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Comparison of circular and polygonal planar orbits for pinhole and slit-slat SPECT

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Abstract

Analytic formulas and Monte Carlo simulation are used to calculate and compare the sensitivity of circular and polygonal orbits at different points in the Field of View (FoV) for both pinhole and slit-slat collimation. Results show that for a given slit-slat collimator an N -sided polygonal orbit tangent to the FoV generally provides average sensitivity lower than the tightest circular orbit consistent with the same aperture angle, but with better spatial resolution that can be traded for sensitivity for a constant-resolution comparison. This generally results in a slight advantage for the polygonal orbit. However, this advantage depends on the clearance that must be allowed between the orbit and the FoV and decreases quickly, vanishing when even a few millimeters of space are left, which in practice is necessary to accommodate mechanical constraints. For a pinhole collimator the advantage for the tangent polygonal orbit is more consistent, but similar conclusions are reached again when clearance is considered. A direct comparison at constant resolution between slit-slat and pinhole collimation in a single transverse plane is shown to be possible with parameters typical of small-animal imaging applications only for detectors with excellent intrinsic resolution; in this case, pinhole collimation is shown to be more sensitive in magnifying geometries, but reduced axial FoV and increased axial blurring should also be considered for a more complete comparison.

1. Introduction

The use of polygonal orbits (often called linear in the literature) has been suggested [1] and evaluated [2] because of the favorable computational speed in the face of equivalent accuracy of reconstruction as compared to classic filtered backprojection. More recently, the technique has been evaluated experimentally in clinical [3] and small-animal [4] settings. Slit-slat collimators combine parallel slats, which provide parallel collimation in the axial direction, with a slit running along the axial direction, which provides collimation much like a pinhole within each of the transverse slices formed by the slats. Slit-slat collimators have been applied in small-animal [4] as well as in cardiac and brain imaging [5]. Their use in combination with polygonal orbits has been reported to result in very substantial sensitivity advantages at constant resolution when compared to a single pinhole [4], although in a recent paper Metzler

et al. have shown that this advantage cannot be ascribed to the use of a slit-slat in place of a pinhole collimator [6].

This paper investigates whether the use of a polygonal instead of a circular orbit offers an advantage. In fact, since, as we shall see, polygonal orbits can be inscribed in a circular orbit while preserving data completeness for the same acceptance angle, a sensitivity advantage can possibly derive from the shorter distance of approach. However, due to truncation, not all points of the Field of View (FoV) contribute to projections at all times during the scan. Further, sensitivity may also be affected because polygonal orbits, especially if tight, see the FoV with incidence angles different from those of a circular orbit. Recent theoretical, computational and experimental work has provided and validated formulas for the resolution and sensitivity of slit-slat collimators [6]-[9]. These formulas, and their pinhole analogues, allow an analytical and numerical investigation of whether the gain in distance of approach overcomes the loss, if any, in incidence angle and dwelling time at different points in the FoV. For generality, both pinhole and slit-slat collimators are compared along circular and polygonal orbits.

In the following, completeness conditions and relevant literature formulas for sensitivity are briefly recalled to determine the orbits that will be compared later. Next, formulas are used to predict sensitivity at the center of the FoV for both collimators. Numerical integration is used to compare average sensitivities for the more relevant case of a disk source for both collimators and orbits. At first, these calculations compare the performance of a given collimator along different orbits and, thus, do not account for the different resolutions that either collimator provides along different orbits. Then, sensitivity results are rescaled to repeat comparisons at constant resolution. Finally, to gain further insight on the differences between the two collimators, an analytical and numerical comparison of performance on a single transverse plane is also presented: the resolution of a pinhole and a slit-slat collimator are matched and sensitivities are then directly compared at constant resolution.

In the next section, polygonal orbits are found to allow complete sampling even in the presence of truncation. It is then possible to consider the limiting case of polygonal orbits tangent to the FoV. Although tangent polygonal orbits can in principle be implemented with a Single-Knife-Edge (SKE) profile [10], the reader should be cautioned that they are not a realistic option because of the space necessary to accommodate, e.g., animal housing, anesthesia and the scanner's bed; they are considered because of theoretical convenience and as an upper performance limit for polygonal orbits. Since the results will show that polygonal orbits provide an advantage only in this case, and that this advantage vanishes when even the minimal (a few millimeters) clearance that is necessary in practical cases is allowed, the conclusion will be reached that polygonal orbits do not provide a sensitivity advantage except for unrealistic imaging scenarios or perhaps specialized applications.

2. Geometry and mathematical preliminaries

In a transverse plane, consider a circular FoV of radius RoV , scanned with a pinhole (or the slit of a slit-slat collimator. See Fig. 1). The pinhole (or slit) is assumed to have a SKE profile (for closer positioning) with a total acceptance angle α and width negligible compared to the size of the orbit for simplicity of the analytical discussion. The numerical and MC calculations below model all physical dimensions precisely.

Two orbits are considered around this FoV: a circular orbit of radius R_c and a regular polygon whose inscribed circle has radius R_p . Clearly $R_p \geq RoV$; in the extreme case $R_p = RoV$ the polygonal orbit is tangent to the FoV. A concrete example of the theoretical convenience of tangent polygonal orbits is found in the next section, where the completeness conditions in the plane of the orbits are investigated: proving that a tangent polygonal orbit is complete proves at once that wider polygonal orbits are complete too.

2.1. Sampling completeness

If the line integrals along all lines passing through all points of the FoV are collected at least once along the orbit, all parallel-beam projections of the object can be obtained by rearranging the data. Well-known theorems can then be applied to state that in the plane of the orbit a complete dataset is acquired [11].

The path of each line integral intersects any convex orbit at two (possibly coincident) points, where the path enters the pinhole (or slit) with incidence angle $\beta_{1,2}$ (see Fig. 2). All line integrals are collected if the minimum of β_1 and β_2 is less than $\alpha/2$ for each line. Thus, completeness is ensured if the maximum of all minimum β 's is less than or equal to $\alpha/2$. This condition applies to all types of orbit.

