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Novel Models of Intertemporal Valuation: Past and Future Outcomes

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Abstract

Temporal discounting refers to the reduction in the present subjective value of an outcome as a function of the temporal distance to that outcome. Though a number of mathematical models have been proposed to describe this time/value relationship, this search has largely excluded insights from the literature on memory decay. This study examines the utility of memory decay models by comparing the fits of four of these models to fits from established temporal discounting models using past and future temporal discounting data. These results (1) suggest that a single model describes valuation of both future and past outcomes, (2) indicate the exponential-power model, from memory decay literature, is statistically superior in fitting discounting data from both past and future outcomes, and (3) support the advancing perspective of the psychological interconnectedness of the future and past.

Temporal discounting refers to the reduction in the present subjective value of an outcome as a function of the delay to its receipt. Studies of temporal discounting provide insight into intertemporal decision-making: how humans (and animals) are able to make decisions when the outcomes of those decisions occur at different times. Because balancing immediate and delayed outcomes is important in many aspects of life, numerous studies have examined the influence of future outcomes on present behaviors. In many human studies of future discounting (e.g., Myerson et al., 2003), participants are presented with dichotomous choices: a reward (typically money) available at some temporal distance in the future or a smaller reward available immediately. The proximal reward is adjusted from trial to trial to determine an indifference point—the present, subjective value that is equivalent to the distal reward. This procedure is repeated with the same distal reward at different temporal distances, resulting in multiple indifference points that are expected to monotonically decrease as a function of temporal distance.

As studies of temporal discounting determine the value of outcomes in the future, recent research has examined the value of outcomes in the past. In a study evaluating the value of outcomes in the past, Caruso, Gilbert, and Wilson (2008) asked participants to imagine having engaged in a task in the past (e.g., mowing someone else's lawn a week ago) and to place a fair and current financial value on having hypothetically completed the task. Using a similar logic, temporal discounting research has been expanded to examine the reduction of valuation as a function of time into the past. As in studies of future discounting, participants were presented with dichotomous choices: one alternative was a distal outcome in the past and the other was proximal (Yi et al., 2006). Discounting future and past outcomes yielded similar

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overall patterns of results. Importantly, in both future- and past-discounting conditions, the monotonically decreasing data were better fit by the hyperbolic model than the exponential model. This result was further supported by Bickel et al. (2008) in a comparison of cigarette smokers and nonsmokers: the hyperbolic model provided a superior fit over the exponential model to the both the past and future data. Further, smokers discounted past outcomes more than nonsmokers, which was congruent with and replicated the well-established result of smokers discounting future outcomes more than nonsmokers (e.g., Baker et al., 2003; Mitchell, 1999)

Mathematical description of the decreasing function observed in future and past discounting is important for understanding how the valuation process is altered because of time. As temporal discounting research expands into paradigms employing brain-imaging technologies (McClure et al., 2004; 2007) and computational modeling (Busemeyer et al., in press; Redish, 2004), an accurate mathematical model of discounting will become increasingly important, allowing greater understanding of intertemporal decision-making and behavior. Determination of accurate discounting models has important practical implication as well. Different models have different theoretical assumptions that if found to be accurate, signify targets for potential interventions to reduce temporal discounting. Furthermore, different models predict different preference reversal points; time points where interventions might effectively prevent drug relapse, diet failure, and other analogous preference reversal events. Finally, determining similarities and/or differences between how outcomes are valued as a function of time in the past or the future provides opportunities to further illuminate possible relationships between the past and future.

Memory Decay Functions as a Source for Discounting Models

Studies of future and past discounting provide some empirical evidence of a connection between future and past events. While the basis for this connection is not yet understood, a number of scientists have proposed that the ability to project into the future is tied to our ability to remember the past. These proposals include the ideas that (a) estimates of the delay to a future event are directly based on the estimate of time already elapsed (Rachlin, 2000), (b) the mental representation of future events also affect the mental representation of past events (Trope & Liberman, 2003), (c) the ability to project oneself into the future is based on the same processes and systems influencing the ability to project oneself into the past (Buckner & Carroll, 2006; Tulving, 1999), and (d) mental representation of a future event is the pre-experiencing of a future event, influenced by the experience of past events (Attance & O'Neill, 2001). Furthermore, the characteristics used to describe episodic memory (recently evolved, late-developing; Tulving, 2002) are consistent with those used to describe the ability to choose delayed rewards (Bickel & Yi, 2009).

