order to compare the vector acoustic method and the potential energy method Gaussian white noise (3dB) was added to the acoustic pressure signal, and the corresponding reconstructed images are shown in Fig. 5. Figs. 5(a) and (b) represent the distribution of potential energy and Lorentz force reconstructed using the potential energy method. Fig. 5(c) is the Lorentz force image obtained by using the vector acoustic method. Both of them work well, the details are shown in Fig. 4(b). The curve of potential energy method looks more coarse, and the vector acoustic method are smoother, but with larger fluctuation.

Considering the time-varying magnetic field induced current inside the human body, from the view of energy conservation, induced electric energy in tissue should be less than that in the stimulating electric coils, i.e. $I_t^2 R_t < I_c^2 R_c$, where the subscript t and c represent tissue and coil. The electric conductivity of muscle and copper are $0.15 \text{ s} / \text{m}$ [24], and $59.6 \times 10^6 \text{ s} / \text{m}$. If the limit of current in human body is 10mA, the driving current in copper coil should be under 100A. Considering the eddy currents exist in three dimensional space and the efficiency of energy transition is much less than 1, the actual driving current in copper coil could go over 1000A. When the driving current goes larger, the noise of radiofrequency (RF) will appear in the sampling data, fortunately due to acoustic propagation delay, the RF noise and the real MAT-MI acoustic signal are separated in time domain.

These algorithms provide estimates of different physical quantity from the acoustic measurements, and indicate that magnetoacoustic tomography with magnetic induction can offer us images of acoustic field, Lorentz force image, induced eddy current image, and potential energy image.

IV. CONCLUSION

The forward mechanism and reconstruction algorithm of MAT-MI are explored using Green's function. The potential energy method and vectored pressure method can reconstruct the Lorentz force distribution directly and give a more accurate conductivity reconstruction. The divergence method can only give conductivity boundary reconstruction. The generated acoustic signal in MAT-MI includes high frequency components generated from the conductivity boundary, and low frequency components generated from the inner part of the conductive object. In order to image the inner structure of object, a wideband transducer with high sensitivity is needed, and the proposed reconstruction algorithm of potential energy and vectored pressure method can be used. If only a narrowband transducer is used, the divergence method is enough to solve the problem.

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Biographies

Rongmin Xia was born in 1972 in Yancheng, Jiangsu, China. He received his M.S. degree from Shanghai University, Shanghai, in radio physics in 1999, and his Ph.D. degree from Shanghai Jiao Tong University, Shanghai, China, in bio-medical engineering in 2002, respectively. Dr. Xia was an associate professor in the Department of Biomedical Engineering of Shanghai Jiao Tong University. He has visited the Department of Biomedical Engineering at the University of Minnesota, and is currently visiting The Methodist Hospital.

His main research interests are in ultrasound, biomedical engineering, and computer-aided measurement systems. Dr. Xia is a grantee of the National Natural Science Foundation of China (No. 30400104). He has been awarded three patents in the ultrasound field and published more than 20 papers.

Xu Li was born in China. He received his Bachelor's and Master's degree in Biomedical Engineering from Zhejiang University, Hangzhou, China in 2001 and 2004 respectively.

He is now a PhD student in the Department of Biomedical Engineering at the University of Minnesota, Minneapolis, MN, USA. His research interest is on magnetoacoustic tomography with magnetic induction, bioimpedance imaging and related imaging methodology.

Bin He (S'87–M'88–SM'97–F'04) is a Distinguished McKnight University Professor and Professor of Biomedical Engineering, Electrical Engineering and Neuroscience at the University of Minnesota at Twin Cities, where he serves as the Director of Center for Neuroengineering. Dr. He's major research interests include neuroengineering, functional biomedical imaging, and bioelectromagnetism. He is the Editor of the book entitled “Neural Engineering”, and serves as an associate editor of multiple international journals including IEEE Transactions on Biomedical Engineering, IEEE Transactions on Neural Systems and Rehabilitation Engineering, and Brain Topography. He is also a member of Editorial Board of IEEE Spectrum, Journal of Neural Engineering, and Clinical Neurophysiology, among others. Dr. He was the recipient of NSF CAREER Award, American Heart Association Established Investigator Award, and is a Fellow of IEEE and American Institute of Medical and Biological Engineering.

Fig. 1.
One-dimensional schematic diagram of configuration of acoustic dipole sources

Fig. 2.
 (a) The amplitude distribution of two-dimensional force vector, (b) the corresponding conductivity distribution, and (c) the distribution of force divergence.

Fig. 3.

(a) and (b) are reconstructed force divergence and conductivity distribution using Eqs. (7) and (13); (c) and (d) are reconstructed force amplitude and conductivity distribution using Eqs. (14) and (16); (e) and (f) are reconstructed potential energy and conductivity distribution using Eqs.(20) and (16).

Fig.4.

(a) and (b) are the force amplitude profile along the horizontal central line across the center of the biggest circular object. The brown solid line is obtained using Eq. (20), the blue dashed line is calculated using Eq. (14), and the red dashed line indicates the original values. (a) is obtained with noise free condition; (b) is obtained with 3dB Gaussian white noise added to the acoustic pressure signal.

Fig. 5. (a) and (b) represent the distribution of potential energy and Lorentz force reconstructed using the potential energy method. (c) is the Lorentz force image estimated by the vector acoustic method.