For a circular orbit, inspection of Fig. 2 shows that R_c must be larger than $RoV \csc(\alpha/2)$ for all line integrals to be collected. This condition also avoids data truncation. For an N -sided polygonal orbit, first consider the case $R_p = RoV$. Inspection of Fig. 2 shows that, for paths intersecting two adjacent sides, $\beta_1 + \beta_2 = \pi (1-2/N)$. The largest minimum β is obtained when $\beta_1 = \beta_2 = \pi (1/2 - 1/N)$, regardless of the polygonal orbit being tangent to the FoV.[†] The condition for a complete dataset is again $\alpha/2 \geq \beta$, i.e. $\alpha \geq \pi (1-2/N)$. Unlike for a circular orbit, in the case of a polygonal orbit this condition does not exclude truncation: data will be truncated, but will still be complete because all line integrals are collected at least once along the orbit.

In the more general and relevant case $R_p \geq RoV$ (polygonal orbits not tangent to the FoV), it is possible to observe that the FoV is a subset of the FoV tangent to the polygonal orbit considered, which is complete if $\alpha = \pi (1-2/N)$; therefore the smaller FoV considered is also complete.

In the following, it is assumed that designs will adopt the minimum α , $\alpha = \pi (1-2/N)$, e.g. to minimize penetration.

2.2. Sensitivity formulas

The sensitivity along the orbit can be evaluated by proper averaging of the point-source sensitivity of the pinhole [12],

$$g_p = \frac{l^2}{4\pi h^2} \sin^3 \theta, \quad (1)$$

or of the slit-slat collimator [7],

$$g_{ss} = \frac{wd^2}{4\pi (d+t) ah} \sin^3 \theta, \quad (2)$$

where the pinhole has been assumed square, with side l ; h is the distance from the point to the plane of the pinhole (or slit); $\theta = \pi/2 - \beta$ is the incidence angle; w is the slit width; and d is the gap between the slats, which have thickness t and height a (the dimension of the slats perpendicular to the plane of the detector).

[†]In the case of non-adjacent sides $\beta_1 + \beta_2 < \pi (1-2/N)$, which does not provide the largest minimum β (unless the relationship expressing the condition of FoV not intersecting the diagonals connecting second-nearest vertices, $R_p > RoV \cos(\pi/N) / \cos(2\pi/N)$, is satisfied, in which case the polygonal orbit is too large and is outside the range considered and described in Section 3.2).

These formulas are recognized to diverge for $h \rightarrow 0$; this is because their derivation assumes $h \gg l$ and $h \gg w$ for the pinhole and slit-slat collimator, respectively. If these conditions are not satisfied, more complex, non-diverging formulas should be used instead. For pinhole collimation, the exact expression of the solid angle subtended by a square opening [13] was used for numerical integration; (21) in [8] was used for slit-slat collimation. Of course, for $h \gg l$ and $h \gg w$, these formulas reduce to (1) and (2), respectively.

2.3. Resolution formulas

The Full Width at Half Maximum (FWHM) resolution formula for a pinhole is the same in both the axial and transaxial direction. It is given by [12]

$$\lambda_p = \sqrt{l^2 \left(1 + \frac{h}{f}\right)^2 + \lambda_i^2 \left(\frac{h}{f}\right)^2}, \quad (3)$$

where f is the distance from the pinhole to the detector (focal length) and λ_i is the intrinsic resolution of the detector, also measured in FWHM. Metzler *et al.* have proposed [6] and verified experimentally [9] that for slit-slat collimation (3) can be used transaxially (with w replacing l), i.e. in the direction collimated by the slit, whereas axially it is possible to use the same resolution expression of a parallel-hole collimator [12]:

$$\lambda_c = \sqrt{d^2 \left(\frac{h+f}{a}\right)^2 + \lambda_i^2}. \quad (4)$$

3. Sensitivity estimates

3.1. Analytical calculation

The formulas of the previous section estimate sensitivity at a point as a function of its location relative to the aperture. The calculation of sensitivity at a point averaged along the orbit is set up as the integral along the path of the orbit (of length L) of point source sensitivity:

$$\langle g \rangle = \frac{1}{L} \oint g[h(\xi), \theta(\xi)] d\xi. \quad (5)$$

For a point source at the center of the FoV, h remains constant along both types of orbits, although at two different values. The average, thus, is only an average over the incidence angle θ , which appears only as $\sin^3 \theta$. For example, for a polygonal orbit and a pinhole, (5) becomes

$$\langle g_p \rangle_{pol} = \frac{l^2}{4\pi h^2} \langle \sin^3 \theta \rangle = \frac{l^2}{4\pi h^2} \frac{1}{h \tan \frac{\pi}{N}} \int_0^{h \tan \frac{\pi}{N}} \sin^3 \theta(\xi) d\xi, \quad (6)$$

in which it was recognized that because of the symmetry of the problem only half of one side of the polygon needs to be considered. With our choice of α , this formula is valid for $N \geq 4$. It is also valid for $N = 3$ if $\alpha \geq 2\pi/3$ is assumed so that the center of the FoV is never truncated. From Fig. 1, with the source at the center, $\sin \theta = h / (h^2 + \xi^2)^{1/2}$; integration of (6) gives

$$\langle g_p \rangle_{pol} = \frac{l^2}{4\pi h^2} \cos \frac{\pi}{N}. \quad (7)$$

For the same orbit and a slit-slat collimator:

$$\langle g_{ss} \rangle_{pol} = \frac{wd^2}{4\pi (d+t) ah} \langle \sin^3 \theta \rangle = \frac{wd^2}{4\pi (d+t) ah} \cos \frac{\pi}{N}. \quad (8)$$

Integration along a circular orbit is trivial for a point source at the center because $\theta = \pi/2$. For a pinhole:

$$\langle g_p \rangle_{cir} = \frac{l^2}{4\pi h^2}; \quad (9)$$

for a slit-slat collimator:

$$\langle g_{ss} \rangle_{cir} = \frac{wd^2}{4\pi (d+t) ah}. \quad (10)$$

In (7) - (10) h represents the distance from the point source to the plane of the pinhole or the slit.