Given the apparent connection between memory and thinking about the future, the extensive literature on modeling memory may provide a useful resource for better understanding intertemporal decision-making. Interestingly, the exponential and hyperbolic models have been proposed to describe both temporal discounting and memory decay (the rate at which a memory is lost as a function of time). Though the hyperbolic model has consistently provided a superior fit to temporal discounting data compared to the exponential model (e.g., Kirby & Markovic, 1995; Madden et al., 1999),¹ a number of other single-parameter models better describe memory decay and may be appropriate for models of temporal discounting.²

¹Two-parameter models reported to provide better fits than the hyperbolic model are Myerson & Green's (1995) hyperboloid, Rachlin's (2006) two-parameter power, and Takahashi et al.'s (2008) Steven's power law models.

²As parameterized with a y-intercept of 1.

To explore potential models of temporal discounting, we reviewed the extensive literature modeling memory decay. The best-fitting model of memory decay is not generally agreed upon, so our selection of models drew on reports comparing numerous models of decay to each other (Rubin & Wenzel, 1996; Wixted & Ebbesen, 1991), and the analysis was limited to models with only one free parameter. The improved fit resulting from additional free parameters necessitates conditioning interpretation of one parameter on set values of the other parameters, which is particularly problematic for longitudinal or population comparisons. We identified six models for inclusion in our analysis of temporal discounting: exponential, hyperbolic, logarithmic, power, exponential-power, and hyperbolic-power.

Potential Discounting Models

The exponential and hyperbolic models predominate in studies of temporal discounting; although the majority of studies have found the hyperbolic function provides a superior fit, we included the exponential function in this study for the sake of comparison to previous results. Furthermore, some research indicates that humans do discount exponentially under certain constraints (Schweighofer et al., 2006). The literature on memory and forgetting support the four remaining models: logarithmic, power, exponential-power, and hyperbolic-power. In particular, Rubin and Wenzel (1996) found that, among 105 potential models of forgetting (including hyperbolic and exponential models), these four models best described 210 published datasets. A previous study of memory decay (Wixted & Ebbesen, 1991) also applied five of our proposed models (hyperbolic-power model was not included). Each panel of Figure 1 displays equivalent discount curves (half-life of 500 time units) for each of the discounting models.

To examine the applicability of the six proposed models to temporal discounting, we reanalyzed a partial data set obtained from temporal-discounting procedures previously reported in Bickel et al. (2008). The six potential models of discounting were fit to data obtained with future- and past-discounting procedures, and the results were compared to determine the best-fitting model of discounting.

METHOD

Participants

Twenty-nine healthy participants (13M, 16F) between 19 and 55 years of age (mean=31.9, SD=11.8) completed all assessments. Participants were free from current drug or alcohol abuse or dependence, with no significant medical or psychiatric disorders. For greater detail on participant characteristics and procedures, please see Bickel et al. (2008).

Procedure

Computerized past and future discounting assessments were completed using a laptop PC running Visual Basic 6.0. Data were collected at individual workstations in a quiet room, under the supervision of research staff. In each trial, participants were required to choose between two hypothetical alternatives: one proximal (e.g., now, 1 hour ago) and one distal (e.g., 1 week from now, 6 months ago). In the future discounting condition, these outcomes were: "Receive [*amount*] right now" and "Wait [*delay*] and then receive [*amount*]." In the past-discounting condition, these outcomes were: "Having gained [*amount*] 1 hour ago" and "Having gained [*amount*] [*delay*] ago." We selected "1 hour ago" as the timing of the proximal outcome in the past-discounting condition to ensure that it was not interpreted as an imminently occurring event ("right now"). Temporally proximate and distal outcomes were presented in command boxes to the left and right of center on the monitor, and the participant used a mouse click to

indicate preference. Following indicated choice on a given trial, the participant confirmed preference before advancing to the next trial.

