Published in final edited form as:

IEEE Trans Ultrason Ferroelectr Freq Control. 2005 October ; 52(10): 1699.

A Finite-Element Method Model of Soft Tissue Response to Impulsive Acoustic Radiation Force

Mark L. Palmeri [Student Member, IEEE], Amy C. Sharma, Richard R. Bouchard, Roger W. Nightingale, and Kathryn R Nightingale [Member, IEEE]

The authors are with Duke University, Department of Biomedical Engineering, Durham, NC 27708

Mark L. Palmeri: mark.palmeri@duke.edu; Amy C. Sharma: ; Richard R. Bouchard: ; Roger W. Nightingale: ; Kathryn R Nightingale:

Abstract

Several groups are studying acoustic radiation force and its ability to image the mechanical properties of tissue. Acoustic radiation force impulse (ARFI) imaging is one modality using standard diagnostic ultrasound scanners to generate localized, impulsive, acoustic radiation forces in tissue. The dynamic response of tissue is measured via conventional ultrasonic speckle-tracking methods and provides information about the mechanical properties of tissue. A finite-element method (FEM) model has been developed that simulates the dynamic response of tissues, with and without spherical inclusions, to an impulsive acoustic radiation force excitation from a linear array transducer. These FEM models were validated with calibrated phantoms. Shear wave speed, and therefore elasticity, dictates tissue relaxation following ARFI excitation, but Poisson's ratio and density do not significantly alter tissue relaxation rates. Increased acoustic attenuation in tissue increases the relative amount of tissue displacement in the near field compared with the focal depth, but relaxation rates are not altered. Applications of this model include improving image quality, and distilling material and structural information from tissue's dynamic response to ARFI excitation. Future work on these models includes incorporation of viscous material properties and modeling the ultrasonic tracking of displaced scatterers.

I. INTRODUCTION

Acoustic radiation force-based imaging modalities are currently being studied by many groups, including: vibroacoustography, kinetic acoustic vitreoretinal examination (KAVE), supersonic imaging, shear wave elasticity imaging (SWEI), and high intensity focused ultrasound (HIFU) ablation monitoring. Vibroacoustography uses frequency-shifted, confocal acoustic beams to generate an oscillating radiation force within tissue and the tissue response is monitored with either a hydrophone [1]–[3] or ultrasonic methods [4]. The KAVE method uses radiation force to generate steady-state stresses within soft gels and the vitreous of the eye, and ultrasonic displacements are monitored to characterize the response of these materials [5]. Supersonic imaging [6] and shear wave elasticity imaging (SWEI) [7], [8] monitor the shear waves generated by short-duration, acoustic radiation force to characterize the shear modulus of the medium. HIFU transducers also are being studied to generate radiation force in conjunction with a piston to track displacements in order to monitor ablation treatment [9]. In addition to soft tissue imaging, radiation force is being used to manipulate ultrasonic contrast agents in vitro and in vivo [10]–[12].

Acoustic radiation force impulse (ARFI) imaging, which is the focus of this paper, uses standard, diagnostic, ultrasound scanners to generate localized impulsive acoustic radiation forces in tissue. The localized, micron-scale tissue displacements generated by radiation force are tracked through time using ultrasonic, correlation-based methods. The dynamic response of tissue to this impulsive radiation force reflects its mechanical properties. ARFI imaging is unique from other radiation force modalities in that a spatially localized distribution of radiation force is applied for a short amount of time ($<100 \mu\text{s}$), and the dynamic response of tissue in the region of excitation (ROE) is monitored, representing the mechanical impulse response of tissue in that region [13]–[15]. Spherical lesions that are stiffer than their surroundings exhibit characteristic dynamic responses to impulsive acoustic radiation force excitation that distinguish them from the surrounding tissue.

This paper presents a finite-element method (FEM) model that accurately simulates the dynamic response of homogeneous tissues of varying stiffnesses, with and without stiffer spherical inclusions, to focused applications of impulsive, acoustic radiation force. The dynamic response of this model has been validated with experimental ARFI data from calibrated tissue-mimicking phantoms. Section II presents a derivation of the governing equations for elastodynamics, along with the derivation of acoustic radiation force in viscoelastic tissues, which are solved using the FEM. Section III provides an overview of the implementation of the FEM to solve these governing elastodynamic equations. Section IV presents a comparison between the dynamic FEM results and experimental data from calibrated phantoms, and demonstrates the dependence on the dynamic behavior of tissue to ARFI excitations on ultrasonic attenuation, density, Poisson's ratio, and elasticity.

