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Ultrasonically-Induced Lorentz Force Tomography

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Abstract

Electrical conductivity can be measured using the ultrasonically-induced Lorentz force. An ultrasonic wave is passed through tissue in the presence of a magnetic field. Moving charges in a magnetic field are subject to the Lorentz force, which acts as the source of current and potential. This paper shows that ultrasonically-induced Lorentz force imaging can be formulated in a way that makes it similar to tomography: an image can be reconstructed using waves propagating in various directions. More specifically, measuring the dipole strength for a particular direction and wavelength is equivalent to measuring the Fourier transform of the conductivity distribution at one point in frequency space. Measurements at a variety of wavelengths and directions are equivalent to mapping the Fourier transform of the conductivity distribution. The conductivity can then be found by an inverse Fourier transform.

Keywords

Ultrasound; Conductivity; Lorentz Force; Tomography; Fourier Transform

Introduction

Wen et al. [7] and Montalibet et al. [3,4] developed a technique to image electrical conductivity using the ultrasonically-induced Lorentz force. An ultrasonic wave is passed through tissue in the presence of a magnetic field. The wave causes the tissue--and the charged ions within the tissue--to move. A moving charge in a magnetic field is subject to the Lorentz force [1]. This force causes positive and negative charge to separate, and acts as the source of current and potential. Montalibet et al. [3] were able to detect the potential produced by a 1.5 MPa, 500 kHz ultrasonic wave in a 0.35 T magnetic field. They used these measurements to produce a one-dimensional image of conductivity gradients in biological tissue.

The goal of this paper is to analyze and extend this method to two or three dimensions, and to perform a preliminary calculation to prove that the method works in principle. We show that ultrasonically-induced Lorentz force imaging can be formulated in a way that makes it similar to tomography: reconstructing an image using waves propagating in various directions. More specifically, measuring the dipole strength for a particular direction and wavelength is equivalent to measuring the Fourier transform of the conductivity distribution at one point in frequency space. Measurements at a variety of wavelengths and directions are equivalent to mapping the Fourier transform of the conductivity distribution throughout frequency space, allowing us to determine the conductivity as a function of position by an inverse Fourier transform.

Methods

Let an ultrasound planar wave pass through tissue in the direction x . At one instant of time, the velocity $\mathbf{u}(x)$ of the tissue in the direction of propagation is

$$\mathbf{u}(x) = u_o e^{ikx} \hat{\mathbf{i}}, \quad 1$$

where u_o is the amplitude of the wave and k is the spatial frequency. Assume the tissue has a conductivity distribution given by

$$\sigma(x, y, z) = \sigma_o + \sigma'(x, y, z), \quad 2$$

where σ_o is a uniform background conductivity and $\sigma'(x, y, z)$ specifies a localized spatial variation about this background. A uniform, steady magnetic field is in the z -direction, $\mathbf{B} = B_o \hat{\mathbf{k}}$. The movement of charge in the magnetic field will give rise to a Lorentz force, producing a current density $\mathbf{J}_L$, which by the right-hand-rule is directed in the y -direction

$$\mathbf{J}_L = u_o B_o e^{ikx} (\sigma_o + \sigma') \hat{\mathbf{j}}. \quad 3$$

If separation of charge occurs, an Ohmic current density $\mathbf{J}_C$ is also present

$$\mathbf{J}_C = -(\sigma_o + \sigma') \nabla V, \quad 4$$

where V is the induced voltage. Continuity of total current, $\mathbf{J}_L + \mathbf{J}_C$, implies

$$\nabla \cdot ((\sigma_o + \sigma') \nabla V) = \nabla \cdot \mathbf{J}_L. \quad 5$$

In a completely homogeneous conductivity distribution ($\sigma' = 0$), there would be currents $\mathbf{J}_L$ inside the object having a strength that is proportional to the conductivity (as opposed to the gradient of the conductivity). These strictly solenoidal currents (zero divergence) have nonzero curl ($\nabla \times$). In an inhomogeneous conductivity distribution, charge accumulates where the gradient of the conductivity is non-zero. The electric field produced by these charges is purely irrotational (has zero curl), but does have a divergence ($\nabla \cdot$), leading to Eq. 5.