In all conditions, the proximal outcome was adjusted (i.e., increased or decreased) from trial to trial, while the distal outcome was held constant. Based on the double-limit algorithm (see Johnson & Bickel, 2002 for comprehensive description), the value of the proximal outcome approached the participant's subjective equivalent of the distal outcome. Briefly, a possible proximal value, occurring in 2% increments of the distal outcome, was initially selected from a range of values between zero and the value of the distal outcome. Based on the participant's choices, this range of possible values decreased across trials until the algorithm determined the equivalence between the proximal and distal outcomes. For future discounting, this procedure was employed with six distal outcomes (\$10, \$100, and \$1000 gains and losses) at each of seven temporal future distances to that option (1 day, 1 week, 1 month, 6 months, 1 year, 5 years, and 25 years). For past discounting, this procedure was employed exactly as for the future condition, except the temporal distances were in the past. Future conditions preceded past conditions; order-of-magnitude conditions were counterbalanced.

Statistical Method

For modeling the expected indifference point (EY) at a given time (t), we considered six functions: exponential, hyperbolic, logarithmic, power, exponential-power, and hyperbolic-power (Table 1). Each model has one free parameter (b) that, when estimated, describes the rate of discounting as a function of time.

For each of the 12 conditions (2 [future/past] $\times 2$ [gain/loss] $\times 3$ [magnitude]), we fitted a participant's indifference points with each of the six models to obtain parameter estimates (b) and residual variances from the regressions (mean square errors; MSEs). Spearman rank correlations were calculated to assess agreement among the discounting-parameter estimates (estimated b) provided by the six models. Our primary focus was to compare models' fits to the datasets; MSEs were utilized for this purpose. We analyzed the MSEs within a four-factor repeated-measures analysis of variance (ANOVA). The factors (levels) were Timeframe (future and past), Sign (gain and loss), Magnitude (\$10, \$100, and \$1000), and Model (the six models in Table 1). We allowed for a general (i.e., unstructured) covariance structure among the models fitted to the same dataset and estimated denominator degrees of freedom of F -tests with the Kenward-Roger method (SAS, 2004).³ On all comparisons involving any two models, we computed bootstrapped probabilities of replicating the effect (p_{rep}), as recommended by Killeen (2005), and we omitted any adjustment for the multiple comparisons because this study is exploratory in nature. Please note that the probability of replication (p_{rep}) reflects the likelihood that a result would be replicated under similar conditions.

For effect sizes, we report the generalized η^2 , proposed and denoted as η_G^2 by Olejnik and Algina (2003). All hypotheses tests were two-sided and conducted with a statistical significance level of 0.05.

Results

Problematic Datasets

Of the six models, three (exponential, exponential-power, and hyperbolic) failed to converge on at least one of the 348 datasets analyzed. Of the 174 past-discounting datasets, seven could not be fitted with the exponential model, and four of those seven did not converge for the

³Though the residuals from this analysis exhibited skewness, a nonparametric bootstrap of the ANOVA resulted in the same statistical decisions for all key model comparisons. We thus present the original ANOVA results.

exponential-power model. The hyperbolic and exponential models each failed to converge on one of the 174 future-discounting datasets. In order to reduce any bias in model comparisons, a dataset that could not be fitted by all six models was excluded from analyses.

Agreement in Discounting Parameter Estimates

All Spearman rank correlations of estimated *bs* coming from pairs of models were positive and highly significant. The various levels of Timeframe, Sign, and Amount had no patterns in the Spearman rank correlations; hence, we marginalized over these effects. Logarithmic and exponential models had the *least* rank agreement in their discounting estimates, with a correlation of 0.975.

Model Comparisons

The ANOVA indicated that MSEs differed among the six models ($F_{5,323}=94.38$, $p<.001$, $\eta_G^2=0.59$). Analyses suggested that comparisons of models depended on whether the discounting was of gains or losses ($F_{5,323}=2.68$, $p=0.022$, $\eta_G^2=0.04$), and possibly on past and future discounting ($F_{5,323}=2.28$, $p=.046$, $\eta_G^2=0.03$). After examining these interactions for each pair of models, we assembled the results below to the extent that they affect comparisons among the best-fitting models. No compelling evidence existed for any remaining two-, three- and four-way interactions (all 9 $ps>.166$ and $\eta_G^2s<.029$).