3.2. Choice of orbits

Once again, it is instructive to consider the theoretical case of a polygonal orbit tangent to the FoV, for which $R_p = RoV$. As previously discussed, mechanical constraints prevent the implementation of tangent orbits of any shape, but usually do not affect the tightest circular orbit, which is not tangent to the FoV and already allows sufficient clearance. Polygonal orbits of inscribed radius $R_p \geq RoV$, thus, were considered along with a single circular orbit of fixed radius $R_c = RoV \csc(\alpha/2)$ assumed as reference. For the same $\alpha = \pi(1-2/N)$ adopted for a polygonal orbit, $R_c = RoV \sec(\pi/N)$. When $R_p = R_c$ both orbits have the same clearance; the polygonal orbit lies entirely outside the circular orbit, which, then, is expected to provide better performance. This expectation can be supported, for example, by substituting the same value for h ($R_p = R_c$) in (7) - (10) and calculating that $\langle g \rangle_{pol} = \langle g \rangle_{cir} \cos(\pi/N)$ for both collimators, at any radius. More in general, $\langle g \rangle_{pol} / \langle g \rangle_{cir} = (R_c / R_p)^n \cos(\pi/N)$, where $n = 1$ for slit-slat collimators and $n = 2$ for pinhole collimators.

For these reasons, the following considers only the polygonal orbits that have inscribed radii in the range $RoV \leq R_p \leq RoV \sec(\pi/N)$. A summary of the orbits is sketched in Fig. 3, which also shows the point-source sensitivity ratio $\langle g \rangle_{pol} / \langle g \rangle_{cir}$ as a function of the clearance D allowed by the polygonal orbit, given by $D \equiv R_p - RoV$ for both a slit-slat and a pinhole collimator. Use of (7) - (10) shows that the ratio $\langle g \rangle_{pol} / \langle g \rangle_{cir}$ is independent of RoV . From its definition and the range of R_p , clearance is found to scale with RoV and, thus, is more conveniently and generally treated via the dimensionless variable $\Delta = D / RoV$. More specifically, $0 \leq \Delta \leq \sec(\pi/N) - 1$. For a point source, breakeven can be calculated to occur for $\Delta = 0$ in the case of a slit-slat collimator and $\Delta = [\sec(\pi/N)]^{1/2} - 1$ for pinholes.

3.3. Numerical integration

In the previous Section a first comparison between orbits has been attempted; it will be discussed more in depth in Section 4.1. It is noted here that several assumptions have been made. First, sensitivity was estimated only at the center of the FoV. A more indicative quantity is often an average over the whole FoV, which can be obtained by considering a disk source instead. Second, the collimator was assumed to be the same along both orbits. Whereas this comparison is relevant to decide on which orbit a pre-existing collimator should be used, an evaluation at constant resolution is also interesting for a direct comparison of different collimator designs. Section 3.5.1 describes how this was implemented.

An analytical treatment may be possible when certain combinations of these hypotheses are assumed, but it was not attempted. Instead, numerical integration was used with sensitivity formulas in their more complex versions that do not diverge for $h \rightarrow 0$, so that demanding cases such as tangential orbits could still be treated reliably for completeness at the minimal expense of having to implement more involved formulas. Two activity distributions were considered: a point source at the center of the FoV (to validate the results of Section 3.1) and a uniform disk covering the whole FoV (i.e. a disk source of radius RoV). For the latter distribution, emission points were chosen at random within the disk for each position of the aperture along an orbit. Next, the program evaluated truncation of the projection of the pinhole (or slit), both at its back side (i.e. facing the detector) and on the detector. Then, sensitivity was calculated according to the non-diverging formula applicable to the collimator considered (see Section 2.2). Finally, all results were summed and divided by the number of emission points within the source and vantage points along the orbit. A number of emission points resulting in less than 1% uncertainty was used. Results for the case of a pinhole scanning a disk source along a square orbit are shown in Fig. 4.

3.4. Monte Carlo simulation

Two sets of MC estimates were also obtained for validation. First, a simple MC program was written in C++ to simulate isotropic emission from a point source. The program calculated the intersection of the path of photons emitted from different points of the sources considered with the planes of the pinhole (or slit) and the detector; for slit-slat collimation, intersection with the slats was also evaluated. Photons passing through the collimator and hitting the detector were counted and sensitivity calculated as the fraction of emitted photons hitting the detector. Second, a GEANT4 program was used to simulate identical conditions for cross validation. GEANT4 runs typically followed 6×10^8 histories, leading to uncertainties of less than 1% in all cases. Scatter and penetration were modeled in neither program.

3.5. Matching resolutions via renormalization

3.5.1. Same collimator, different orbits—The main goal of this study is to determine if sensitivity is higher along a polygonal or a circular orbit for a given collimator. For a fair comparison, it is necessary to match resolution along both orbits. This is possible only approximately, because matching resolution at all points of the FoV everywhere along the orbit is impossible. Although the reconstructed point-spread function is not isotropic [3], for the point at the center of the FoV the geometric FWHM of the projection of an ideal point source on the detector is constant along both orbits. For these reasons, resolution was matched at the center of the FoV. The implications of this approximation were verified in the case of collimators having $w = 1$ mm, $l = 1$ mm, $RoV = 15$ mm, $\lambda_i = 3.5$ mm, $d = 1$ mm, $a = 90$ mm, $f = 100$ mm and $t = 0.1$ mm. These values are suitable for a mouse-imaging application. The detector was assumed to have a transaxial width of 398.72 mm and an axial length of 242.08 mm (Picker Prism 3000XP. Philips Medical Systems, Andover, MA). Figure 5 shows that matching resolution at the center also approximately matches resolution throughout the FoV and that a very minor adjustment is needed axially for the slit-slat collimator considered.

The orbits considered are those described in Section 3.2. Transaxial resolutions are matched by calculating the pinhole (or slit) width $l_{cir}(w_{cir})$ that results in the same resolution for the circular orbit as the resolution provided by the pinhole (or slit) width $l(w)$ along the corresponding polygonal orbit. $l_{cir}(w_{cir})$ is substituted with $h = R_c = RoV \sec(\pi/N)$ into (3); similarly $l(w)$ is substituted with $h = R_p$ also into (3), thus obtaining one equation for the circular and one for the polygonal orbit. $l_{cir}(w_{cir})$ is obtained by equating and solving for $l_{cir}(w_{cir})$. Finally, l_{cir} is used in place of l in (1) with $h = R_c = RoV \sec(\pi/N)$ to calculate the sensitivity along the circular orbit of a pinhole having the same resolution as a pinhole of width l scanning the polygonal orbit.

