Sensory constraints on auditory identification of the material and geometric properties of struck bars

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A computational formula is derived for estimating the constraints limited auditory sensitivity imposes on auditory identification of the material and geometric properties of struck bars. The formula combines a model of the transverse motion of the bar with empirical psychometric functions to map out “null” regions in the bar’s physical parameter space where changes in the frequency, amplitude, and decay of partials are likely below threshold for detection. Parameters of the physical space include bar density, Young’s modulus, fluid and viscoelastic damping factors, bar length, and bar cross-sectional area (as related to bar shape and hollowness). The formula is used to estimate the possible effect of limited sensitivity in past studies on the auditory identification of bar attributes. The results suggest that sensitivity may, indeed, have played a role in some studies, and that apparent discrepancies in results may be understood based on whether the predominant source of damping was internal or external to the bar. The formula identifies conditions representing an expected bound on identification performance and thereby may be used to aid in the design of future studies for which the struck bar is the sound source of choice.

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I. INTRODUCTION

Most of us at one time or another have tapped on an object to determine from the sound whether it is hollow or solid, or made of metal or wood. What information in the sound allows us to do this? It is a simple question, but one that is currently at the center of a debate concerning one of the most important challenges for contemporary psychoacoustics—understanding how we perceive the world through sound. Historically, the debate has involved competing theories as to whether the listeners have access to *invariant* acoustic information that would allow them to uniquely identify object attributes, such as material composition or hollowness (cf. Gibson, 1966; Ullman, 1980; Micheals and Carello, 1981; Neuhoﬀ, 2004; and Lutfi, 2008). In the parlance of literature, invariant acoustic information for an object attribute changes only with that attribute; it is unchanged for variation in all others. Invariant acoustic information for material, for example, would not change with the size of the object, its shape, or the manner in which the object is suspended and struck. As such it would allow a listener to accurately identify material despite the many forms the sounding object or event may take.

Much effort, naturally, has been devoted to determining what might serve as invariant acoustic information for different source attributes. Conjecture, for the most part, has been based on physical-acoustic analyses of rudimentary, freely vibrating sources with few degrees of freedom (homogenous, isotropic bars, and plates). The basic attributes of bars and plates considered thus far include material composition (Wildes and Richards, 1988; Lutfi and Oh, 1997), hollowness (Lutfi, 2001), size (Gaver, 1988; Lutfi, 2008), shape (Kac, 1966; Lakatos *et al.*, 1997; Kunkler-Peck and Turvey, 2000),

mallet hardness (Freed, 1990), and point of contact (Lutfi and Liu, 2007). The relevant acoustic relations identified in each case have been proposed as a likely basis for listener judgments regarding these attributes, but such speculation has so far received only spotty empirical support. In the identification of material and in the detection of hollowness, for example, listeners are often found to base their judgments on simple differences in the frequency or duration of sounds (Lutfi and Oh, 1997; Lutfi, 2001; Giordano and McAdams, 2006), or to be overly influenced by the highest-level partials in the sounds (Lutfi *et al.*, 2008). Individual differences in the cues listeners use to make decisions have been reported (Lutfi and Liu, 2007), and it is not uncommon for studies investigating very similar conditions to come to quite different conclusions regarding the role of different sources of acoustic information (cf. Lutfi and Oh, 1997; Klatzky *et al.*, 2000).

The apparent discrepancies in the outcomes of these studies are not well understood, but in attempts to explain them one consideration has been largely overlooked—it is whether the invariant acoustic information for identification is always above the listeners’ threshold for detection. Human listeners, we know, are capable of detecting quite small changes in the frequency, amplitude, and decay of sounds (Jesteadt *et al.*, 1977; Wier *et al.*, 1977; Schlauch *et al.*, 2001), but sensitivity for the relations among these parameters, such as might constitute the invariant acoustic information for different source attributes, has not been widely studied (cf. Stoelinga and Lutfi, 2008). Lutfi (2001) considered the role of limited sensitivity in the auditory detection of the hollowness of struck bars. From an analysis of theoretical equations of bar motion, the invariant acoustic information was determined to be a specific relation between the

frequency and decay of partials. Sensitivity to this relation was gauged by adding small random perturbations to the frequency and decay of partials so as to simulate the effects of sensory (internal) noise. For the specific conditions of the study, the simulations suggested that the relation would not yield a significant detection advantage over simple differences in frequency alone. This outcome was corroborated by data from listeners showing trial-by-trial judgments of holowness to be most strongly correlated with frequency. Further simulations by [Lutfi \(2000\)](#) suggested that there are likely other conditions involving judgments of material and size where invariant acoustic relations are similarly vulnerable to sensory noise. [Giordano and McAdams \(2006\)](#) considered the role of limited sensitivity in the judgment of material from the impact sounds of large suspended plates. A cochlear model was used to evaluate the audibility of changes in various descriptors of these sounds, including a specific relation between frequency and decay deemed to be invariant acoustic information for material (cf. [Wildes and Richards, 1988](#)). Listener judgments were found to be correlated with this relation, but only for judgments made across gross material categories (steel-glass vs wood-plexiglass). Judgments within material categories, where the acoustic differences were likely more subtle, were correlated more strongly with simple differences in frequency.

The acoustic information that is available about a source must be limited to some extent by the ear's finite ability to resolve changes in the frequency, amplitude, and decay of individual partials. What these limits are and how they might have influenced past results are not known, nor would it be practical for all cases of potential interest to attempt an empirical determination of these limits. On the other hand, human sensitivities to individual changes in frequency, amplitude, and decay are fairly well documented in studies. By considering these data within the context of a viable model of the resonant source, it should be possible to provide a reasonable estimate of the effects of sensitivity on the identification of source properties for a broad range of conditions of interest. In the present study, we take this approach to map out the regions of a large physical parameter space for which corresponding changes in frequency, amplitude, and decay are likely to be below a listener's threshold for detection. We refer to these as null regions (NRs) on the premise that a change not reliably detected is not likely to participate in a higher order relation constituting invariant acoustic information for the listener.

II. ANALYSIS

A. Model of the resonant source

As a first step in the analysis we require a viable model of the resonant source that can be used to identify the acoustic effects of manipulations in source parameters and their likely effect on sensitivity. We choose for this purpose a homogenous, isotropic bar, struck with a mallet. The struck bar has been the sound source of choice in many studies because its acoustic properties (at least prior to propagation)

are fairly well understood and because it represents, in principle, a large class of sound-producing objects encountered in everyday listening.