Past discounting—We calculated pairwise differences between the exponential-power model (i.e., the overall best-fitting model) and each other model in the context of past discounting (Figure 1). A mean MSE difference of 0 indicates that the model was equal to the exponential-power model in fitting indifference points, a mean difference less than 0 indicates that the model was superior to the exponential-power model (not observed), and a mean difference greater than 0 indicates that the model was inferior to the exponential-power model. The exponential-power model provided the best fit to past-discounting datasets (Figure 1, first panel), and provided a statistically better fit to all but the hyperbolic-power model ($t_{327}=1.73$, $p=.084$, $p_{\text{rep}}=0.744$; Table 2).

Future discounting—As observed with the past-discounting datasets, the exponential-power model provided a better fit than all other models fitted to future-discounting datasets (Figure 1, second panel) and was statistically so for all but the hyperbolic-power model ($t_{327}=1.72$, $p=.087$, $p_{\text{rep}}=0.777$; Table 2). The comparison of fits between exponential-power and hyperbolic-power models depended on whether the datasets were of discounting gains or losses ($F_{1,327}=4.67$, $p=.031$, $p_{\text{rep}}=0.893$). For discounting losses, the hyperbolic-power model was estimated to fit slightly better than the exponential-power ($t_{327}=0.15$, $p=.881$, $p_{\text{rep}}=0.497$), but for discounting gains, the exponential-power model was statistically a better fit than the hyperbolic-power ($t_{327}=2.60$, $p=.010$, $p_{\text{rep}}=0.940$).

Overall discounting—Across the levels of Timeframe, Sign, and Amount, the exponential-power model provided a statistically lower MSE than all of the other models (Figure 1, third panel). Even though the comparisons between exponential-power and hyperbolic-power models depended on Sign ($F_{1,327}=4.67$, $p=.031$, $p_{\text{rep}}=0.523$), when averaging past and future discounting, the exponential-power model was statistically a better fit than the hyperbolic-power model for gains ($t_{327}=3.30$, $p=.001$, $p_{\text{rep}}=0.905$) and slightly better for losses ($t_{327}=0.20$, $p=.845$, $p_{\text{rep}}=0.540$).

Past/Future Correlations

Establishing a statistical relationship between discount rates for future and past outcomes would provide additional support for conceptualization of a common process for past and future

discounting. Thus, Spearman correlations were conducted with discount rates from comparable past and future conditions using the exponential-power model (Table 3). The correlation between past and future discounting was positive in all cases and was statistically significant in all gains conditions, one losses condition, and all combined conditions (Figure 2).

Discussion

The present analysis compared potential models of intertemporal valuation informed by previous research on temporal discounting and memory decay. Of the six single-parameter models compared, the exponential-power model, with little dispute, best fit past- and future-discounting data. Additionally, the correlations between the past and future gains conditions were all significantly positive. Inconclusive results in the losses condition were generally consistent with previous research on the discounting of losses. This analysis demonstrates that the exponential-power model accurately describes the quantitative process of temporal discounting; furthermore, it supports the growing viewpoint that the future and past are deeply related.

Beyond the strong evidence provided here based on comparing models of discounting, the exponential-power model is theoretically appealing because it synthesizes divergent paradigms regarding the discounting process, and it is consistent with the current understanding of neurocorrelates of discounting behavior. Economists and psychologists continue to disagree on the form of discounting, with economists advocating that the decay of discounting is constant over time (i.e., exponential discounting) and psychologists advocating that the decay slows over time (i.e., hyperbolic discounting); the proposed exponential-power model suggests that both disciplines are correct.

