Determination of exact reconstruction regions in composite-circling cone-beam tomography

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Image reconstruction of a short portion of a long object using longitudinally truncated cone-beam data is important for major medical computed tomography (CT) applications, especially cardiac CT. Cardiac CT is an essential imaging tool for the diagnosis and therapeutic assessment of heart defects, cardiovascular diseases, and lung diseases, but it is still limited by suboptimal image quality. Recently, saddle-curve and composite-circling scanning modes have been proposed to solve this problem using exact reconstruction formulas. However, saddle-curve scanning involves both circular and linear motions, while composite-circling scanning conveniently involves just circular motions. Because saddle-curve scanning is difficult to implement mechanically, composite-circling scanning provides another, hopefully easier, approach: An x-ray focal spot in an x-ray tube is rotated on a plane facing the heart, while the x-ray tube and possibly the detector are simultaneously rotated on the gantry plane. This article determines regions for chord-based exact reconstruction in the composite-circling scanning mode and compares them to those in the saddle-curve scanning mode. For different scanning parameter combinations, this article finds the largest sphere centered at the origin that can be embedded inside the exact reconstruction region. This article also shows that the embedded spheres become larger when the x-ray focal spot rotates at variable speeds, allowing the scanning curve to cover a larger object. In summary, this article derives guidelines for prototyping a new cardiac CT scanner to meet the goals of reducing radiation dose and increasing spatial and temporal resolution. © 2009 American Association of Physicists in Medicine. [DOI: [10.1118/1.3158733](https://doi.org/10.1118/1.3158733)]

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I. INTRODUCTION

X-ray computed tomography (CT), introduced in 1973,¹ immediately generated great excitement in the medical field for its diagnostic possibilities, and since then it has been widely used to detect various disorders and diseases. It has revolutionized clinical imaging and become a cornerstone of radiology departments. Different types of scanning modes for x-ray CT have been developed, including fan-beam spiral scanning, cone-beam spiral scanning, and saddle-curve scanning.¹⁻⁵ Currently in clinical applications, cone-beam CT is used most extensively. Despite the great importance of cone-beam CT and the major efforts devoted so far to applying cone-beam geometry to clinical CT scanning, cone-beam image reconstruction algorithms are far from being fully developed. Cone-beam CT scanning still needs to be significantly improved in terms of spatial, contrast, and temporal resolution, image artifacts, and radiation dose.

One of the most important clinical applications is cardiac imaging. Cardiovascular diseases are pervasive with high mortality and morbidity rates and tremendous social and healthcare costs. For example, atherosclerosis, endothelial dysfunction, and coronary artery disease remain major health problems in the United States.⁶⁻⁸ There are urgent needs for high quality cardiovascular imaging that cannot be met by

current imaging technologies due to the small size of the heart and the high speed of heartbeats. Cardiac imaging as an example of a special type of the long object problem is still a major obstacle in current cardiovascular research and clinical applications. This particular long object problem (which may also be referred to as a quasishort object problem as in Ref. 9) is closely related to, but still different from, the common long object problem. While in the former problem one reconstructs only a short internal region (a heart) inside a long object (a patient), in the latter problem one reconstructs a long portion (a vascular tree) inside a long object (a patient). The practical implication is that one can use saddlelike scanning to solve the former problem optimally and spiral scanning to solve the latter problem optimally.

As explained in Ref. 9, the use of the saddlelike curve and recently developed cone-beam tomography theory gives an exact solution to this problem. Here the object to be reconstructed is “short,” and yet it is embedded in a large object. In this sense, it is a special type of the long object problem but not a short object problem because of the data truncation. The common saddle-curve scanning involves both circular and linear motions: The x-ray source is rotated in a circle around the object while it is also moved linearly back and forth.^{2,4,16} However, the common saddle-curve scanning mode is difficult to implement in practice, so a composite-

circling scanning mode was recently developed.^{3,9} The composite-circling scanning mode (Fig. 2 of Ref. 9) involves two circular motions: The x-ray tube is rotated in a circle around the object, while the x-ray focal spot is also rotated in a plane facing the object.⁹ In contrast to common saddle-curve cone-beam scanning, the x-ray focal spot undertakes a circular motion in a plane facing the short object to be reconstructed instead of a linear motion.