For a pinhole, this procedure will also match axial resolution. For a slit-slat collimator, axial resolution is matched by calculating the ratio $(d/a)_{cir}$ which results in the same resolution for the circular orbit as the resolution provided by the ratio d/a along the corresponding polygonal orbit. To maximize the sensitivity of the collimator, the resulting $(d/a)_{cir}$ is assumed to be implemented with the maximum a_{cir} compatible with the geometry, which is a , so that $d_{cir} = (d/a)_{cir}a$. This choice maximizes the ratio $d_{cir} / (d_{cir}+t)$ appearing in the expression of the sensitivity of the resolution-matching collimator, where t was assumed to be the same as in the non-matched case. To calculate the sensitivity along the circular orbit of a slit-slat collimator having the same resolution as a slit-slat collimator with slit aspect ratio d/a and slit width w scanning the polygonal orbit, $d_{cir} / (d_{cir}+t)$, $(d/a)_{cir}$ and w_{cir} are substituted into (2) for $d / (d+t)$, d/a and w , respectively, with $h = R_p$.

Since l_{cir} , $d_{cir} / (d_{cir}+t)$, $(d/a)_{cir}$ and w_{cir} are multiplicative factors, numerical estimates of resolution-matched sensitivity can be obtained by rescaling non-matched sensitivities, i.e. by dividing by l^2 , $d / (d+t)$, d/a and w^2 and multiplying by l_{cir}^2 , $d_{cir} / (d_{cir}+t)$, $(d/a)_{cir}$ and w_{cir}^2 .

3.5.2. Different collimator, same orbit—A direct comparison between pinhole and slit-slat collimation is also possible, but requires additional considerations. As long as the slit width w equals the pinhole size l , transaxial resolution is the same for a pinhole and a slit-slat collimator; to match the axial resolution as well, it is necessary to equate (3) and (4) and solve for some free parameter, chosen here to be the slit gap d , which is then defined to be the slit gap d_{ax} resulting in the same axial resolution characterizing the pinhole:

$$d_{ax} \equiv \frac{a}{f} \sqrt{w^2 + \frac{h-f}{h+f} \lambda_i^2}. \quad (11)$$

This shows that it is impossible to match the axial resolution if the argument of the radical is negative. From a physical point of view, this results from the magnification $m \equiv f/h$ reducing (i.e. improving) both the intrinsic and the geometric component of axial resolution in the case of the pinhole [see (3)]. A slit-slat collimator lacks magnification and, thus, can improve its axial resolution only by narrowing the slit gap d : this may not be enough to match the performance of the pinhole. Note that, unlike the pinhole collimator, the axial resolution of the slit-slat collimator is limited to the intrinsic resolution of the detector [see (4)]. All these considerations are reflected in (11), which is real for

$$\frac{w^2}{\lambda_i^2} \geq \frac{m-1}{m+1}. \quad (12)$$

In these cases, d_{ax} can be used to replace d in (2) to obtain sensitivity with axial resolution matching that of the pinhole. From (12), this is always possible, of course, in a non-magnifying geometry ($m \leq 1$), as is obvious from the discussion above, and if $w \geq \lambda_i$, i.e., in intuitive terms, if the resolution of the pinhole is not too fine. If $w < \lambda_i$, it is possible to place a point source sufficiently close to the aperture that magnification is so high that the axial resolution of the pinhole collimator improves too much for the slit-slat collimator to match the improvement. With these observations, from (1) and (2) the ratio of the average sensitivities over the entire FoV of a pinhole and slit-slat collimator is, for orbits far enough from the FoV for (1) and (2) to apply everywhere,

$$\frac{\langle\langle g_p \rangle\rangle}{\langle\langle g_{ss} \rangle\rangle} = w a \frac{d_{ax} + t}{d_{ax}^2} \frac{\int_0^{R_p \tan \frac{\pi}{N}} \iint_{FoV} \frac{\sin^3 \theta}{h^2} dA d\xi}{\int_0^{R_p \tan \frac{\pi}{N}} \iint_{FoV} \frac{\sin^3 \theta}{h} dA d\xi}, \quad (13)$$

where the outermost integral averages sensitivity over a polygonal orbit having N sides [see (5)].

If resolution is matched at the center of the FoV, it is possible to replace d_{ax} with (11) and reach

$$\frac{\langle\langle g_p \rangle\rangle}{\langle\langle g_{ss} \rangle\rangle} = f \frac{d_{ax} + t}{d_{ax}} \left[1 + \frac{1 - m_0}{1 + m_0} \frac{\lambda_i^2}{w^2} \right]^{-\frac{1}{2}} \frac{\int_0^{R_p \tan \frac{\pi}{N}} \iint_{FoV} \frac{\sin^3 \theta}{h^2} dA d\xi}{\int_0^{R_p \tan \frac{\pi}{N}} \iint_{FoV} \frac{\sin^3 \theta}{h} dA d\xi}, \quad (14)$$

where $m_0 \equiv f/R_p$, which is magnification at the center of the FoV. Since transverse resolution is also matched, it makes sense to define $\lambda \equiv \lambda_p = \lambda_c$ [for whose definitions see (3) and (4)].

Next, it is possible to derive from (3) that $w^2/\lambda_i^2 = (m_0^2 \lambda^2 / \lambda_i^2 - 1)/(1 + m_0)^2$, which can be substituted into (12) and (14) to replace w with λ if $m_0 \geq \lambda_i / \lambda$. Further, if $t \ll d_{ax}$, e.g. in the common case of low energy,

$$\frac{\langle\langle g_p \rangle\rangle}{\langle\langle g_{ss} \rangle\rangle} = \left[\frac{R_p \int_0^{R_p \tan \frac{\pi}{N}} \iint_{FoV} \frac{\sin^3 \theta}{h^2} dA d\xi}{\int_0^{R_p \tan \frac{\pi}{N}} \iint_{FoV} \frac{\sin^3 \theta}{h} dA d\xi} \right] \sqrt{\frac{m_0^2 - \frac{\lambda_i^2}{\lambda^2}}{1 - \frac{\lambda_i^2}{\lambda^2}}}, \quad (15)$$

with (12) yielding the condition $\lambda \geq \lambda_i$. The factor in the brackets is only a function of the shape of the orbit (i.e. N) and the ratio $r \equiv RoV/R_p$, and is indicated with $C(r, N)$, where, by its definition, $0 \leq r \leq 1$. We did not seek an analytical expression for C in the general case of polygonal orbits because, if one exists, it is likely to be rather complex. For these orbits only numerical results were pursued. Valuable insights were gained from analytical treatment of the particular case of circular orbits, in which $N \rightarrow \infty$ and the average over the orbit becomes unnecessary. The Appendix provides a summary of the derivation of an analytic form for C

(r, ∞) . Further, if the aperture angle α is assumed to be the minimum compatible with no truncation, r and α are directly connected because $r = RoV/R_c = \sin(\alpha/2)$ (see Section 3.2) and C becomes a sole function of α , so that

$$\frac{\langle\langle g_p \rangle\rangle}{\langle\langle g_s \rangle\rangle} = C(\alpha) \sqrt{\frac{m_0^2 - \frac{\lambda_i^2}{\lambda^2}}{1 - \frac{\lambda_i^2}{\lambda^2}}}, \quad (16)$$

where $C(\alpha)$ is used as shorthand for $C(r, \infty)$.