The specific model of the bar described here is based on the work of [Chaigne and Doutaut \(1997\)](#). The Chaigne–Doutaut model is principally the same model used for sound synthesis in several of the studies cited in the Introduction, except that it provides a more refined account of viscoelastic losses in the bar and a more realistic representation of the bar-mallet interaction (for psychophysical evaluations, see [Lutfi et al., 2005](#); [McAdams et al., 2004](#)). The model gives the transverse displacement w over time at each point x along the length bar in response to an applied force density $F(x, x_0, t)$ at point x_0 . Only the transverse displacement is considered, as the longitudinal and torsional components of the motion typically radiate little sound. The displacement function is given by the fifth-order differential equation

$$\frac{\partial^2 w}{\partial t^2}(x, t) = \frac{1}{\rho S} \frac{\partial^2 M}{\partial x^2}(x, t) - \gamma_B \frac{\partial w}{\partial t}(x, t) + F(x, x_0, t), \quad (1a)$$

$$M(x, t) = -EI \left(1 + \eta \frac{\partial}{\partial t} \right) \frac{\partial^2 w}{\partial x^2}(x, t), \quad (1b)$$

where ρ is mass density of the bar, S is its cross-sectional area, M is the bending moment, γ_B is the fluid (air) damping coefficient, E is Young's modulus, I is the moment of inertia of the cross section, and η is the viscoelastic damping coefficient. The values of ρ , E , and η in this case are dictated by the particular material of the bar and are measured empirically. The force density term $F(x, x_0, t)$ is given by a model of the interaction between bar and mallet in which the impact force is governed by Hertz's law and the inertial force of the mallet by Newton's second law. [Chaigne and Doutaut \(1997\)](#) used the finite difference method to provide numerical time-domain solutions to the differential equations and showed these to be in excellent agreement with physical measurements on wood bars. As they noted, however, the model does not apply to most metal bars where other sources of damping, particularly those resulting from thermoelastic losses, play a complex role (cf. [Cremer, 2005](#)).

The Chaigne–Doutaut model is best suited for our purposes, but numerical solutions are computationally prohibitive for all points of interest in the physical space. Fortunately, we do not require complete solutions, only a few basic acoustic descriptors of these solutions. Expressions describing the dependence of these descriptors on the physical properties of the bar can be derived analytically by imposing a few minor constraints in the application of the model. To begin, we limit consideration to cases where the bar is lightly damped. The solution to Eq. (1) is then of the form

$$w = \sum_n A_n e^{-t/\tau_n} \sin(2\pi f_n t), \quad (2)$$

where f_n , A_n , and $\tau_n \gg 1/f_n$ denote, respectively, the frequencies, initial amplitudes, and decay moduli of individual partials making up the sound. Expressions for the frequencies in this case are derived in standard acoustic texts (cf. [Morse and Ingard, 1968](#)). For example, for the cylindrical bar clamped

at one end and struck at the other, the frequencies are

$$f_n = \frac{\pi r}{4L^2} \sqrt{\frac{E}{\rho}} \beta_n^2, \quad (3)$$

where $\beta_1=0.597$, $\beta_2=1.494$, and $\beta_{n>2}=n-\frac{1}{2}$ determine the frequency ratio of successive partials.

Consider next the amplitudes A_n . Their role in the identification of bar properties has been mostly neglected in literature because the changes in amplitude are generally small compared to those that result from differences in the manner of impact and other factors unrelated to bar properties. The changes are considered here, however, as we have found points in the physical parameter space where they may prove to be a useful cue. So as to constrain the behavior of the amplitudes the force density is approximated by a delta pulse applied to one end of the bar, $f(x, x_0=L, t)=\delta$. Though far from a realistic representation of the complexities of the bar-mallet interaction, the constraint is considered justified inasmuch as the present focus is on the properties of the bar, not the impact, which is expected to convey little if any information about bar properties.¹ The derivation of the amplitudes for the cylindrical bar clamped at one end is given by Morse and Ingard (1968). For a total force P of the impulse applied to the free end of the bar,

$$A_n = (-1)^{n-1} \frac{PL}{\pi^3 r^3} \sqrt{\frac{32}{\rho E}} \beta_n^2. \quad (4)$$

For the decay moduli τ_n , we consider separately the cases for which the predominant source of damping is external (fluid) and internal (viscoelastic). There are two reasons for this apart from expediency. First, of the two sources of damping in the model, only the viscoelastic component is directly related to the material of the bar. Later, in fact, we will discuss work in which a quantity related to the viscoelastic damping coefficient η is identified as a source of invariant acoustic information for material (Wildes and Richards, 1988). Second, empirical measurements by Chaigne and Doutaut (1997) on wood bars show the viscoelastic damping to be, by far, the predominant source of damping, fluid damping having but a small influence at the lowest frequencies, $f_n < 400$ Hz. Considering only the viscoelastic component the model gives, consistent with measurements,

$$\tau_n^{(v)} = \frac{1}{2\pi^2 \eta f_n^2}, \quad (5)$$

where (v) denotes the effect of viscoelastic losses on τ_n [cf. Eqs. 18 and 19 of Chaigne and Doutaut, 1997]. In the present application the values of η for different materials are taken from tables published by Cremer (2005).

For comparison to this model, we consider an alternative form of fluid damping proposed by Morse and Ingard (1968) in which the damping by air is given by a fixed force per length of the bar. Unlike Chaigne–Doutaut’s model of fluid damping, this model yields an expression for the decay moduli, which depends critically on the material and geometric properties of the bar over a wide frequency range. The relation is

$$\tau_n^{(a)} = c \left(\frac{\pi \rho}{r^2 f_n^3} \right), \quad (6)$$

where (a) denotes the effect of air damping and $c=16 \times 10^8$ is an air damping factor. This model has been used in the psychoacoustic studies of Lutfi and Oh (1997) and Lutfi (2001). It yields similar values of τ to the Chaigne and Doutaut (1997) model in some cases, but quite different values in others. Beyond increasing the generality of the analysis, the specific advantage of considering the two forms of damping separately is that it greatly simplifies the analysis. Inspection of Eqs. (3)–(6) reveals that the information in successive partials is redundant, the acoustic parameters being scaled in each case by a constant factor β_n raised to a constant power. This means that sensitivity to changes in individual partials of the sound, and thus sensitivity to change in the sound as a whole, can be estimated using a single computational formula; only the values of the parameters of this formula need change with the different partials. Empirical support for this approach is later given in Sec. IV, and possible exceptions are considered in Sec. V.