The benefits of the composite-circling mode have been discussed before.⁹ Since the electromechanical control is more challenging in converting a motor-driven rotation into a linear oscillation than staying with the circular motion, one can use the composite-circling mode to rotate an x-ray source and the associated detector collimator instead of moving both or either of them back and forth as required by the standard saddle curve scanning mode. Currently, either classic e-beam scanning or XinRay technology allows fabrication of a source focal spot in either circular or linear motion but the detector collimator is still needed to reject scattering. Hence, we prefer rotating the 2D detector collimator for that purpose rather than using a longitudinally oscillating x-ray source and aligning the collimator during a linear motion.

The composite-circling mode actually suggests a wide family of scanning trajectories to solve the cardiac CT problem, with the standard saddle-curve as a special case. Thus, we believe that this new mode seems to deserve further investigation for theoretical possibilities as well, and may lead to novel CT architectures in the future.

The detector size requirement is essentially the same as that in the case of standard saddle-curve cone-beam scanning if we use a stationary detector. However, since we are proposing to use a rotating detector and associated collimator, the detector size would be the same as in the case of circular cone-beam scanning.

According to Yu and Wang,⁹ a composite-circling curve is given by $y(t)=(y_1(t), y_2(t), y_3(t))$ with

$$\begin{aligned} y_1(t) &= R \cos t - r \sin t \sin mt, \\ y_2(t) &= R \sin t + r \cos t \sin mt, \\ y_3(t) &= r \cos mt \end{aligned} \quad (1)$$

for $0 \leq t \leq 2\pi$, where R is the radius of the x-ray tube rotation. Within the x-ray tube, there is also a small rotating circle with radius r over the object. The rotation speed along the small circle is m times of the speed of the x-ray tube rotation, where $m \geq 2$. When $m=2$, this composite-circling scanning curve is similar to the generalized saddle curve, which as defined by Pack *et al.*² as the intersection of two surfaces,

$$S_1 = \{(x, y, z) | z = f(x)\} \quad \text{and} \quad S_2 = \{(x, y, z) | z = g(x)\}, \quad (2)$$

with

$$f''(x) > 0 \text{ everywhere} \quad \text{and} \quad g''(y) < 0 \text{ everywhere}. \quad (3)$$

We note, however, that when a composite-circling curve (1) with $m=2$ is written as the intersection of S_1 and S_2 as in Eq.

FIG. 1. Wedge-shaped region when $y'(s_1)$, $y'(s_2)$, and chord L are not coplanar.

(2), condition (3) does not necessarily hold. The case of $R = 57$ cm, $r = 20$ cm, and $m = 2$ is such an example.

In order to reconstruct a region exactly from cone-beam projections, chords are commonly used.^{10,11} A chord is a line segment with two endpoints on the curve. Points on chords can be exactly reconstructed and easily implemented using the most recently developed techniques (see Refs. 2 and 13–15 and references listed in Ref. 12). We will focus on chord-based exact reconstruction in this paper and only point out that other reconstruction schemes may require the weaker condition of Tuy (cf. Ref. 17). The Tuy condition based region can be simply figured out as the convex hull of the trajectory.

The region of chord-based exactly reconstruction consists of all chords. To determine this region, all the chords on the region's boundary need to be considered. In this paper, we first determine the shape of the exact reconstruction regions inside composite-circling scanning curves using different parameters to graph the region's surfaces. Secondly, we compute the radii of the largest spheres centered at the origin inside composite-circling scanning curves and compare them to those of spheres inside saddle curves to see which type of curves can scan larger objects. Thirdly, we modify the original composite-circling scanning curves by varying the speed of the focal spot rotation to increase the exact reconstruction region. Finally, we discuss other relevant issues.

II. METHODOLOGY

II.A. Composite-circling scanning curves with constant speed

Let L be a chord, and let $y(s_1)$ and $y(s_2)$ be the endpoints of L . Let $y'(s_1)$ and $y'(s_2)$ be tangent vectors of the scanning curve at $y(s_1)$ and $y(s_2)$, respectively. If $y'(s_1)$, $y'(s_2)$, and L are not coplanar, then L cannot be on the boundary of the region because L is contained within a 3D wedge-shaped region consisting of chords (Fig. 1). Therefore, the boundary surface of the region of chord-based exact reconstruction consists of the chords L from $y(s_1)$ to $y(s_2)$ such that L , $y'(s_1)$, and $y'(s_2)$ are coplanar (Fig. 2).

In order to identify all such boundary chords and form the boundary surface, the radius of the largest ball embedded within the exact reconstruction region is calculated. This radius is simply the minimum distance from all boundary chords to the origin for the cases on hand.