$C(\alpha)$ can be used as a rescaling factor for the ratio of the sensitivities. The normalized ratio of the sensitivities depends only on m_0 and λ_i/λ . This relationship is shown in Fig. 6.

The expression of $C(\alpha)$ is cumbersome. A useful approximation is $C(\alpha) = 1.3743 + 0.5758 \Delta\alpha + 0.3381 \Delta\alpha^2$, where $\Delta\alpha = \alpha - \pi/2$ is expressed in radians. Figure 7 shows the range of applicability of this approximation, which is accurate to 1% between 61.9° and 116.6° , to 5% between 42.3° and 135.0° and to 10% between 29.2° and 146.1° .

Figure 7 also shows numerical results for square, hexagonal and octagonal orbits, which are found not to deviate significantly from the circular orbit case, especially for practical values of r .

4. Results and discussion

4.1. Same collimator, different orbits

Figure 3 (right) shows the ratio of the point-source sensitivity along a hexagonal orbit to the sensitivity along the reference circular orbit as the hexagon's size increases from tangent to the FoV to tangent to the reference circular orbit. Numerical calculations agree with analytical estimates and provide a first validation of the methods. As expected, a polygonal orbit performs best when it can be kept close to the FoV. For a given slit-slat collimator, there is no advantage over the circular orbit even if it were possible to scan along a polygonal orbit tangent to the FoV: along this orbit performance would be equivalent for the two orbits. Some advantage is present for a pinhole, but, as clearance increases, sensitivity along the polygonal orbit decreases, as expected from the inverse relationship between sensitivity and distance from the FoV. Since the circular orbit already allows a larger clearance, its radius remains fixed to the only radius considered for circular orbits. Thus, the sensitivity ratio decreases. Figure 3 also shows that, if a clearance of a little more than 1 mm must be allowed about a 15-mm-radius FoV, the polygonal orbit loses all advantage as breakeven with a circular orbit is reached. The faster decrease for a pinhole derives from its sensitivity being proportional to h^{-2} and, thus, decreasing more than that of a slit-slat collimator (h^{-1}) for increasing clearance. It is also expected, and Fig. 3 confirms, that for the maximum clearance considered (i.e. that allowed by the circular orbit) polygonal orbits have worse performance than circular orbits, confirming that the clearance range considered includes the breakeven point, if present. Figure 4 presents similar results for the more relevant case of a disk source and the most favorable case for the polygonal orbit (square orbit for a pinhole – see Table I). Note how the advantage vanishes if only 3 mm of clearance must be allowed. This is true regardless of resolution matching, which Fig. 4, unlike Fig. 3, considers along with the non-matched case.

These first results are indicative of typical behavior and confirm that the sensitivity at the extremes of the clearance range considered includes most of the information conveyed by plots such as Fig. 3 and Fig. 4. Instead of producing plots for the many cases considered, the

sensitivity for different orbits and collimators for both ends of the range was summarized in Table I, which gives the absolute value of sensitivity for the polygonal orbit for both a point source at the center of the FoV and a disk source. The collimator parameters assumed are those mentioned in Section 3.5.1. The next two columns provide the relative sensitivity $\langle g \rangle_{pol} / \langle g \rangle_{cir}$ when collimator parameters are the same on both orbits. The last two columns list the same ratio after rescaling sensitivity to match resolution as described in Section 3.5.1. All MC results were within statistical uncertainty ($<1\%$) from the numerical results of Table I and, thus, are not reported.

Inspection of Table I suggests several observations. We consider first the case in which resolution is not matched. First, absolute sensitivity generally increases with N . This is expected for the circular orbit because R_c decreases and for the polygonal orbit because the corners of the polygonal orbits come closer to the FoV. Second, a polygonal orbit always gives better sensitivity than the corresponding circular orbit for a pinhole and $\Delta = 0$; under the same conditions a polygonal orbit gives at most the same sensitivity as a circular orbit for slit-slat collimation. Third, the prediction that at the center of the FoV the sensitivity of a slit-slat collimator is the same along a circular or a tangent polygonal orbit is confirmed. Also, relative sensitivity for a pinhole and a point source at the center of the FoV is indeed predicted by $\langle g_p \rangle_{pol} / \langle g_p \rangle_{cir} = \sec(\pi/N)$ for $\Delta = 0$; for maximum clearance, $\langle g \rangle_{pol} / \langle g \rangle_{cir} = \cos(\pi/N)$ for both collimators. Lastly, the prediction that for maximum Δ the sensitivity ratio is less than 1 is verified.

Results at constant resolution for different orbits show that polygonal orbits always provide a sensitivity advantage at constant transverse (and axial for a slit-slat collimator) FoV for $\Delta = 0$. Axial resolution was not matched across collimators and was, e.g., 1.32 mm and 3.73 mm at 15 mm, respectively for a pinhole and a slit-slat collimator along the largest hexagonal orbit. In the small-animal imaging configuration considered, the advantage is modest for a slit-slat collimator (at most 24%), but potentially significant for pinholes (as much as 146% in the case of a pinhole on a tangent square orbit around a disk source).

4.2. Different collimator, same orbit

According to (12), with the values of the parameters used for the case study presented here, resolution at the center of the FoV can be matched for different collimators on the same orbit only if $w > 3.01$ mm (for $\lambda_i = 3.5$ mm) or $\lambda_i < 1.16$ mm (for $w = 1$ mm). To allow a direct comparison between collimators the case was made of a detector with better intrinsic resolution than the clinical Anger camera considered so far. $\lambda_i = 1$ mm was assumed and the slit-slat sensitivity data of Table I scaled by dividing by $d^2/(d+t)$ and multiplying by $d_{ax}^2/(d_{ax}+t)$. The results for polygonal and circular orbits are both reported in Table II, again in the case of point and disk activity distributions. Sample MC data (for $\Delta = 0$) are also shown to validate the rescaling procedure.