The theoretical insight offered by the exponential-power model is that discounting may indeed occur exponentially, but that humans' (and animals') experience of time is not linear. Specifically, the model proposes that perception of time is described by a power function, with time raised to the $\frac{1}{2}$ power (the hyperbolic-power model, the second-best model in the present analyses, also proposes a power scaling of time). To wit, this model is not unlike the proposed δ and β systems of McClure et al. (2004; 2007). Based on studies of brain activation during intertemporal choice using functional magnetic resonance imaging (fMRI), McClure and colleagues have proposed an exponential discounting process (δ) with a second process to give relatively extra weight to immediate rewards (β); analogously, the exponential-power model submits that outcomes are discounted exponentially, but time is scaled to give relatively more weight to outcomes with very short delays (i.e., relatively immediate rewards). McClure did not specify whether the weighting process occurs because of Weber-Fechner Law (which specifies a logarithmic scaling of time) or Steven's Power Law, (which specifies power scaling of time). The relatively poor fit of the Power model of the present analyses, which is a reduced version of an exponential discounting model with time scaled logarithmically, suggests that time is scaled according to Steven's Power Law for models of intertemporal valuation (the logarithmic model is not the appropriate point of comparison because it implies linear discounting with time scaled logarithmically).

Certainly, acceptance of the exponential-power model as the appropriate descriptor of discounting behavior requires further study. Validation of the model would have important implications for a vast array of suboptimal human behaviors that can be described as preference reversals—circumstances where an individual has a choice between a small reward that is relatively proximal and a larger reward that is relatively distal. When both rewards are far away, individuals frequently prefer the larger, distal reward. As time passes and both proximal and distal rewards approach, however, preference frequently switches to the smaller, proximal reward. Preference reversals have been proposed as the mechanism that explains why the

alcoholic plans to stop drinking (larger, more distal reward) tomorrow, but continues to drink (smaller, more proximal reward) when tomorrow arrives (Rachlin, 2000).

Though all the models tested here (with the exception of the exponential and power models) predict preference reversals, they predict that the preference reversals will occur at different times: e.g., as shown in Figure 1, the hyperbolic model predicts that the preference reversal will occur more distal to the rewards than the exponential-power model predicts. When \$100 is available at time = 0 and \$80 is available at time = 200, the hyperbolic and exponential-power models predict that preference reversals (the point at which the lines cross) would occur at times = 494.5 and 303.5 time units to the \$100, respectively. Given that the \$100 and \$80 occur 200 time units apart, preference reversals would occur 294.5 and 103.5 time units from the \$80, respectively. The timing of this preference reversal is important because it indicates an individual's temporal point of vulnerability to preferring the smaller, more proximal reward. Knowledge of the temporal location of a potential preference reversal could allow a temporally targeted intervention to prevent such an occurrence, greatly improving ability to maintain drug abstinence and prevent relapse, compliance to a diet and exercise regimen, or reinforcing savings and limiting impulsive spending. Thus, the nearly 3-fold difference in the timing of the preference reversal between the hyperbolic and exponential-power models of discounting for this example is substantial. Our finding that the exponential-power model is a better model of temporal discounting suggests that the time of vulnerability is not as temporally distant to the outcomes as suggested by the hyperbolic model by a wide margin. This will need to be empirically tested in the future.

In addition to the proposal of a new model of discounting, the present study also contributes to the developing perspective that time into the past and future are dimensions of psychological distance (Trobe & Liberman, 2003) that are affected by the same systems and processes (Buckner & Carroll, 2007); load on working memory previously was found to affect intertemporal decision-making (Hinson, Jameson, & Whitney, 2003). The observation that a single, intuitively appealing, theoretically justified discounting model accounts for discounting of past and future outcomes further advances the postulation that the way we make decisions about the future is influenced by the way we perceive the past and, possibly, that the way we remember the past is influenced by the way we think of the future.

The primary limitations of the present analysis are procedural. First, we used only hypothetical monetary outcomes in the choice procedures, which allowed for the use of the past discounting assessment as well as comparison with the future discounting assessment. We have observed the same pattern of results with data from discounting of real money gains, but those data are not included here because of procedural differences from the analyzed datasets. Furthermore, a substantial body of research indicates that discounting for real and hypothetical money rewards show no systematic differences (Lagorio & Madden, 2005), and are highly correlated (Johnson & Bickel, 2002; Madden, Begotka, Raiff, & Kastern, 2003). Additionally, no differences in brain activation have been observed in an fMRI study of temporal discounting between real and hypothetical outcomes (Bickel et al., 2009). However, the possibility of differences remains. Second, the order of future/past conditions was not counterbalanced. Though previous research has shown that order of future/past discounting conditions does not account for the observed relationship (Yi et al., 2006), we also cannot rule out a possible order effect in the present study.