FIG. 2. A surface chord L where $y'(s_1)$, $y'(s_2)$, and L are coplanar.

To find all boundary chords, we use an algorithm with two main loops. The outside loop takes s from 0 to 2π in small steps of length h . Inside this s -loop, we run a t -loop, with t going from 0 to 2π in small steps of the same length h . To determine if a chord from $y(s)$ to $y(t)$ is a boundary chord, we use the following process.

First, compute the tangent vectors of the curve. We need the tangent vectors to verify that $y'(t_1)$, $y'(t_2)$, and the chord from $y(t_1)$ to $y(t_2)$ are coplanar. Set

$$y'(t) = (y_1'(t), y_2'(t), y_3'(t)).$$

Then,

$$y_1'(t) = -R \sin t - mr \sin t \cos mt - r \cos t \sin mt,$$

$$y_2'(t) = R \cos t + mr \cos t \cos mt - r \sin t \sin mt,$$

$$y_3'(t) = -rm \sin mt.$$

Let $y'(t_1)$ be equal to (a_1, b_1, c_1) , let $y'(t_2)$ be (a_2, b_2, c_2) , and let (a_0, b_0, c_0) be the difference in vectors representing the endpoints of the chord. Next we determine whether

$$\begin{vmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0, \quad (4)$$

i.e., whether the three vectors $y'(t_1)$, $y'(t_2)$, and (a_0, b_0, c_0) are coplanar. Note that the absolute value of the determinant in Eq. (4) equals the volume of the parallelepiped determined by $y'(t_1)$, $y'(t_2)$, and (a_0, b_0, c_0) , which is a multiple of the volume of the wedge-shaped region in Fig. 1. This provides a proof of the arguments at the beginning of this section.

We note that while any boundary chord satisfies Eq. (4), the converse may not hold for general scanning curves or even for Eq. (1), together with Eqs. (6) and (7) below, when m is large. When $m=2, 4$, and 6, however, one can verify that Eq. (4) characterizes boundary chords using the symmetry of the scanning curves.

If Eq. (4) is true, then the distance from the line through $y(t_1)$ and $y(t_2)$ to the origin is determined. If $y(t_1) = (x_1, y_1, z_1)$ and $(t_2) = (x_2, y_2, z_2)$, the equation for the line passing through $y(t_1)$ and $y(t_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

Then, its distance to the origin is expressed as

$$d = \frac{\begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}.$$

Hence,

$$d^2 = \frac{(y_1 z_2 - y_2 z_1)^2 + (x_1 z_2 - x_2 z_1)^2 + (x_1 y_2 - x_2 y_1)^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

We need to find the smallest d , which is the radius of the largest ball inside the curve that can be covered entirely by chords. Note that this argument may not work for an arbitrary scanning curve, but it works for the curves considered in this paper.

Because of these properties, the boundary chords are symmetric in several ways. For example, let the ratio of the two circles' angular frequencies be $m=4$. Then, the parametric equations for the curve are

$$x = R \cos t - r \sin t \sin 4t,$$

$$y = R \sin t + r \cos t \sin 4t,$$

$$z = r \cos 4t.$$

The boundary of the exact reconstruction region consists of four surfaces (Fig. 3). The equations for the surface that consists of lines parallel to the y axis [Fig. 3(a)] are

$$x = R \cos t - r \sin t \sin 4t,$$

$$y = (R \sin t + r \cos t \sin 4t)v,$$

$$z = r \cos 4t,$$

where $-1 \leq v \leq 1$ and $0 \leq t \leq \pi$. On the other hand, the equations for the surface that consists of lines parallel to the x axis [Fig. 3(b)] are

$$x = (R \cos t - r \sin t \sin 4t)v,$$

$$y = R \sin t + r \cos t \sin 4t,$$

$$z = r \cos 4t,$$

where $-1 \leq v \leq 1$ and $-\pi/2 \leq t \leq \pi/2$. The equations for the surface that consists of lines in the $\pi/4$ direction [Fig. 3(c)] are

$$x = (R \cos t - r \sin t \sin 4t) \frac{v+1}{2} + \left(R \cos \left(\frac{3\pi}{2} - t \right) - r \sin \left(\frac{3\pi}{2} - t \right) \sin 4 \left(\frac{3\pi}{2} - t \right) \right) \frac{1-v}{2},$$