The results of Table II show a consistent sensitivity advantage for the pinhole. This derives from the need to use very closely spaced slats to match axial resolution. However, it must be re-emphasized that this comparison involves only the plane of the orbit and that the two collimators have different axial FoV and data completeness properties, which should be considered for a comprehensive comparison.

4.3. Other considerations

The conclusions above hold in a small-animal imaging geometry. The analysis developed in Section 3.5.2 has removed scale factors and is thus suitable to attempt a more general conclusion. Inspection of Fig. 6 indicates that pinholes have better sensitivity for the same resolution in a magnifying geometry; the opposite is true in a minifying geometry. Application

of this qualitative general principle, however, should take into account the factor $C(r, N)$. For circular orbits and typical values of α (say, $60^\circ < \alpha < 120^\circ$) $1.15 < C(\alpha) < 1.79$. Since $C(\alpha) \geq 1$, the advantage of the pinhole may be present also in some minifying conditions. From (16), the breakeven magnification for which sensitivity is the same for both collimators is

$$m_0 = \sqrt{\frac{1}{C^2} \left(1 - \frac{\lambda_i^2}{\lambda^2} \right) + \frac{\lambda_i^2}{\lambda^2}}. \quad (17)$$

For polygonal orbits, C takes on systematically lower values, but remains consistently larger than 1. Thus, the result does not change qualitatively. Quantitatively, the advantage of the pinhole in minifying conditions will still be present, but will not be as large as for circular orbits.

Absolute sensitivity increases with increasing N and, thus, increasing α . For this reason, it is advantageous to use orbits with the largest α practical for the application at hand. This is well known for pinholes and circular orbits. For triangular orbits, $\alpha = 60^\circ$; this is the largest practical aperture only in specialized applications. If a larger aperture is still practical, its use provides higher sensitivity. If $\alpha = 90^\circ$ can be used, as in most applications, a square orbit becomes available and provides better sensitivity: for a pinhole and a disk source, the parameters used in this study yield a sensitivity of 2.50×10^{-4} and 3.87×10^{-4} , respectively for triangular and square tangent orbits. For comparison, the sensitivity of the triangular tangent orbit for $\alpha = 60^\circ$ is 1.82×10^{-4} . For this reason, triangular orbits were not considered further in this study; however, they may be of interest under different assumptions.

Numerical results not reported for brevity show that values of α larger than those necessary for completeness increase the sensitivity of circular orbits (because of the smaller radius) more than that of polygonal orbits (because of reduced truncation). Thus, the choice made, for each polygon, of the smallest α compatible with completeness is consistent with the best-case scenario for polygonal orbits assumed throughout.

Tangent polygonal orbits were considered in spite of the difficulty or impossibility of their implementation for their theoretical interest as the orbits favoring the sensitivity of polygonal orbits the most. Circular orbits were found to be a better alternative possibly in all practical cases. Another potential drawback of tangent polygonal orbits is the difficulty to perform stable image reconstruction throughout the FoV. In light of the sensitivity considerations, that tend to exclude the use of tangent polygonal orbits anyway, this aspect was not pursued any further.

A complete comparison involves several aspects: sensitivity and resolution are perhaps the most popular, but others, such as FoV and sampling completeness, are equally important. As already pointed out, in exchange for the limited axial resolution, the slit-slat collimator offers a much longer axial FoV, which is also completely sampled. For the configuration considered, the axial FoV at the center of the transaxial FoV was 51.4 mm at most (circular orbit circumscribed to a square orbit itself tangent to the FoV, $RoR = 21.2$ mm) for a pinhole and the full axial FoV of the detector, 242.08 mm, for the slit-slat collimator. Further, long axial FoVs favor slit-slat collimation because sensitivity remains constant on all transaxial planes, unlike in pinhole collimation.

5. Conclusions

The choice of the best collimator for a given application should consider all of the parameters involved. Slit-slat collimation appears to be a competitive option when a relatively long axial

FoV is needed; this is likely to include the case of mid-size objects, such as single-organ studies in adults, and mid-size animals. For small animals, pinhole (or multi-pinhole) collimation may be a better option, depending on the axial FoV needed.

The sensitivity of a pinhole and a slit-slat collimator were compared along polygonal and circular orbits. The performance of the same collimator along different orbits shows that polygonal orbits could result in sensitivity advantages only in the case of the pinhole collimator and only on very tight polygonal orbits. However, the advantage vanishes if, as is likely, a clearance of even only a few millimeters must be allowed to accommodate mechanical constraints such as animal pallet, heating and housing equipment. When comparisons are performed at constant resolution, the potential sensitivity advantage increases and could also be present for the slit-slat collimator, but the same observation on the necessary clearance leads to the same conclusion that it is unlikely that the advantage will be achieved in realistic cases. In summary, polygonal orbits offer only a theoretical advantage that will be difficult to achieve in practice.

Considering penetration complicates the theoretical model, which can be validated with straightforward changes in MC simulation. Modeling penetration for the common case of ^{99m}Tc does not change qualitatively the results.

A theoretical comparison of a pinhole and a slit-slat collimator with the same resolution on the same orbit is possible for small-animal resolutions only for detectors with excellent intrinsic resolution. The comparison shows that the pinhole is likely to outperform the slit-slat collimator in magnifying geometry regardless of the shape of the orbit. This comparison assumed a volume of interest with a limited axial extent. If an imaging volume axially longer is needed, two potential disadvantages should be considered: reduced axial FoV and increased axial blurring outside the central plane due to data incompleteness. Their investigation is outside the scope of the analysis presented here.