B. Determination of null regions

Having specified a model of the resonant source, the next step is to determine regions of the physical parameter space given by the model for which changes are likely to be below the listener’s threshold for detection; these are the NRs mentioned earlier. For this purpose, three threshold levels corresponding to three values of d' ($=1, 2$, or 3) are estimated from empirical psychometric functions (PFs) relating d' to changes x in the frequency, amplitude, and decay of pure tones (Abel, 1972; Dai, 1995; Buus and Florentine, 1991; Freyman and Nelson, 1986; Jesteadt and Bilger, 1974; Jesteadt and Sims, 1975; Jesteadt *et al.*, 1977; Wier *et al.*, 1977). The form of the PF typically used in these studies and the one that describes the data well is a power law

$$d' = ax^b, \quad (7)$$

where a and b are, respectively, the intercept and slope of the function fitted to the data in log-log coordinates (Green and Swets, 1966; Egan *et al.*, 1969; Buus and Florentine, 1991). For a given stimulus ϕ the magnitude of change has been variously expressed as $\Delta\phi$, $\Delta\phi/\phi$, and $\ln(1+\Delta\phi/\phi)$. However, where $0 < \Delta\phi/\phi \ll 1$, as it is for the discrimination of frequency, amplitude, and decay, these different representations can be shown to yield essentially equivalent fits to the data with an appropriate selection of a and b .² For consistency we choose a single measure of change to describe the PFs from all studies,

$$x = \ln \left(1 + \frac{\Delta\phi}{\phi} \right), \quad (8)$$

where $\Delta\phi/\phi$ is Weber’s fraction. We also choose the slope of the PF for each ϕ to be constant across its range. For the discrimination of tone amplitude ($\phi=A$) the slope of the PF given by Eqs. (7) and (8) is found to be essentially constant over most of the audible range of A and just slightly less than 1, $b=0.94$ (Buus and Florentine, 1991).³ Similarly, for the

discrimination of frequency ($\phi=f$), a constant slope of $b = 0.72$ appears to describe the data reasonably well (Jesteadt and Sims, 1975; Dai, 1995; Jesteadt and Bilger, 1974). Comparable data on PF slopes for the discrimination of decay are lacking; however, it seems likely that the slopes would follow closely those for the discrimination of overall duration (T), given that a change in duration accompanies any change in decay. Indeed, as we discuss shortly, the agreement between duration discrimination thresholds for signals with and without exponential decay is quite good. Abel (1972) showed the values of ΔT (for $d' = 1$) derived from the PFs for the discrimination of tone duration to be well predicted by the corresponding change in energy over a range of $T = 0.16$ –960 ms. Based on this result we assume the PFs for the discrimination of tone decay to behave like those for the discrimination of tone amplitude described above.

For the intercepts of the PFs we use the differential thresholds ($\Delta\phi$) obtained in studies corresponding to a given value of d' . For the discrimination of frequency the most extensive data are provided by Wier *et al.* (1977). These authors showed that at moderate sensation levels (40 dB SL) the relation

$$\Delta f = 10^{0.026\sqrt{f}-0.533} \quad (9)$$

accurately describes the data over a wide range of frequencies (0.2–8.0 kHz), deviating only a few Hz at levels near the threshold of audibility. Jesteadt *et al.* (1977) provided complementary data for the discrimination of tone intensity (I). They showed threshold ΔI to be essentially independent of frequency and well described by the relation $\Delta I = 0.463(I/I_0)^{0.928}$, where I_0 is the threshold for detection as reported in their study. Converting this to a differential threshold for amplitude yields

$$\Delta A = 0.680(A/A_0)^{0.928}, \quad (10)$$

where A_0 is the detection threshold in amplitude. Van Heuven and Van Den Broecke (1979) conducted the most extensive study involving the discrimination of pure-tone decay. However, they used a method of adjustment procedure, which precludes a measure of d' . We rely, instead, on the data of Schlauch *et al.* (2001), who used an objective three-alternative, forced-choice procedure to measure thresholds. The signals in the study of Schlauch *et al.* (2001) were noise bursts instead of tones; however, our informal analyses indicate that the threshold estimates from the two studies are in reasonably good agreement if one assumes the equivalent of a low false-alarm rate in the study of Van Heuven and Van Den Broecke (1979). The relevant data from Schlauch *et al.* (2001) are given in their Fig. 7 (panel 5, open symbols) where threshold $\Delta T/T$ is plotted against $T = 10$ –100 ms on log-log coordinates. A linear regression provides an excellent fit to these data yielding the relation $\Delta T = 0.93T^{0.535}$. Schlauch *et al.* (2001) chose the time constants for decay to be one-fifth of the total duration T of their signals. Hence, converting this relation, we have

$$\Delta\tau = 0.439\tau^{0.535}, \quad 2 \leq \tau \leq 20 \text{ ms}. \quad (11a)$$

For longer duration decay constants we use the ΔT thresholds of Abel (1972). This seems justified inasmuch as the

FIG. 1. Comparison of the best-fitting curves to the duration discrimination (ΔT) thresholds of Abel (1972) (triangles and squares) and Schlauch *et al.* (2001) (circles). In the Schlauch *et al.* (2001) study the signals decayed exponentially with decay constant equal to one-fifth the duration of the signal. In the Abel (1972) study the signals were gated on and off rectangularly.

reported fits to her data agree remarkably well with those of Schlauch *et al.* (2001) over the same range of durations (cf. $\Delta T = 1.09T^{0.449}$ and $T = 0.16$ –80 ms). The curves corresponding to these fits accounted for more than 97% of the variance in the data and are shown in Fig. 1. Note that two curves were fitted to the data to account for a break point near $T = 100$ ms. The curve for the longer durations $T = 100$ –1000 is given by the relation $\Delta T = 0.38T^{0.722}$. To estimate the corresponding relation for $\Delta\tau$ we take one-fifth the duration as before since this previously yielded excellent agreement to the Schlauch *et al.* (2001) data at the shorter durations in Fig. 1. We have

$$\Delta T_{1/5} \rightarrow \Delta\tau = 0.243\tau^{0.722}, \quad 20 < \tau \leq 200 \text{ ms}. \quad (11b)$$

This and the other relations giving the slopes and intercepts of the PFs used in this analysis are summarized in Table I.