FIG. 3. Boundary surfaces of the region of reconstruction for $R=57$, $r=10$, and $m=4$.

$$y = (R \sin t + r \cos t \sin 4t) \frac{\nu+1}{2} + \left(R \sin \left(\frac{3\pi}{2} - t \right) + r \cos \left(\frac{3\pi}{2} - t \right) \sin 4 \left(\frac{3\pi}{2} - t \right) \right) \frac{1-\nu}{2},$$

$$z = r \cos 4t,$$

where $-1 \leq \nu \leq 1$ and $-\pi/4 \leq t \leq 3\pi/4$. Here the lines connect points at t and $3\pi/2 - t$. The equations for the surface that consists of lines in the $-\pi/4$ direction [Fig. 3(d)] are

$$x = (R \cos t - r \sin t \sin 4t) \frac{\nu+1}{2} + \left(R \cos \left(\frac{\pi}{2} - t \right) - r \sin \left(\frac{\pi}{2} - t \right) \sin 4 \left(\frac{\pi}{2} - t \right) \right) \frac{1-\nu}{2},$$

$$y = (R \sin t + r \cos t \sin 4t) \frac{\nu+1}{2} + \left(R \sin \left(\frac{\pi}{2} - t \right) + r \cos \left(\frac{\pi}{2} - t \right) \sin 4 \left(\frac{\pi}{2} - t \right) \right) \frac{1-\nu}{2},$$

FIG. 4. Composite-circling motions at (a) a constant speed v_r of the focal spot rotation and (b) a variable speed v_r , where v_R is the speed of gantry rotation and v_{comp} is the speed of the resulting composite-circling motion.

$$z = r \cos 4t, \quad (5)$$

where $-1 \leq \nu \leq 1$ and $-3\pi/4 \leq \pi/4$. So the lines connect points at t and $\pi/2 - t$.

II.B. Composite-circling scanning curves with variable speeds

When $m=2$, the constant-speed scanning curve works well because the radius of the largest embedded sphere is almost the same as the radius of the focal spot rotation when $r \leq 15$ cm. When $m=3$ and $m=5$, the radius becomes 0 because the boundary surfaces pass through the origin (Fig. 6). Thus the cases of odd m do not need to be considered for chord-based exact reconstruction. When $m=4$ and $m=6$, we propose varying the speed of the focal spot rotation and the associated detector rotation so that it rotates faster on top and slower at the bottom, according to $v_r = 4t + \sin 4t$ for $m=4$ and $v_r = 6t + \sin 6t$ for $m=6$.

Compared to composite-circling motion with a constant speed, as illustrated in Fig. 4(a), the variable-speed rotation of the focal spot and detector creates a flatter bottom on the scanning curve, as shown in Fig. 4(b), which allows a larger sphere to fit inside the curve. The scanning curve is now given by the following parametric equations:

$$x = R \cos t - r \sin t \sin(mt + \sin mt),$$

$$y = R \sin t + r \cos t \sin(mt + \sin mt),$$

$$z = r \cos(mt + \sin mt), \quad (6)$$

where $0 \leq t \leq 2\pi$ and $m=4, 6$. Figure 5 is such an example.

III. RESULTS

III.A. Largest embedded spheres within composite-circling scanning curves

The most clinically relevant parameter is the radius of the largest sphere embedded in the exact reconstruction region since it shows how large of an object can be covered by the CT scanner. Ideally, such an embedded sphere should contain and just contain the cardiac region to be reconstructed. We performed extensive numerical simulations with the results

FIG. 5. Composite-circling trajectory when $R=57$ cm, $r=16$ cm, and $m=4$ at a variable speed v_r .

summarized in Table I, where r represents the radius of the focal spot rotation and r_s is the radius of the largest sphere centered at the origin embedded in the exact reconstruction region. We set $R=57$ cm in all cases because it is commonly used in clinical CT scanners.

III.B. Largest embedded spheres within m -fold saddle curves

For comparison with composite-circling scanning curves, we also studied the m -fold saddle curve

$$\begin{aligned}x &= R \cos t, \\y &= R \sin t, \\z &= r \cos mt\end{aligned}\quad (7)$$

for $0 \leq t \leq 2\pi$. When $m=2$, Eq. (7) is the common saddle curve. As in the case of common saddle curve, the m -fold saddle-curve scanning also involves both circular and linear motions: The x-ray source is rotated in a circle around the object while it is also moved linearly back and forth. When $m=3$ and $m=5$, the radius of the largest sphere inside the exact reconstruction region is 0, which is the same as it is for composite-circling scanning curves. When $m=2$, $m=4$, and $m=6$ (with $R=57$ cm in all cases), the results are shown in Table II.