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Appendix

Since both integrals in (14) have analytic expressions for circular orbits, it is possible to have an exact expression for $C(\alpha)$. Briefly, the integrals can be calculated in the Cartesian coordinates h and $x = h \cot \theta$. Integration over x is simple; the second integration involves reduction to elliptic integrals of the first, second and third kind. The definitions herein adopted are:

$$F(\phi, k) \equiv \int_0^\phi \frac{d\alpha}{\sqrt{1 - k^2 \sin^2 \alpha}}, \quad (18)$$

$$E(\phi, k) \equiv \int_0^\phi \sqrt{1 - k^2 \sin^2 \alpha} d\alpha, \quad (19)$$

and

$$\Pi(\phi, n, k) \equiv \int_0^\phi \frac{d\alpha}{(1 - n \sin^2 \alpha) \sqrt{1 - k^2 \sin^2 \alpha}}. \quad (20)$$

The result of the integration involves only the case $\phi = \pi/2$. Thus, complete elliptic integrals can be used; following standard notation, these are $F(\pi/2, k) \equiv K(k)$ and $E(\pi/2, k) \equiv E(k)$. Finally, with $u = \csc(\alpha/2)$ and $k = 2 u^{1/2}/(u+1)$:

$$C(\alpha) \equiv \frac{R_c \iint_{\text{FoV}} \frac{\sin^3 \theta}{h^2} dA}{\iint_{\text{FoV}} \frac{\sin^3 \theta}{h} dA} = \frac{3u^2}{(u+1)^2} \cdot \frac{-(u+1)^2 E(k) + (3u^2+1) K(k) + 2u(1-u) \Pi(\frac{\pi}{2}, \frac{2}{1+u}, k)}{(u^2+1) E(k) - (u-1)^2 K(k)}. \quad (21)$$

This is a function of the only parameter α and is shown in Fig. 7, which also shows the results of numerical integration for validation. It is interesting to note that for a point source at the center of the FoV the integrals are readily obtained and yield $C = 1$ (independent of α). Thus, the finding that $C(0) = 1$ is expected, because for $\alpha = 0$ the aperture is infinitely far from the FoV and a geometry equivalent to that of a point source is obtained.

Taylor expansion of (21) about, e.g., $\alpha = 90^\circ$ can be obtained numerically. The resulting expression was given in Section 3.5.2 and is also shown in Fig. 7.

At higher energy, or if the slats must be kept very close to match axial resolution, the ratio $(d_{ax+t}) / d_{ax}$ in (14) should also be considered. It can be incorporated analytically in the expression of $C(\alpha)$, which then becomes a function of the ratio t/d_{ax} as well.

References

1. Edholm PR, Herman GT. Linograms in image reconstruction from projections. *IEEE Trans Med Imag* 1987;MI-6:301–307.
2. Edholm PR, Herman GT, Roberts DA. Image reconstruction from linograms: implementation and evaluation. *IEEE Trans Med Imag* 1988;MI-7:239–246.
3. Walrand S, van Dulmen A, van Rossem H, Pauwels S. Acquisition of linograms in SPET: implementation and benefits. *Eur J Nucl Med* 2002;29:1188–1197.
4. Walrand S, Jamar F, de Jong M, Pauwels S. Evaluation of novel whole-body high-resolution rodent SPECT (linoview) based on direct acquisition of linogram projections. *J Nucl Med* 2005;46:1872–1880. [PubMed: 16269602]
5. Rogers WL, et al. SPRINT: A stationary detector single photon ring tomography for brain imaging. *IEEE Trans Med Imag* 1982;MI-1:63–68.
6. Metzler SD, Accorsi R, Novak JR, Ayan AS, Jaszczak RJ. On-axis sensitivity and resolution of a slit-slat collimator. *J Nuc Med* 2006;47:1884–1890.
7. Wieczorek H, Goedicke A. Analytical model for SPECT detector concepts. *IEEE Trans Nuc Sci* 2006;53:1102–1110.
8. Accorsi R, Novak JR, Ayan AS, Metzler SD. Derivation and validation of a sensitivity formula for slit-slat collimation. *IEEE Trans Med Imag* 2008;27:709–722.
9. Novak JR, Ayan AS, Accorsi R, Metzler SD. Verification of the sensitivity and resolution dependence on the incidence angle for slit-slat collimation. *Phys Med Biol* 2008;53:953–966. [PubMed: 18263951]
10. Accorsi R, Metzler SD. Resolution effective diameters for asymmetric-knife-edge pinhole collimators. *IEEE Trans Med Imag* 2005;25:1637–1646.
11. Orlov SS. Theory of three dimensional reconstruction. *Sov Phys Crystallogr* 1975;20:312–314.

12. Anger, HO. Radioisotope cameras. In: Hine, GJ., editor. Instrumentation in Nuclear Medicine. Vol. 1. New York, NY: Academic Press; 1967. p. 485-552.
13. Gotoh H, Yagi H. Solid angle subtended by a rectangular slit. Nucl Instrum Methods 1971;96:485–486.

Fig. 1. Circular (dotted) and polygonal (dashed) orbit about a round FoV (solid). The case $N = 6$ (hexagonal orbit) was considered. To illustrate orientation relative to the object, a pinhole is shown at two different points along the polygonal orbit. Note that the distance to the plane of the aperture h is different for the two orbits.

Fig. 2. Truncation conditions for orbits (dotted) around the circular field of view of radius RoV (solid). Line integrals (dashed) intersect convex orbits at two points (possibly coincident). Dash-dot lines indicate the centerline normal to the plane of the aperture. (Left) Circular orbit of radius R_c (dotted circle). (Right) Regular polygon with $N = 8$ sides (octagon, dotted lines).

Fig. 3.

(Left) Orbits considered about a circular FoV (solid) of radius RoV . The case $N = 6$ (hexagonal orbit) was considered. The tightest circular orbit (dots) for $\alpha = \pi (1-2/N) = 120^\circ$ has radius $R_c = RoV \sec(\pi/N) = 2 RoV / \sqrt{3}$. A range of polygonal orbits from the tangent polygonal orbit (dots, $R_{p,min} = RoV$) to the orbit internally tangent to the circular orbit (dash, $R_{p,max} = R_c$) covers the shaded area. The generic polygonal orbit in this region is denoted by dash-dots. (Right) Plot of the ratio of the point-source sensitivities $\langle g \rangle_{pol} / \langle g \rangle_{cir}$ as a function of dimensionless clearance $\Delta \equiv D / RoV = R_p / RoV - 1$, which ranges from 0 to $\sec(\pi/N) - 1 \approx 0.1547$. Numerical data (isolated points) validate the analytical model (lines) for two collimators and two different sizes of the FoV (15 and 30 mm). Resolution matching was not considered at this stage.