TABLE I. Summary of the parameters of the PFs used in the analysis. Values of the intercepts a are computed from the data in columns 3, 4, and 6 according to the relation $a = d'[\ln(1 + \Delta\phi/\phi)]^{-b}$. Columns 5 and 7 give, respectively, the range of values over which the intercepts a and slopes b were measured in the referenced studies.

ϕ	Studies	$\Delta\phi$	d'	Range	b	Range
f	a–d	$10^{0.026\sqrt{f}-0.533}$	0.78	0.2–8.0 kHz	0.72	0.2–1.0 kHz
A	d–f	$0.680(A/A_0)^{0.928}$	0.78	5–80 dB SL	0.94	30–90 dB SPL
τ	g	$0.439\tau^{0.535}$	1.35	2–20 ms	...	...
	h*	$-0.248\tau^{0.722}$	0.95	20–200 ms	0.94	0.03–200 ms

^aWier *et al.*, 1977.

^bDai, 1995.

^cJesteadt and Bilger, 1974.

^dJesteadt and Sims, 1975.

^eBuus and Florentine, 1991.

^fJesteadt *et al.*, 1977.

^gSchlach *et al.*, 2001.

^hAbel, 1972.

*Estimates based on $T_{1/5}$.

TABLE II. Values of exponents p_i from Eqs. (3)–(6) relating physical to acoustic parameters, used to compute NRs according to Eq. (15).

	E	ρ	η	L	r
f	1/2	−1/2	...	−2	1
A	−1/2	−1/2	...	1	−3
$\tau^{(v)}$	−1	1	−1	4	−2
$\tau^{(a)}$	−3/2	5/2	...	6	−5

We are now in a position to derive an analytic expression giving d' for the change in any one of the three acoustic parameters $\phi \in \{f, A, \tau\}$ corresponding to the change in any combination of the five physical parameters $\theta \in \{\rho, E, \eta, L, r\}$. For this we note that the general form of the relation between ϕ and θ given by Eqs. (3)–(6) is

$$\phi = \lambda \prod_i \theta_i^{p_i}, \quad (12)$$

where λ is a constant and the p_i are the exponents on the physical parameters corresponding to ratios of small integers (given in Table II). The values of θ_i in this case do not depend on one another; hence, for any given $\theta_{i=k}$,

$$\phi = \gamma \theta_k^{p_k}, \quad (13a)$$

$$\phi + \Delta\phi = \gamma(\theta_k + \Delta\theta_k)^{p_k}, \quad (13b)$$

where γ is a new constant. Substituting these expressions into Eq. (8) yields

$$x = p_k \ln \left(1 + \frac{\Delta\theta_k}{\theta_k} \right), \quad (14)$$

which upon rearranging terms from Eq. (7) gives

$$\Delta\theta_k = \theta_k \left[\exp \left(\frac{d'}{ap_k^b} \right)^{1/b} - 1 \right]. \quad (15)$$

In what follows the values of a , b , and d' from the studies referenced in Table I and the exponents p_i from Eqs. (3)–(6) (and listed in Table II) are used to compute null regions for each acoustic parameter in the physical parameter space according to Eq. (15). For material the physical parameter space included three variables: bar density ρ , Young's modulus E , and the coefficient of viscoelasticity η . For geometry there were two variables: bar radius r and bar length L . The effect of two other aspects of bar geometry, shape and hollowness, are principally the same as for bar radius and so are treated along the way. Table III summarizes all variables included in the analysis and their definitions, while Table IV gives the specific values of the physical parameters used. The values of ρ and E in Table IV correspond nominally to wood (oak), glass, aluminum, and plastic (plexiglass) as given in published tables (Cremer, 2005, pp. 191 and 196). So too, the values of η can be considered only nominal as there is much reported variability in their measurement (Cremer, 2005). Finally, the values of r and L were chosen for these materials to produce frequencies and decay moduli that extended over most of the audible range above 400 Hz, where, as earlier noted, the predominant source of damping in the Chaigne and Doutaut (1997) model is viscoelastic.

TABLE III. Definitions of variable quantities and their units.

Symbol	Definition (unit)
f	Frequency (Hz)
A	Amplitude
$\tau_n^{(a)}$	Decay modulus, fluid damping model (s)
$\tau_n^{(v)}$	Decay modulus, viscoelastic damping model (s)
E	Young's modulus (dyn/cm ²)
ρ	Mass density (g/cm ³)
L	Bar length (cm)
r	Bar radius (cm)
κ	Radius of gyration (cm)
η	Coefficient of viscoelasticity (ns)
c	Air damping factor
ϕ	Generic symbol for acoustic parameter
θ	Generic symbol for physical parameter
a	Intercept of the psychometric function
b	Slope of the psychometric function
d'	Index of discriminability
p_i	Exponents relating acoustic to physical parameters

III. ANALYTIC RESULTS

A. Null regions for material properties

Example NRs in the $\rho \times E$ parameter space are given by the color-shaded regions in Fig. 2; red, green, and blue, respectively, for frequency, amplitude, and decay. The decay is for the fluid damping model; viscoelastic damping is considered later. Increasing values of $d' = 1-3$ are represented by shade from dark to light for both increments and decrements in the acoustic parameters. The NRs are for the first partial ($n=1$) of a small glass (Gl) and wood (Wd) bar, left and right panels, respectively (note the cross-over of the NRs at the points for glass and wood). The NRs give the predicted discriminable changes in the first partial corresponding to a change in the material of the bar from glass or from wood. For example, in going from glass to aluminum, the change in frequency is estimated to be highly discriminable ($d' \gg 3$), whereas the change in decay is not ($d' < 3$). The properties of the other bars selected for investigation are listed in Table IV. These properties were selected so that the first partial, at least, would fall within the range for which there were psychophysical data relevant to the discrimination of decay. Over this range the different geometries had little noticeable effect on the NRs. The largest effect overall was on fluid damping in going from the small to the thick wood bar; the corresponding NR ($d'=3$) for the thick wood bar is given by the blue dashed line in the right panel of Fig. 2.

The NRs of Fig. 2, as noted, are for the first partial. To compute the NRs for the complex as a whole one would next need to determine the sensitivities to changes in the remaining audible partials and combine these in some fashion to estimate sensitivity to changes in the complex as a whole (cf. Moore *et al.*, 1984; Gockel *et al.*, 2007). Though clearly doable, our analyses indicate that the outcome would hardly differ from that for the first partial alone. Figure 3 demonstrates the reason why. It shows by shade the NRs ($d'=3$) for the first three partials of the glass and wood bar (the darkest shade representing the lowest frequency partial, and the

TABLE IV. Values of physical and acoustic parameters corresponding to the reference bars represented by the NRs in Fig. 1. Amplitude of the reference is fixed at 40 dB SL. The estimates of the Weber fractions, $\Delta\theta/\theta$, for the physical parameters in columns 9–13 is based on the NR for frequency for $d'=1$.