II. BACKGROUND

A. Viscoelastic Dynamics

To a first-order approximation, at low excitation frequencies ($<100 \text{ kHz}$), soft tissues can be described as linear, elastic, isotropic solids. The balance of linear momentum for such a material can be expressed as (Table I):

$$\sigma_{ij,j} + \rho b_i = \rho f_i, \quad (1)$$

where $\sigma_{ij,j}$ [N/m^3] represents the divergence of the stress tensor, ρ [kg/m^3] is the material density, b_i [m/s^2] represents an external, steady-state acceleration (e.g., gravity), and f_i [m/s^2] represents an externally applied acceleration (e.g., radiation force). The constitutive equation that relates stress to strain in a linear, elastic solid is given by:

$$\sigma_{ij} = \lambda \Delta \delta_{ij} + 2\mu \varepsilon_{ij}, \quad (2)$$

where λ and μ represent the Lamé constants for the material [16]. The strain (ε_{ij}) is defined as the symmetric displacement gradient using the following relationship:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (3)$$

where u_i represents particle displacement. Note that both σ_{ij} and ε_{ij} are symmetric tensors. Δ represents the material dilatation given by:

$$\Delta = u_{i,i} = \varepsilon_{ii}. \quad (4)$$

Time-independent body forces, such as gravity, can be neglected in (1) because this linear equation can be decomposed into elastostatic and elastodynamic components that are separable [16]. To derive an expression for displacement in the absence of external body forces (b_i), the constitutive relationship (2) is substituted into the balance of linear momentum expression (1), with $b_i = 0$ and $f_i = \ddot{u}_i$, to yield:

$$\sigma_{ij,j} = \lambda \Delta u_i + \mu (u_{i,jj} + u_{j,ji}), \quad (5)$$

$$\rho \ddot{u}_i = (\lambda + \mu) u_{i,jj} + \mu u_{j,ji}. \quad (6)$$

The displacement (6) can be written in vector form as [17]:

$$(\lambda + \mu) \nabla (\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} = \rho \ddot{\vec{u}}. \quad (7)$$

The displacement field in a solid can be decomposed such that [17]:

$$\vec{u} = \nabla \psi + \nabla \times \vec{W}, \quad (8)$$

where ψ represents the dilatational (scalar, time-dependent, volume change) components of displacement that occur in the direction of wave propagation (longitudinal), and $\vec{W}$ represents the equivoluminal components of displacement that occur transversely to the direction of wave propagation. These scalar and vector displacement fields can be determined by using a Helmholtz decomposition [17]. Substituting (8) into (6) yields:

$$\nabla \left[(\lambda + 2\mu) \nabla^2 \psi - \rho \frac{\partial^2 \psi}{\partial t^2} \right] + \nabla \times \left[\mu \nabla^2 \vec{W} - \rho \frac{\partial^2 \vec{W}}{\partial t^2} \right] = 0. \quad (9)$$

The displacement components shown in (9) are separable, and each of them takes the form of a wave equation [17]. For example, in one direction ($\hat{x}$), these equations can be expressed as:

$$0 = \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c_L^2} \frac{\partial^2 \psi}{\partial t^2}, \quad (10)$$

$$0 = \frac{\partial^2 \vec{W}}{\partial x^2} - \frac{1}{c_L^2} \frac{\partial^2 \vec{W}}{\partial t^2}, \quad (11)$$

$$c_L = \sqrt{\frac{(\lambda + 2\mu)}{\rho}}, \quad (12)$$

$$c_T = \sqrt{\frac{\mu}{\rho}}. \quad (13)$$

The constants c_L and c_T represent the longitudinal and transverse (shear) wave speeds, respectively. Assuming constant tissue density, these equations reveal c_L to be proportional to the material uniaxial modulus given by $\sqrt{\lambda+2\mu}$, which in fluids is commonly approximated by the bulk modulus of the medium, and c_T to be proportional to $\sqrt{\mu}$, where μ is the material shear modulus. The elastic wave speeds, c_L and c_T , also are commonly expressed in terms of the Young's modulus (E) and the Poisson's ratio (ν) as follows [17]:

$$c_L = \sqrt{\frac{(1-\nu)E}{(1+\nu)(1-2\nu)\rho}}, \quad (14)$$

$$c_T = \sqrt{\frac{E}{2(1+\nu)\rho}}. \quad (15)$$

In solids, attenuation of waves occurs due to the viscous properties of materials. Discretely, viscoelasticity can be represented by a combination of Maxwell and Voigt lumped-parameter systems. In continua, the time-dependent behavior of the material can be represented by complex Lamé constants, λ^* and μ^* . These frequency-dependent material parameters tend to be empirically determined for soft tissue. For longitudinal waves established in a viscoelastic solid during ultrasonic wave propagation, (12) becomes:

$$c_L^* = \sqrt{\frac{\lambda^* + 2\mu^*}{\rho}}. \quad (16)$$

A solution to (10) using the expression for the longitudinal wave velocity in (16) has the form:

$$\vec{\psi} = A e^{-\alpha x + j(\omega t - kx)} \hat{x}, \quad (17)$$

where A is the original wave amplitude, and k represents the wave number (ω/c_L^*). The attenuation coefficient for the material, when excited at ultrasonic frequencies, is represented by α and can be represented as a function of λ^* and μ^* , as determined empirically by the frequency-dependence of the material. Given these expressions, the particle velocities ($\vec{v} = j\omega \vec{\psi}$) can be represented by a sinusoid that is phase shifted by 90° with respect to the displacement function:

$$\vec{v} = j\omega A e^{-\alpha x + j(\omega t - kx)} \hat{x}. \quad (18)$$

B. Linearly Viscous Fluids and Acoustic Radiation Force

The generation of acoustic radiation force occurs through a transfer of linear momentum from an acoustic wave to the propagation medium. This process has been thoroughly described in

the literature for both gases and fluids in which energy-loss mechanisms are present, either through the absorption or reflection of acoustic energy [18]. The derivation of acoustic radiation force cannot be directly obtained from the stress-strain relationships for a purely elastic material because no energy-loss mechanisms are described that would allow for a change in wave momentum. Also, at ultrasonic frequencies, soft tissues do not support shear stresses; thus, they often are described as viscous fluids. Therefore, expressions that describe radiation force in linearly viscous fluids are extended to viscoelastic solids herein. Biological tissues have viscoelastic properties that are strongly frequency-dependent. At low excitation frequencies (<100 kHz), the materials can be regarded as purely elastic; and at ultrasonic excitation frequencies, acoustic wave attenuation is appreciable and leads to heating and generation of acoustic radiation force, so the tissues can be regarded as viscous fluids [19].

1. Linearly Viscous Fluids—In a linearly viscous (Newtonian) fluid, the constitutive relationship can be expressed as:

$$\sigma_{ij} = -p\delta_{ij} + \lambda_f \Delta \delta_{ij} + 2\mu_f D_{ij}, \quad (19)$$

where D_{ij} represents a rate of deformation tensor defined as:

$$D_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \quad (20)$$

where v represents the particle velocity, $\Delta = D_{ii}$, λ_f is a fluid Lamé constant, and μ_f represents the viscosity of the fluid. The latter two terms on the right-hand side of (19) represent the viscous stress tensor and are dependent on the rate of deformation (D_{ij}); p is the scalar pressure that does not depend on the rate of deformation.

To arrive at the Navier-Stokes equations of motion for an incompressible Newtonian fluid, the total particle acceleration is defined as [20]:

$$a_i = \dot{v}_i + v_j v_{i,j}, \quad (21)$$

where the first term represents the local particle acceleration, and the second term represents the convective acceleration of the particle. Substituting (19) and (21) into the balance of linear momentum equation (1) yields the Navier-Stokes equation [20]:

$$-\rho \dot{v}_i + \mu_f \nabla^2 v_i + \rho b_i = \rho(\dot{v}_i + v_j v_{i,j}). \quad (22)$$

As was shown in the equations of dynamics for a viscoelastic solid, the solution for the velocity field created by acoustic wave propagation in a linearly viscous fluid (22) also can be represented by (18). The dynamics of a viscoelastic solid, therefore, can be viewed as a combination of the properties of an elastic solid and a linearly viscous fluid [21].

2. Acoustic Radiation Force—The derivation of radiation force comes from the fact that the viscous (μ_f) terms in (22), along with an additional bulk viscosity term (μ'_f), represent energy loss described by a frequency-dependent attenuation coefficient (α) [19].