If the deviations of the conductivity distribution are small compared to the background, then we can ignore σ' compared to σ_o on the left-hand-side of Eq. 5, obtaining Poisson's equation

$$\nabla^2 V = \frac{u_o B_o}{\sigma_o} \frac{\partial \sigma'}{\partial y} e^{ikx}. \quad 6$$

Following a standard multipole expansion for the potential [1], the dipole term is given by

$$V = \frac{1}{4\pi\sigma_o} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}, \quad 7$$

where the dipole is

$$\mathbf{p}(k) = -u_o B_o \iiint \mathbf{r} \frac{\partial \sigma'}{\partial y} e^{ikx} dx dy dz. \quad 8$$

If we integrate by parts in the y -direction and assume that σ' is localized, so it goes to zero at large distances, then the y -component of the dipole strength becomes

$$p_y(k) = u_o B_o \int e^{ikx} \left[\iint \sigma' dy dz \right] dx. \quad 9$$

In this paper, we consider only a 2-dimensional slab of tissue that is uniform and of thickness δ in the z direction, giving

$$p_y(k) = u_o B_o \delta \int e^{ikx} \left[\int \sigma'(x, y) dy \right] dx. \quad 10$$

The integral in x has the form of a Fourier transform.

The two-dimensional Fourier transform of the conductivity distribution is defined as

$$\widehat{\sigma'}(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma'(x, y) e^{i(k_x x + k_y y)} dx dy. \quad 11$$

If we let $k_y = 0$, this becomes

$$\widehat{\sigma'}(k_x, 0) = \int_{-\infty}^{\infty} e^{ik_x x} \left[\int_{-\infty}^{\infty} \sigma'(x, y) dy \right] dx. \quad 12$$

Thus, comparing Eqs. 10 and 12, we find that measurement of $p_y(k)$ provides information about the two-dimensional Fourier transform of the conductivity distribution at the point in frequency space $k_x = k$ and $k_y = 0$.

This analysis is very similar to the method of two-dimensional image reconstruction from projections by Fourier transform, often used to reconstruct computed tomography or magnetic resonance images [2]. By analogy to CT, there is nothing special about the direction that we chose for the ultrasonic wave. Applying the ultrasonic wave in another direction and measuring the dipole moment perpendicular to that direction will provide information about the conductivity distribution along another line in frequency space. By taking measurements along many directions and using different wavelengths, we can map out the entire frequency plane. Once this information is known, a two-dimensional inverse Fourier transform will yield an image of the conductivity distribution.

Results

We tested this algorithm by a four-step procedure: 1) calculate $V(r, \phi)$ numerically using Eq. 6 for various values of k and θ , 2) estimate the dipole strength $p(k, \theta)$ from the calculated V in each case, 3) use a bilinear interpolation to obtain $p(k_x, k_y)$ from $p(k, \theta)$, and 4) perform a 2D inverse Fourier transform of $p(k_x, k_y)$ to obtain the conductivity $\sigma'(x, y)$.

1. Calculate $V(r,\varphi)$ numerically

We assume the tissue is circular, with radius 20 mm (Fig. 1a). The outer boundary of the tissue is sealed. The potential is solved using a finite difference approximation of Eq. 6 in polar coordinates (r, φ) , and overrelaxation. The space step is $\Delta r = 0.2$ mm and $\Delta \varphi = 2^\circ$ (200 points in the r -direction, and 180 points in the φ -direction).

The source term in Eq. 6 depends on the values k and θ , and on the conductivity distribution $\sigma'(x,y)$. As an example, consider the case of a Gaussian distribution for σ' (Fig. 1b),

$$\sigma'(x,y) = C \exp \left[-\frac{x^2 + (y - y_0)^2}{a^2} \right], \quad 13$$

where $C = 1 \text{ S m}^{-1}$, $y_0 = 3$ mm, and $a = 3$ mm. Figure 1c shows the real part of the source term for $k = 1.18 \text{ mm}^{-1}$ (corresponding to a wavelength of 5.33 mm) and $\theta = 50^\circ$. This source term results in the potential distribution in Fig. 1d.