Bar	ρ (g/cm ³)	E (dyn/cm ² × 10 ¹¹)	η (ns)	$L \times r$ (cm × cm)	f_1 (Hz)	$\tau_1^{(a)}$ (ms)	$\tau_1^{(v)}$ (ms)	$\Delta E/E$ × 10 ⁻³	$\Delta\rho/\rho$ × 10 ⁻³	$\Delta\eta/\eta$ × 10 ⁻³	$\Delta L/L$ × 10 ⁻⁵	$\Delta r/r$ × 10 ⁻³
Lg Wd	0.7	1.00	100	20 × 6	1587	25	201	5.7	5.6	5.6	1.9	2.8
Sm Wd	0.7	1.00	100	10 × 3	3174	12	50	7.6	7.6	7.6	0.6	3.8
Tk Wd	0.7	1.00	100	15 × 6	2821	4	64	7.1	7.0	7.1	0.9	3.5
Lg Gl	2.5	6.00	30	20 × 9	3085	5	177	7.5	7.4	7.4	1.2	3.7
Sm Gl	2.5	6.00	30	10 × 3	4114	20	100	9.4	9.3	9.4	0.6	4.7
Tk Gl	2.5	6.00	30	15 × 6	3657	7	126	8.5	8.4	8.4	0.9	4.2
Sm Al	2.7	7.20	10	9 × 3	5354	10	177	12.4	12.2	12.4	0.5	6.2
Tk Al	2.7	7.20	10	15 × 6	3855	7	34	8.9	8.8	8.8	0.9	4.4
Lg Pl	1.1	0.60	200	20 × 9	1471	21	117	5.6	5.6	5.6	1.9	2.8
Sm Pl	1.1	0.60	200	10 × 3	1961	81	66	6.0	6.0	6.0	0.8	2.8
Tk Pl	1.1	0.60	200	15 × 6	1743	29	83	5.8	5.8	5.8	1.3	2.9

lightest shade the highest). The geometries of the bars in this case ($r=1.5$ and $L=20$ cm) were selected so that all three partials would fall within the audible range.⁴ What is evident from the figure is that the NRs are either unchanged (as for amplitude), or they become much larger with increasing order of the partial (for frequency and decay). The increase in the NRs for frequency and decay is a general result and is to be expected since the respective Weber fractions are larger for the higher frequencies and shorter decays associated with the higher order partials [cf. Eqs. (9), (11a), and (11b)]. The significant point to be made here is that changes in the few higher order partials that are audible will contribute little if at all to overall sensitivity if changes in the first partial are already below threshold for detection. For simplicity of presentation we focus in what follows on the NRs for the first partial alone as a limiting case. A subsequent empirical test is provided in Sec. IV.

Returning to Fig. 2, several gross features of the NRs are worth noting. First, the NRs for frequency are by far the smallest. This is due to the fact that small changes in bar density and elasticity produce relatively large changes in frequency [cf. Eq. (3)] while, at the same time, the Weber fraction for the discrimination of frequency is exceedingly small, $\Delta f/f < 0.005$. Table IV shows what is predicted to be the

smallest detectable change in either material density or elasticity ($d'=1$) based on the NRs for frequency. The smallest detectable change in each case is of the same order of magnitude as the Weber fraction for the discrimination of frequency. The implication of the small frequency NRs is that changes in frequency could provide a robust cue for material when differences in material are small. This might go some way in explaining the results of the aforementioned studies where listeners have been observed to base their judgments of material on simple differences in frequency (Lutfi and Oh, 1997; Giordano and McAdams, 2006). We will return to this point in Sec. V.

A second feature to note is that the NRs for amplitude are roughly orthogonal to the NRs for frequency and decay (although not shown, this is also true for decay due to viscoelastic damping). As such, changes in amplitude, in special circumstances, could provide an independent source of information about material. A case in point is given in the left panel of Fig. 2 where the change from glass to aluminum produces a greater increase in d' for amplitude than it does for $\tau_1^{(a)}$. This specific case was, in fact, investigated by Lutfi and Oh (1997). They used the fluid damping model to synthesize impact sounds of glass, aluminum, and quartz bars, quartz being very similar to aluminum in mass density and elasticity. Listeners were able to distinguish among these materials, but three of four listeners participating in the discrimination of glass from aluminum gave a significant,

FIG. 2. NRs for changes in frequency (red), amplitude (green), and decay based on fluid damping (blue) are plotted as a function of bar density, ρ , and elasticity, E , for glass (Gl) and wood (Wd) bars in the left and right panels, respectively. The locations of plastic (Pl) and aluminum (Al) bars in this space are also shown (see Table IV). Shading of the NRs from dark to light represents increasing values of d' from 1 to 3. The NRs are for the small (Sm) bars in Table IV. The NRs are essentially unchanged for the other bar geometries in Table IV with the exception of the thick wood bar, for which the NR for decay ($d'=3$) is given by the dashed line.

FIG. 3. NRs ($d'=3$) for the first three partials of the glass and wood bar, represented by shade with the darkest shade representing the lowest frequency partial and the lightest shade representing the highest frequency partial. The geometries of the bars ($r=1.5$ and $L=20$ cm) were selected so that all three partials would fall within the audible range.

FIG. 4. NRs (blue) for changes in decay (viscoelastic damping) are plotted as a function of the viscoelastic damping factor η and bar length L for plastic (PI), wood (Wd), glass (Gl), and aluminum (Al) bars (see Table IV). Shading from dark to light represents increasing values of d' from 1 to 3 as before.

though not predominant, weight to amplitude. All three listeners participating in the discrimination of glass from quartz gave a significant weight to amplitude with one giving a predominant weight to amplitude. Unfortunately, in many other studies, the role of amplitude cannot be gauged because the amplitudes are either not reported or the levels of signals are equalized in some manner.