Fig. 4.

Sensitivity ratio for a pinhole, disk source and square orbit relative to the reference circular orbit as a function of the clearance $D \equiv R_p - RoV$ and the dimensionless clearance $\Delta \equiv D / RoV = R_p / RoV - 1$. The baseline values (circles) at $\Delta = 0$ and $\Delta = \sec(\pi/4) - 1 = 0.4142$ (maximum Δ) correspond to those found in Table I. Resolution matching follows Section 3.5.1. The RoV assumed was 15 mm. All parameters are indicated at the beginning of Section 3.5.1.

Fig. 5.

Transaxial (left, from (3)) and axial (right, from (4)) resolutions as a function of position in the FoV with nominal (top, w and d/a) and adjusted parameters (bottom, w_{cir} and $(d/a)_{cir}$) for a circular (solid) and a polygonal (dash-dot) orbit. The aperture is located at negative distances from the center of the FoV (a circle of radius 15 mm). The case of a square orbit ($N=4$) is shown. Parameters were adjusted to have the circular orbit match the resolution of the polygonal orbit, which is the same at the top and bottom. For a pinhole, transaxial and axial resolutions coincide and are both read on the left. The right column shows axial resolution for the slit-slat collimator: with the adjusted parameters, the two curves are not easily distinguished.

Fig. 6.

Normalized ratio of the sensitivities of a pinhole and a slit-slat collimator as a function of magnification at the center of the FoV (16). Both transaxial and axial resolutions were matched. A circular orbit and $d_{ax} \gg t$ were assumed. The dash-dot and dash lines indicate the curve for $\lambda_i / \lambda = 0$ and 0.99 , respectively. Horizontal dotted lines show $1/C(\alpha)$ for five values of α : their intersection with the other lines indicates the breakeven magnification, for which $\langle g_p \rangle = \langle g_{ss} \rangle$; see (17).

Fig. 7.

Exact (solid, 21) vs. approximate (dotted) expression of the coefficient $C(r, \infty) = C(\alpha)$. Isolated dots show the result of numerical integration. Isolated squares, crosses and diamonds show the result of numerical integration for square $[C(r, 4)]$, hexagonal $[C(r, 6)]$ and octagonal $[C(r, 8)]$ orbits, respectively. The top axis shows $\alpha = 2 \arcsin r$ and is meaningful only for the case of circular orbit.

Table I
Absolute and Relative Sensitivities of Different Orbits and Collimators for Different Activity Distributions in the FoV.

Orbit	Absolute Sensitivity (polygonal orbit)					Relative sensitivity ^a		Relative sensitivity, resolution adjusted	
	Δ^b	N	$\alpha(^{\circ})$	Center	Disk	Center	Disk		
Pinhole									
Square	0	4	90	$2.49 \cdot 10^{-4}$	$3.97 \cdot 10^{-4}$	1.41	1.75	1.98	2.46
	0.4142			$1.25 \cdot 10^{-4}$	$1.35 \cdot 10^{-4}$	0.71	0.60	0.71	0.60
Hexagon	0	6	120	$3.06 \cdot 10^{-4}$	$6.29 \cdot 10^{-4}$	1.15	1.49	1.29	1.67
	0.1547			$2.29 \cdot 10^{-4}$	$3.36 \cdot 10^{-4}$	0.87	0.79	0.87	0.79
Octagon	0	8	135	$3.26 \cdot 10^{-4}$	$7.34 \cdot 10^{-4}$	1.08	1.32	1.15	1.40
	0.0824			$2.79 \cdot 10^{-4}$	$4.75 \cdot 10^{-4}$	0.92	0.86	0.92	0.86
Slit slit									
Square	0	4	90	$3.78 \cdot 10^{-5}$	$3.13 \cdot 10^{-5}$	1.00	0.89	1.24	1.10
	0.4142			$2.67 \cdot 10^{-5}$	$2.38 \cdot 10^{-5}$	0.71	0.67	0.71	0.67
Hexagon	0	6	120	$4.64 \cdot 10^{-5}$	$4.06 \cdot 10^{-5}$	1.00	0.98	1.08	1.05
	0.1547			$4.02 \cdot 10^{-5}$	$3.67 \cdot 10^{-5}$	0.87	0.88	0.87	0.88
Octagon	0	8	135	$4.95 \cdot 10^{-5}$	$4.27 \cdot 10^{-5}$	1.00	0.99	1.04	1.03
	0.0824			$4.57 \cdot 10^{-5}$	$4.07 \cdot 10^{-5}$	0.92	0.94	0.92	0.94

^aSensitivity is relative to the sensitivity along the circular orbit, i.e. $\langle g>_{pol}/\langle g>_{cir}$.

^bThe maximum clearance is given by Δ R_{oV} with $\Delta = \lfloor \sec(\pi/N) - 1 \rfloor$. The R_{oV} considered was 15 mm.

Table II
Pinhole Sensitivity Relative^a to a Slit-Slat Collimator for the Same Resolution,^b Orbit and Different Activity Distributions in the FoV.

Orbit	Relative sensitivity (polygonal)				Relative sensitivity (circular)	
	A	N	$\alpha(^{\circ})$	Center	Disk	Disk
Theory						
Square	0	4	90	15.9	30.5	13.0
	0.4142			9.5	11.5	13.0
Hexagonal	0	6	120	15.9	37.4	22.8
	0.1547			12.8	20.5	22.8
Octagonal	0	8	135	15.9	41.4	29.7
	0.0824			14.1	27.0	29.7
Relative difference from Monte Carlo (GEANT4) ^c						
Square	0	4	90	0.01	0.00	-0.02
Hexagonal	0	6	120	0.01	0.00	0.01
Octagonal	0	8	135	-0.01	0.01	0.00

^aThe table reports $\langle g_p \rangle / \langle g_{ss} \rangle$ for $\lambda_i = 1$ mm and $RoV = 15$ mm.
^bAxial and transaxial resolutions were matched at the center of the FoV.
^cThe table reports $[(\langle g_p \rangle / \langle g_{ss} \rangle)MC - (\langle g_p \rangle / \langle g_{ss} \rangle)num] / (\langle g_p \rangle / \langle g_{ss} \rangle)num$. The uncertainty of this ratio due to MC statistics is ± 0.03 or less.