TABLE I. Radius (cm) of the largest embedded sphere in the chord-based exact reconstruction region for the composite-circling curve with $R=57$ cm.

r (cm)	5	10	15	20
$m=2$	5.0	10.0	14.9	16.4
$m=4$	5.0	8.0	6.0	2.7
$m=6$	5.0	3.9	0.23	0

TABLE II. Radius (cm) of the largest embedded sphere in the chord-based exact reconstruction region for the m -fold saddle curve with $R=57$ cm.

r (cm)	5	10	15	20
$m=2$	5.0	10.0	15.0	20.0
$m=4$	5.0	10.0	14.9	17.8
$m=6$	5.0	10.0	12.4	13.4

FIG. 6. Exact reconstruction region for the composite-circling curve with $R=57$ cm, $r=10$ cm, and $m=3$.

III.C. Comparison between composite-circling and m -fold saddle curves

In both composite-circling and m -fold saddle-curve scanning, when $m=3$ and $m=5$, the radius of the largest sphere is 0 because the boundary surface passes through the origin, as shown in Fig. 6. Therefore, composite-circling curves and m -fold saddle curves are not preferred for chord-based reconstruction when $m=3$ and $m=5$.

According to Tables I and II, both composite-circling curves and m -fold saddle curves provide large reconstruction regions when $m=2$ (Fig. 7). When $m=4$ and $m=6$, however, the composite-circling curves have smaller but adequate reconstruction regions. As seen in Figs. 8 and 9, the reconstruction regions become smaller because composite-circling curves make sharper edges at the bottom of reconstruction regions.

III.D. Largest embedded spheres within composite-circling scanning curves at variable speeds

As shown in Table III below, varying the speed of the focal spot rotation and the associated detector rotation can increase the radius of the largest sphere inside the exact reconstruction region significantly. Although the radii are still not as large as those for saddle curves, the differences become insignificant for most medical applications. The exact reconstruction regions with the largest embedded spheres are shown in Figs. 10 and 11.

IV. DISCUSSIONS AND CONCLUSION

In this paper, we showed that the radii of the largest embedded spheres centered at the origin inside the exact recon-

FIG. 7. Exact reconstruction region for the composite-circling curve with $R=57$ cm, $r=20$ cm, and $m=2$.

FIG. 8. Exact reconstruction region for the composite-circling curve with $R=57$ cm, $r=10$ cm, and $m=4$. (a) Top view. (b) Bottom view.

struction regions are slightly smaller in the case of composite-circling curves than those in the case of saddle curves. As shown by the graphs, composite-circling curves have sharper points and their region's surfaces have sharper edges on the bottom, causing the radii of the largest spheres in the regions to decrease. Nevertheless, the spheres inside the regions in the composite-circling mode are adequate for clinical cardiac CT, and the composite-circling trajectories

FIG. 9. Exact reconstruction region for the composite-circling curve with $R=57$ cm, $r=6$ cm, and $m=6$. (a) Top view. (b) Bottom view.

TABLE III. Radius of the largest embedded sphere for the $R=57$ cm at a variable speed.

r (cm)	5	10	15	20
$m=2$	5.0	10.0	15.0	20.0
$m=4$	5.0	10.0	15.0	13.5
$m=6$	5.0	10.0	7.1	2.77

provide another, hopefully easier, implementation option. While with conventional saddle-curve scanning, one can have 1D collimation, with composite circling scanning, one can have 2D collimation since the detector and collimator can be rotated in synchrony. However, both modes are potentially useful.

The largest region of reconstruction occurs when $m=2$. Note that the average human heart measures 12 cm in length and 9 cm in breadth. The heart can thus be covered using a composite-circling curve without varying the scanning speed when $r=10$ and $m=2$.