In order to provide a more challenging test of the algorithm, we consider a conductivity distribution $\sigma'(x,y)$ that is the sum of five Gaussians

$$\sigma'(x,y) = \sum_{n=1}^5 C_n \exp \left[-\frac{(x - x_n)^2 + (y - y_n)^2}{a_n^2} \right], \quad 14$$

where the constants C_n , a_n , x_n , and y_n are given in Table 1. Figure 2a plots $\sigma'(x,y)$. We calculate the potential for 16 spatial frequencies $k_i = (i-1) \Delta k$, where $\Delta k = 0.393 \text{ mm}^{-1}$, in 18 directions $\theta_j = (j-1) \Delta \theta$, where $\Delta \theta = 10^\circ$.

2. Estimate the Dipole Strength

The potential distribution in Fig. 1d has two contributions: a part that has a spatial distribution very similar to the source function, and a part more distant from the source that has approximately a dipole pattern. Our algorithm requires determination of the dipole strength. We obtain a measure of the relative strength of the dipole by measuring the potential difference between opposite sides of the circular tissue, perpendicular to the direction of propagation of the ultrasonic wave (the dots in Fig. 1a). These measurements yield dipole values $p(k, \theta)$ at each k and θ .

3. Bilinear Interpolation

In order to perform an inverse Fourier transform, we need the dipole strength for a regular array of k_x and k_y values. Therefore, we use a bilinear interpolation to obtain $p(k_x, k_y)$ from $p(k, \theta)$, for 32 values (for both positive and negative spatial frequencies) using $\Delta k_x = \Delta k_y = 0.393$

mm^{-1} . Values of k_x and k_y corresponding to $k = \sqrt{k_x^2 + k_y^2}$ greater than 5.89 mm^{-1} are outside the range measured and are therefore set to zero.

4. Inverse Fourier Transform

Once we have $p(k_x, k_y)$, we take the inverse Fourier transform to obtain $\sigma'(x,y)$ [5]. The result is shown in Fig. 2b.

Discussion

The qualitative agreement between Figs. 2a and 2b is proof that in principle our method can be used to image the conductivity distribution. The method would be subject to many problems that arise when using Fourier techniques: aliasing, ringing at sharp discontinuities of conductivity, limitations on the spatial resolution caused by an upper limit of the values of the spatial frequency, etc. These problems are not immediately apparent in Fig. 2, because the conductivity distribution we consider is smooth, and therefore does not contain a lot of high frequency information.

Proof in principle does not imply that the method will be practical. We foresee several limitations of this method, some evident in our calculation and some not.

1. We did not calculate the conductivity distribution σ' , but instead a quantity proportional to it. In other words, we did not obtain the proportionality constant. This is not a fundamental limitation, but arises because we use the potential difference across the tissue (ΔV between the two dots in Fig. 1a) as a relative measure of the dipole strength. In principle, one could fit the measured potential to the dipole distribution and obtain the dipole strength, as is commonly done in electroencephalography and magnetoencephalography studies [2]. Once the dipole strength is known, one could divide by the magnetic field strength, the amplitude of the ultrasonic wave, and the tissue thickness (Eq. 10) and thereby obtain the actual conductivity distribution σ' , including the scale factor.
2. We assumed that the conductivity distribution could be divided into two parts: a uniform background conductivity and a spatially varying part that is localized. The value of the background conductivity did not play any role in our calculation (although it would enter the calculation if determining the scale factor, because it arises in Eq. 7 relating the potential to the dipole moment). If the spatially varying part is not localized, near field contributions may contribute to the measured potential, making it difficult to relate the potential to the dipole strength. Being able to obtain the dipole moment from the potential is therefore crucial for the method. Interestingly, this method should be more accurate if the potential is measured farther from the source, ensuring a purer dipole field, than if measured nearer. Apparently a radius of 20 mm in our calculation was sufficient to image a conductivity distribution that was localized within a radius of about 8 mm.
3. Our calculation was for a two-dimensional conductivity distribution. Equation 7 is appropriate for a three-dimensional geometry, so any attempt to use Eq. 7 to relate the dipole strength to the measured potential would require a full 3D analysis. One could obtain a 3D image either by using an ultrasonic wave that is focused and localized in the z direction (analogous to MRI or CT scans), or using plane waves in all three directions to obtain the 3D Fourier transform.
4. We assume that the ultrasonic wave is a plane wave with no change in amplitude as it propagates. Clearly the mechanical properties of the tissue could reflect or scatter the wave: this is in fact the basis for ultrasonic imaging [2]. Moreover, as frequency increases attenuation of the ultrasonic wave becomes greater. Such modifications to the ultrasonic wave will need to be studied before ultrasonically-induced Lorentz force tomography could become practical. In many cases, the transmitted wave is not dramatically affected by reflections. For instance, the acoustic impedance of muscle is 1.23 times the acoustic impedance of fat, implying that 99% of the incident energy of a 1 MHz ultrasonic wave on a muscle-fat boundary is transmitted and only 1% is reflected [2]. (Other tissues, such as bone, will have a much larger effect.) One can speculate about ways around this issue. If an ultrasound image was obtained without