Finally we note that for glass, in the left panel of Fig. 2, both aluminum and wood fall in or near the NR for $\tau^{(a)}$. This means that if mass density and elasticity were the only factors affecting decay, as is true for $\tau^{(a)}$, then a listener might often detect a change in frequency corresponding to the difference in these materials without detecting the corresponding change in decay. A similar situation might be thought to exist for $\tau^{(v)}$; however, $\tau^{(v)}$ also depends on the material parameter η whose effect is not evident from Fig. 2. Figure 4 shows the corresponding NRs for $\tau^{(v)}$ in the parameter space given by η and L . The figure reveals significant separation among the NRs plotted in this space, indicating clearly discriminable differences in $\tau^{(v)}$ for these materials. The Weber fractions for η based on the discriminable changes in frequency are listed in Table IV and are of the same order of magnitude as those for the discrimination of frequency. Viscoelastic damping as given by η is one of a number of internal damping factors that has figured prominently in the analysis of invariant acoustic information for material (Wildes and Richards, 1988). We return to this point in Sec. V.

B. Null regions for geometric properties

Figure 5 shows the NRs for changes in the parameter space given by $L \times r$, color- and shade-coded as before. In the left panel the bar is relatively large (Lg), having dimensions $L=20$ and $r=6$; in the right panel a thick (Tk) bar is represented with dimensions $L=15$ and $r=6$. The locations of the small (Sm) bars of Fig. 2 are also indicated. The NRs are for the glass bar; however, the differences between these NRs and those for the other materials in Table IV were barely noticeable.

FIG. 5. NRs for changes in frequency (red), amplitude (green), and decay based on fluid damping (blue) are plotted as a function of bar length L and radius r for large (Lg) and thick (Tk) glass bars in the left and right panels, respectively. The location of the small (Sm) bars in this space is also shown (see Table IV). Shading of the NRs from dark to light represents increasing values of d' from 1 to 3.

Certain features of the NRs in Fig. 5 are similar to those for material. As was the case for material, the NRs for frequency are smallest. This again is to be expected as small changes in geometry, or material, tend to produce large changes in frequency where the Weber fraction is smallest. The corresponding Weber fractions for length and radius are given in Table IV. Here, the Weber fractions for length are clearly the smallest, being two orders of magnitude smaller than those for any of the other four physical parameters. This is expected from Eq. (3), where length has the largest-valued exponent and so produces proportionally the largest changes in frequency. A second common feature with material is that the NRs for amplitude, though not orthogonal to, cross the other NRs at considerable angles, particularly in the local region of the bar reference. The relation is such that, while a change in bar length is likely to produce a more discriminable change in frequency or decay than in amplitude, a change in bar radius with length may produce a more discriminable change in amplitude. This reinforces the point made earlier that amplitude in special cases could serve as a cue for bar properties.

In addition to bar length and radius, Fig. 5 provides estimates of sensitivity to changes in two other features of bar geometry: hollowness and shape. These properties affect the cross-sectional area of the bar [S in Eq. (1)] and so their acoustic effect is principally the same as that for bar radius. The specific effect is given by the radius of gyration κ , which is a measure of how the cross-sectional area of the bar is distributed about its center axis. For the cylindrical and rectangular bar $\kappa=r/2$ and $\kappa=r/\sqrt{12}$, respectively, where r is bar radius. For the hollow cylindrical bar $\kappa=\frac{1}{2}\sqrt{r^2+s^2}$, where s is the inner radius of hollowness (Morse and Ingard, 1968). Sensitivity to specific changes in each of these cases can be determined from the NRs in Fig. 5 by scaling the abscissa according to these expressions, but within the acoustic range investigated the general conclusions will be the same as for bar radius.

IV. EMPIRICAL RESULTS

Momentarily we consider how the general features of the NRs described above might account for some apparent discrepancies in past studies. First, however, we offer some preliminary data testing specific predictions of the NRs for

the different effects of fluid and viscoelastic damping on the identification of bar material. The experiment demonstrates an intended use of Eq. (15) as a vehicle for identifying conditions of interest. It also tests the earlier premise that sensitivity to changes in the first partial can be taken to reflect sensitivity to changes in the signal as a whole.

In a standard two-interval, forced-choice procedure with feedback listeners were presented the impact sounds (partials $n=1-3$ present) of the small glass and aluminum bars ($r=3$ and $L=10$ cm), those represented in Figs. 2 and 4. The listeners were asked to judge on each trial which of the two sounds was produced by the aluminum bar. In different conditions the impact sounds were synthesized according to either the fluid or the viscoelastic damping model. As mentioned earlier, both models yield a specific relation between frequency and decay as invariant acoustic information for material (more is said about this in Sec. V). However, the NRs of Figs. 2 and 4 predict that only for viscoelastic damping will the relation be above threshold for detection. For viscoelastic damping aluminum falls well outside the NRs for both frequency (Fig. 2) and decay (Fig. 4), but for fluid damping aluminum falls within the same NR for decay (Fig. 2). The prediction is that listeners will base their judgments of material on the combination of frequency and decay when the sounds are synthesized according to the viscoelastic damping model, but will base their judgments predominantly on frequency when the sounds are synthesized according to the fluid damping model.

A standard perturbation analysis procedure was used to test these predictions (see Berg, 1990 and Lutfi, 2001). On each presentation a small normal deviate was added to the length of the bar ($\sigma_L=0.2$ and 1.0 cm, respectively, for fluid and viscoelastic damping), and in the case of viscoelastic damping, to η ($\sigma_\eta=10^{-8}$ s). The latter perturbation served to decouple the perfect correlation between f and $\tau^{(v)}$, which was necessary for the analysis. It was not so great as to prevent the listener from performing perfectly if they based their judgments on $f\tau^{(v)}$. Correlations between each listener's judgments and the resulting perturbations in f , τ , and $f\tau$ were computed by multivariate regression using the glmfit routine of MATLAB with a logistic link function. The correlations, based on the average of 400 trials, were then converted to relative decision weights by normalizing with respect to the sum of their absolute values (cf. Berg, 1990). The listeners were one male and two female University of Wisconsin students (ages 28–29). All had previous extensive experience in the task and all had normal hearing by standard audiometric tests (ANSI S3.6-1996, 1996).