To improve the imaging performance of composite-circling scanning curves, we proposed varying the speed of the focal spot rotation and the associated detector rotation. We obtained encouraging results, with the scanning curve able to cover larger objects. Varying the speed is not necessary, but it is an interesting possibility. For example, with the CNT-based x-ray source technology, many focal spots can be distributed along a circle and fired rapidly and arbitrarily to simulate varying the rotation speed of the focal spot. Although the variable rotating rate is not necessary, it can be implemented using the XinRay technology (<http://www.xinraysystems.com/>). We acknowledge that rotating the detector at a variable speed under the proposed scheme is much harder to implement. Hopefully, the detector and collimator may become lighter in the future (we may also speculate a possibility of electronically controlling collimation somehow). Currently, we are actively exploring further possibilities, such as optimal variable-speed characteristics, multisource configuration, and extensions beyond the constraints imposed by chords. We will also conduct further studies on the formulation of exact and efficient algorithms for image reconstruction in this scenario.

This approach to cone-beam CT of short objects has significant advantages over existing cardiac CT methods and

FIG. 10. Exact reconstruction region when $R=57$ cm, $r=16$ cm, and $m=4$ at a variable speed (bottom view).

FIG. 11. Exact reconstruction region when $R=57$ cm, $r=11$ cm, and $m=6$ at variable speeds (bottom view).

over common saddle-curve-oriented systems with respect to both engineering implementation and clinical applications.

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¹G. N. Hounsfield, "Computerized transverse axial scanning (tomography) Part I. Description of system," *Br. J. Radiol.* **46**, 1016–1022 (1973).

²J. D. Pack, F. Noo, and H. Kudo, "Investigation of saddle trajectories for cardiac CT imaging in cone-beam geometry," *Phys. Med. Biol.* **49**(11), 2317–2336 (2004).

³G. Wang and H. Yu, "Scanning method and system for cardiac imaging," Patent disclosure submitted to *Virginia Tech. Intellectual Properties* (July

2007).

⁴H. Yu *et al.*, "Exact BPF and FBP algorithms for nonstandard saddle curves," *Med. Phys.* **32**(11), 3305–3312 (2005).

⁵W. A. Kalender, "Thin-section three-dimensional spiral CT: Is isotropic imaging possible?," *Radiology* **197**(3), 578–580 (1995).

⁶J. Chen *et al.*, "In vivo imaging of proteolytic activity in atherosclerosis," *Circulation* **105**, 2766–2771 (2002).

⁷P. L. Huang, "Endothelial nitric oxide synthase and endothelial dysfunction," *Current Hypertension Reports* **5**, 473–480 (2003).

⁸P. L. Huang, "Unraveling the links between diabetes, obesity, and cardiovascular disease," *Circ. Res.* **96**, 1129–1131 (2005).

⁹H. Yu and G. Wang, "Cone-beam composite-circling scan and exact image reconstruction for a quasi-short object," *Int. J. Biomed. Imaging*, Volume **2007**, (2007), Article ID 87319, 10 pages.

¹⁰P.-E. Danielsson, P. Edholm, J. Eriksson, M. Magnusson Seger, "Towards exact 3D-reconstruction for helical cone-beam scanning of long objects. A new detector arrangement and a new completeness condition," *International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine*, Pittsburgh, PA, 25–28 June 1997 (Bristol Inst. of Physics Publ., 1998), pp. 141–144.

¹¹K. C. Tam, "Method and apparatus for converting cone beam x-ray projection data to planar integral and reconstructing a three-dimensional computerized tomography (CT) image of an object," U.S. Patent No. 5,257,183 (1995).

¹²G. Wang *et al.*, "Topical review: Approximate and exact cone-beam reconstruction with standard and nonstandard spiral scanning," *Phys. Med. Biol.* **52**(6), R1–R13 (2007).

¹³A. Katsevich, "Theoretically exact filtered backprojection-type inversion algorithm for spiral CT," *SIAM J. Appl. Math.* **62**, 2012–2026 (2002).

¹⁴A. Katsevich, "A general scheme for constructing inversion algorithms for cone beam CT," *Int. J. Math. Math. Sci.* **21**, 1305–1321 (2003).

¹⁵A. Katsevich, "An improved exact filtered backprojection algorithm for spiral computed tomography," *Adv. Appl. Math.* **32**, 681–697 (2004).

¹⁶C. Bontus, R. Proksa, and T. Kohler, "New saddle trajectories for CT," *IEEE Nuclear Science Symposium Conference Record* (Piscataway, NJ: IEEE Service Center, 2006), Vol. 4, pp. 2309–2310.

¹⁷S. Cho, D. Xia, C. A. Pelizzari, and X. Pan, "Exact reconstruction of volumetric images in reverse helical cone-beam CT," *Med. Phys.* **35**(7), 3030–3040 (2008).