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Figure 1.

All six models are parameterized to have an ED50 of 500 time units, thus the same discount rate. Solid curves are for a delayed reward valued at \$100. Dotted curves are for a delayed reward valued at \$80. Preference reversals are indicated with a vertical hash. Preference reversals are mathematically impossible for the exponential and power models.

Figure 2.

Comparison of exponential-power model with other models (pairwise differences of MSEs and 95% confidence intervals) for past and future conditions and overall (average of both conditions). Pairwise difference above 0 indicates that the comparison model provided a worse fit than the exponential-power model. Models are abbreviated as HYP, hyperbolic; HP, hyperbolic-power; EXP, exponential; POW, power; and LOG, logarithmic.

Figure 3. Past plotted by future ranked parameter estimates from the exponential-power model for gains (filled symbols), losses (unfilled), and combined conditions. Circles, triangles, and squares correspond to discounting of \$10, \$100, and \$1000, respectively. The line of equality is included.

Table 1

Candidate Discounting Models and Equations

Model	Equation
Exponential	$EY = e^{-bt}$
Hyperbolic	$EY = 1/(1 + bt)$
Logarithmic	$EY = -b \log_e(t)$
Power	$EY = t^{-b}$
Exponential-Power	$EY = e^{-b\sqrt{t}}$
Hyperbolic-Power	$EY = 1/(1 + b\sqrt{t})$

Table 2Pairwise Differences (p_{rep}) in MSE for Past and Future Conditions, and Averaging Over Both Conditions

Model 1 – Model 2		Past	Future	Overall
EP	EXP	−6.19* (0.957)	−7.02* (0.99)	−6.61* (0.989)
EP	HP	−1.51 (0.744)	−1.48 (0.777)	−1.49* (0.779)
EP	HYP	−2.23* (0.849)	−2.4* (0.916)	−2.31* (0.905)
EP	LOG	−12.49* (0.992)	−9.82* (>.999)	−11.15* (>.999)
EP	POW	−8.33* (0.949)	−13.14* (>.999)	−10.73* (0.999)
EXP	HP	4.68 (0.786)	5.55* (0.887)	5.11* (0.863)
EXP	HYP	3.97 (0.995)	4.62* (>.999)	4.3* (0.999)
EXP	LOG	−6.3* (0.77)	−2.79 (0.691)	−4.55* (0.775)
EXP	POW	−2.13 (0.611)	−6.12* (0.818)	−4.13 (0.732)
HP	HYP	−0.71 (0.561)	−0.92 (0.608)	−0.82 (0.584)
HP	LOG	−10.98* (>.999)	−8.34* (>.999)	−9.66* (>.999)
HP	POW	−6.82* (0.977)	−11.66* (>.999)	−9.24* (>.999)
HYP	LOG	−10.27* (0.923)	−7.42* (0.945)	−8.84* (0.964)
HYP	POW	−6.1* (0.82)	−10.74* (0.976)	−8.42* (0.944)
LOG	POW	4.16* (0.83)	−3.32 (0.957)	0.42 (0.545)

Note: Values have been multiplied by 1000 to eliminate place-holding 0s.

* Statistically significant

Abbreviations: EP, exponential-power; EXP, exponential; HP, hyperbolic-power; HYP, hyperbolic; LOG, logarithmic; POW, power.

Table 3

Spearman Correlations Between Discount Rates of Corresponding Past/Future Conditions Obtained With the Exponential-Power Model (e.g., Past \$10 Gains and Future \$10 Gains)

Amount	Gain	Loss	Combined
\$10	.65**	.24	.44**
\$100	.66**	.54*	.59**
\$1000	.66**	.33	.51**

*
 $p < .01$

**
 $p < .001$