The results of the experiment are given in Fig. 6 where the obtained decision weights (symbols representing different listeners) are compared to predictions in each condition (horizontal lines). Also shown (legend) is the predicted and obtained proportion of correct responses. Though the obtained weights were not exclusively for one or the other cue, the results are in general agreement with the predictions. All listeners show a predominant weight on f for the fluid damping model and a predominant weight on $f\tau$ for the viscoelastic damping model. Moreover, all listeners as predicted perform more poorly with the fluid damping model. The results

FIG. 6. Obtained decision weights of listeners (symbols) are given for the two-interval, forced-choice distinction between the impact sounds of the small glass and aluminum bars. Left and right panels show, respectively, the results when the sounds were synthesized according to the fluid and viscoelastic damping models. Predictions for the data in each case were derived from the NRs of Figs. 2 and 4 and are given by the short horizontal lines in each panel.

support the use of the first partial as an index of sensitivity for changes in the complex as a whole, although it is always possible to use Eq. (15) to estimate the additional influence of the higher order partials. They also demonstrate how Eq. (15) might be used as practical tool in selecting conditions where one wishes to examine the role of limited auditory sensitivity or where one wishes to select conditions for which limited auditory sensitivity can reasonably be ruled out as a factor.

V. DISCUSSION

We return now to consideration of the possible influence of auditory sensitivity on the availability of invariant acoustic information for bar attributes in past studies. Wildes and Richards (1988) provided the seminal work on the invariant acoustic information for material. Their analysis focuses on the damping due to internal friction created within the object as it vibrates. Internal friction may have several sources, viscoelastic processes being one, but the degree of damping resulting collectively from these sources is commonly measured by the material's coefficient of internal friction, $\tan \varphi$. The coefficient gives the ratio of elastic moduli associated with the stress and strain acting on the object. Wildes and Richards (1988) recognized that, as a unique property of materials, $\tan \varphi$ could convey invariant acoustic information for material through its effect on the decay of partials $\tau^{(e)}$. Citing literature on anelastic solids (Zener, 1948), they proposed a model in which $\tau^{(e)}$ at any frequency f is linearly related to $\tan \varphi$ as follows:

$$\tan \varphi = \frac{1}{\pi f \tau^{(e)}}. \quad (16)$$

Thus, assuming that internal friction is the predominant source of damping, a listener who has access to the frequency f and decay $\tau^{(e)}$ of any one partial could, in theory, recover a measure of internal friction that would allow for a material classification. That measure would be a frequency-independent decay $f\tau^{(e)}$, corresponding to the number of cycles required for the partial to decay to $1/e$ of its original value.

Other cues for material, since implicated in studies, can be shown to be closely related to $f\tau^{(e)}$. In the Chaigne and

Doutaut (1997) model, for example, on which the present analysis is based, internal friction is determined by the viscoelastic damping factor η —a unique material property. From Eq. (5) the corresponding acoustic information for material is given by $f\tau^{(v)}$ for any partial (Note that unlike $\tan \varphi$, η has units of seconds; hence, f is not squared in this relation). Also, because both Eqs. (5) and (16) dictate an inverse relation between frequency and decay, the spectral centroid of the impact sound will shift to lower frequencies over time. McAdams *et al.* (2004) identified this cue as a correlate of η in listeners' dissimilarity ratings of impact sounds. Given the close relation among these cues a strong correlation of listener judgments with any one would likely yield a strong correlation with the others. It probably matters little therefore that most studies, in fact, have restricted their analyses to $f\tau^{(e)}$. Collectively, these studies show that listeners are, indeed, able to judge the material of bars and plates from their emitted sound, regardless of whether the sound is live, recorded, or synthesized based on a physical model (Gaver, 1988; Lutfi and Oh, 1997; Kunkler-Peck and Turvey, 2000; Carello *et al.*, 1998; Tucker and Brown, 2003; Giordano and McAdams, 2006). However, the results are mixed as to whether listeners' judgments are based on $f\tau_e$. Using synthesized sounds of struck clamped bars and a forced-choice classification of material, Lutfi and Oh (1997) found the frequency f of partials to be a better predictor than $f\tau_e$ of the listeners' trial-by-trial judgments. Hermes (1998) reported a similar outcome for both free- and forced-choice classifications of synthesized impact sounds. Klatzky *et al.* (2000), on the other hand, used synthesized sounds of struck, clamped bars, very similar to those of Lutfi and Oh (1997), and reported greater reliance of judgments on $f\tau_e$ than on f for both material classification and similarity judgments of material. Their results appear to be corroborated by Avanzini and Rocchesso (2001) using synthesized impact sounds, and by Tucker and Brown (2003) using recordings of sounds from loosely suspended plates, although Tucker and Brown (2003) only analyzed for $f\tau_e$. Giordano and McAdams (2006) obtained forced-choice judgments of material using live sounds generated by loosely suspended plates. They found judgments to be more strongly correlated with f when materials were confused (as in the case of steel vs glass and wood vs plastic), but more strongly correlated with $f\tau_e$ when they were not (as in the case when comparisons were made across these material pairs). They suggested based on this outcome that f may mediate judgments for materials having similar physical properties, whereas $f\tau_e$ may mediate judgments for materials having very different physical properties.

Many of the apparent discrepancies among these studies may be understood by considering the predicted effect of auditory sensitivity based on Eq. (15). A key prediction is that differences in decay associated with different materials are less likely to be detected if the predominant source of damping is external to the object (e.g., fluid) as opposed to internal (e.g., viscoelastic). Klatzky *et al.* (2000) and Avanzini and Rocchesso (2001) synthesized their sounds using precisely the internal damping model proposed by Wildes and Richards (1988) [Eq. (16) above]. These studies involved classification of wood and glass, among other mate-

rials, which according to the NRs of Fig. 4 would be clearly identifiable based on $f\tau^{(v)}$. Lutfi and Oh (1997), on the other hand, used the fluid damping model given by Eq. (6). Their study involved the classification of glass and aluminum, among other materials, which according to the NRs of Fig. 2 would produce a clearly discriminable change in f , but a much less discriminable change in $\tau^{(a)}$. Hermes (1998), whose results were similar to those of Lutfi and Oh (1997), also used a means of synthesizing sounds that did not include the influence of internal damping.