a magnetic field present, the mechanical effects on the signal might be determined and somehow subtracted out of the conductivity image. One way to do this might be to use the measured ultrasonic wave amplitude for $\mathbf{u}(x)$ in Eq. 1 instead of a plane wave. Moreover, if attenuation is exponential along the path of the wave, its effect might be incorporated into the analysis in a way similar to how the exponential absorption of x-rays is accounted for in CT reconstruction.

5. This method of image construction assumes that both the amplitude and the phase of the Fourier transform can be measured. Obtaining the phase information may be difficult, because it assumes that the phase of the ultrasonic wave is known at the time that the dipole is measured. At the least, this will require careful timing of the measurement with respect to the wave.
6. The method uses continuous wave, rather than pulsed wave, ultrasound. It is essential for our method that the wave be present simultaneously throughout the tissue. This approach is different than that proposed by Wen et al. [7] and Montalibet et al. [3,4], who both used pulsed ultrasound to record reflections at conductivity gradients in a manner more analogous to traditional ultrasound imaging.
7. Tissue anisotropy can produce additional effects during Lorentz force imaging that are not included in our analysis [6].
8. We did not include effects of noise in our simulation. However, Fourier methods for image reconstruction are used widely and have been studied extensively, particularly in magnetic resonance imaging [2]. Noise is a problem in these applications, but not an insurmountable one.

Despite these limitations, we believe that our analysis provides a valuable proof-of-principle that ultrasonically-induced Lorentz force tomography is possible. Wen et al. [7] and Montalibet et al. [3,4] showed that the method is feasible experimentally, and our computations (Fig. 2) show that under a somewhat idealized geometry the method works. Further research is necessary to determine if this new type of tomography will be useful in practice and what are the relative strengths and weaknesses of the method compared to other techniques to measure electrical conductivity.

Acknowledgments

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Figure 1.

a) Schematic diagram of the tissue. The arrow in the direction θ shows the direction of the ultrasonic wave. The circle has a radius of 20 mm, and the dots indicate where the potential difference is measured. b) The conductivity distribution given by Eq. 13. c) The real part of the source term in Eq. 6 for $k = 1.18 \text{ mm}^{-1}$ and $\theta = 50^\circ$. d) The real part of the potential produced by this source term. In the gray scale images, white is positive, black is negative, and gray is zero. In d), the gray scale saturates in the central region in order to better show the dipole potential far from the source.

a)

b)

Figure 2.

a) The conductivity distribution given by Eq. 14, using the parameters in Table 1. b) The conductivity distribution calculated using the algorithm presented in this paper, with 16 values of k and 18 values of θ . In both cases, the gray scale extends from the maximum to minimum value of the conductivity, with white being positive, black negative, and gray zero. Only the central $16 \text{ mm} \times 16 \text{ mm}$ region of the tissue is shown.

Table 1

Parameters used in the conductivity distribution (Eq. 14)

n	C_n	a_n (mm)	x_n (mm)	y_n (mm)
1	1.5	4	-1	1
2	-2.0	2	1	2
3	2.0	1	2	-3
4	-1.5	1	0	-1
5	1.0	2	-2	-1