Though Eq. (15) was developed specifically for bars, the predicted difference in sensitivity to the source of damping may also account for the different results obtained in the studies using real sounds emitted by plates. Plates, by virtue of their geometry, are subject to significant damping by air, generally more so than bars. Moreover, in the studies using plates, the plates have had large surface areas, thus increasing the expected influence of fluid damping. Giordano and McAdams (2006), for example, reported the values of f_1 for nearly all their plates to fall below 400 Hz, well within the region where fluid damping is expected to have a significant impact for bars according to the model of Chaigne and Doutaut (1997). As fluid damping begins to play a more prominent role, the expectation from Fig. 2 is that confusions would first occur between the generic categories of glass and metal (aluminum) since both fall within the same NR for decay. Finally, the NRs of Fig. 2 suggest that as the difference in materials to be classified becomes large amplitude would become a salient cue. Giordano and McAdams (2006), in fact, observed that, related to amplitude, "peak loudness" was as highly correlated with listener judgments in such cases.⁵

The results of studies investigating auditory perception of size and shape are somewhat more difficult to interpret than those for material. To begin, these studies find that size and shape are not nearly as well judged from sound as material (Gaver, 1988, 1993a, 1993b; Tucker and Brown, 2003; Kunkler-Peck and Turvey, 2000; Carello *et al.*, 1998; Ottaviani and Rocchesso, 2004; Rocchesso, 2001; Lakatos *et al.*, 1997). This may have to do with the fact that, unlike material, information about size and shape can be conveyed through the spatial dimension of sound, which is lacking in studies where the sounds were recordings (cf. Lutfi, 2008). Somewhat better, though still rather poor, performance has been observed in studies using live sounds (Kunkler-Peck and Turvey, 2000; Carello *et al.*, 1998). Also there is considerably more variation across these studies in the manner in which the sounding objects were excited to vibrate. In the Gaver (1988) study, for example, the objects were bars struck with a mallet as they rested on a carpeted surface; in the Carello *et al.* (1998) study they were rods that were dropped on a hard floor and allowed to bounce. Notwithstanding these complications, two general statements can be made from the present analysis regarding the identification of bar size and shape. First, because the acoustic effect of these properties according to the model is determined strictly by the bar's length L and radius of gyration κ , there are no additional parameters that might be expected to yield better sensitivity for size and shape than is indicated by the NRs of

Fig. 5. Notably, this was not the case for material, where differences in η yielded clearly discriminable changes in $\tau^{(v)}$ beyond those of the other two material parameters, E and ρ . This distinction, in fact, might go some way in explaining the generally better performance observed for the identification of material in past studies. Second, the NRs for frequency in Fig. 5 would support the informal conjecture of past authors that pitch is a salient cue for bar size, or at least bar length (Gaver 1988, 1993a, 1993b). As earlier noted, the estimated Weber fraction for a change in bar length based on the change in frequency is by far the smallest of all physical parameters investigated.

Compared to size and shape, one aspect of bar geometry that does not have a significant spatial acoustic component is hollowness. Lutfi (2001) analyzed the invariant acoustic information for the hollowness of a cylindrical bar using the fluid damping model given by Eq. (6). Consistent with the foregoing analysis for material, the solution was given by a specific relation between the frequency and decay for any one partial. In the same study listeners were asked to judge hollowness in a two-alternative, forced-choice task using synthesized sounds of hollow and solid bars of different materials and sizes. A regression analysis of each listener's trial-by-trial responses showed for half of the listeners a strong correlation with the relation between frequency and decay, but for the other half of the listeners a stronger correlation with f alone. Simulations of an internal noise model applied to these conditions suggested that the reliance on f reflected the robust nature of this cue with respect to sensory noise. The hypothesis can be checked by applying Eq. (15). Recall that the acoustic effect of hollowness for the cylindrical bar is given by $\kappa = \frac{1}{2}\sqrt{r^2 + s^2}$, where s is the inner radius of hollowness. For the solid aluminum bar used in the study ($L = 10$ cm and $r = 2$ cm), Eq. (15) yields a value of $\Delta\kappa = 0.1$ cm for $d' = 3$ based on $\Delta\tau^{(a)}$, and a value of $\Delta\kappa = 0.03$ based on Δf . The actual value of $\Delta\kappa$ in the study was 0.1 cm ($s = 1$ cm) well outside the NR for f , but right on the border of the NR for $\Delta\tau^{(a)}$. A similar outcome is obtained for the wood bar in that study. The predictions of Eq. (15) thus generally support the outcome of the programed simulations by Lutfi (2001).

We conclude by noting some shortcomings of the analysis. First, because the analysis is based on data measuring sensitivity to changes in single tones, it does not take into account possible interactions that can occur among partials (e.g., mutual masking, combination tones, beats, and the like). This is not likely to be a problem for the widely spaced transverse modes of the homogenous bar [cf. Eq. (3)], and even for plates there are likely to be resolved partials that adequately convey information about source properties. Still, there are degenerate cases (e.g., square plates) where few if any partials are likely to be separately resolved. In these cases, Eq. (15) would not apply. Second, the derivation of Eq. (15) assumes that sensitivity to one acoustic parameter is not affected by the value of the others. This, in fact, is largely true over much of the parametric range investigated (Jesteadt et al., 1977; Wier et al., 1977; Abel, 1972; Freyman and Nelson, 1986). The one notable exception is for the discrimination of frequency where threshold Δf can be an order of

magnitude greater for durations below 20 ms than above (Freyman and Nelson, 1986). In the present study the durations were never less than 20 ms, but in applications where they are a correction to Eq. (15) would have to be imposed. Finally, the psychophysical data on which the analysis is based are somewhat weak for the discrimination of the longer decay constants ($\tau > 100$), where our estimates have been taken from data on the discrimination of duration.⁶ Future studies may help to correct this shortcoming. Notwithstanding these limitations, Eq. (15) should provide, at least, a preliminary workable framework for estimating the bound limited sensitivity places on the auditory identification of rudimentary bar attributes.

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¹One can, of course, imagine conditions where the manner of impact could provide information about bar material or geometry, as, for example, where the surface area of contact is affected by bar shape. Generally, however, we expect the effects of bar material and geometry on the acoustics to be the primary source of information about these properties.

²This simply arises from the fact that for $0 < \Delta\phi/\phi \ll 1$, $\ln(1 + \Delta\phi/\phi) \approx \Delta\phi/\phi$.

³Some previous studies have reported different results. However, Buus and Florentine (1991) pointed to the small range of intensity increments used in these studies as the likely reason for the discrepancies.

⁴Note that for all three partials to fall within the audible range the decay of the first partial had to exceed that for which there are available psychophysical data on the discrimination of decay. The outcome was the same, however, when the decay of the first two partials fell within the range of the psychophysical data.

⁵In Giordano and McAdams, 2006 the peak levels of signals ranged from 54 to 72 dB sound pressure level (SPL). Loudness was computed by passing signals through a bank of auditory filters, compressing the output of each filter and then summing the individual loudness of the partials according to the model of Zwicker and Fastl (1999).

⁶An additional complicating factor here is that durations of the sounds can be so long that, in practice, they would need to be truncated. This, in fact, has been the approach taken in a number of past studies.

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