In a linearly viscous fluid, the radiation force generated by a propagating plane wave is proportional to the time-average change of particle velocity in the direction of wave propagation [19]. Using a perturbative expansion of (1) with respect to density (ρ), pressure

(p), and particle velocity $\vec{v}$, radiation force can be related to the change in momentum from the second-order terms in the expansion such that the second-order balance of linear momentum can be expressed as [18]:

$$\vec{F} = \nabla p_2 - \mu_f \nabla^2 \vec{v}_2, \quad (23)$$

$$\vec{F} = \rho \langle \vec{v}_1 \nabla \cdot \vec{v}_1 + \vec{v}_1 \nabla \vec{v}_1 \rangle, \quad (24)$$

where $\langle \rangle$ represents the time-average quantity, v_1 and v_2 are the first and second-order terms in the expansion of particle velocity, p_2 is the second-order pressure term, and the divergence of particle velocity is assumed to be negligible. $\vec{F}$ in (24) represents a time-average transfer of momentum from the wave to the material. For a plane wave, the expression for radiation force in (24) can be reduced to:

$$\vec{F} = 2\rho \langle \vec{v} \vec{v}_x \rangle. \quad (25)$$

The direction of this radiation force occurs in the direction of wave propagation (i.e., the Poynting vector for the acoustic wave) [18]. Substituting (18) into (25), the magnitude of this acoustic radiation force can be expressed as [18]:

$$|\vec{F}| = A^2 e^{-2\alpha x} \rho \alpha. \quad (26)$$

Therefore, at any given spatial location, the magnitude of acoustic radiation force, F [kg/(s²cm²)], is:

$$|\vec{F}| = \frac{2\alpha I}{c}, \quad (27)$$

where F is in the form of a body force, c [m/s] is the sound speed, α [Np/m] is the absorption coefficient of the tissue, and I [W/cm²] is the temporal average intensity at that spatial location.

C. Acoustic Radiation Force Impulse Imaging

ARFI imaging is a radiation, force-based imaging method that studies the local mechanical properties of tissue and uses short duration (<100 μ s), high-intensity, acoustic pulses to generate localized displacements in tissue. The tissue recovery response is monitored using ultrasonic, correlation-based methods. Images of two-dimensional regions of interest (ROIs) are generated by sequentially interrogating multiple spatial locations, as is done in color Doppler imaging. During the application of high-intensity pulses, energy is absorbed by the tissue that results in the generation of acoustic radiation force and tissue displacement.

The spatial distribution of the radiation force field is determined by the acoustic parameters of the transmitter along with the attenuation and sound speed of the tissue. In soft tissues, the majority of ultrasonic attenuation occurs through absorptive mechanisms [19]; and (27) can be used to model radiation force fields associated with complex intensity field geometries by computing the temporal average intensity at points in a three-dimensional ROI. The shape of

this intensity field is dependent on the transducer focal configuration, which can be characterized by the dimensionless f-number ($F/\#$) of the system:

$$F/\# = \frac{z}{d}, \quad (28)$$

where z is the acoustic focal length, and d is the active aperture width. The result is a continuous, variable-magnitude, body force applied throughout the tissue within the geometric shadow of the transducer.

D. Finite-Element Methods

Analytically solving for the dynamic behavior of tissue in response to geometrically complicated distributions of impulsive body forces is nontrivial. Mathematical models using the Khokhlov-Zabolotskaja-Kuznetsov (KZK) equation of the mechanical response of tissue to acoustic radiation force in dissipative media have been developed, but they cannot easily model a variety of transducer configurations and acoustic material properties [8], [33], [35]. The analysis of stress and displacement fields in continua lends itself to numerical methods. Lumped-parameter systems containing mass, elastic, and viscous components are very useful for modeling the dynamic behavior of discrete masses, but they become more cumbersome when used to model deformations in three dimensions. Bercoff *et al.* [6] and Walker [34] have used simulated acoustic beam intensity patterns and Greens function analysis to simulate the displacement fields that result from acoustic radiation force excitation of solids. In this paper, the dynamic displacement fields associated with ARFI imaging were modeled using FEM to solve the three-dimensional, weak-form equations of elastodynamics [22].

III. METHODS

A. FEM Pre-Processing

1. Mesh Generation—A three-dimensional, rectangular, solid mesh was assembled using eight-noded, linear, elastic, brick elements (HyperMesh, Altair Computing Inc., Troy, MI). The mesh extended 7.5 mm in elevation, 35 mm laterally, and 25 mm axially. The elements within the geometric shadow of the transducer (17 mm laterally by 2.7 mm in elevation, for all depths) were uniform cubes with a node spacing of 0.2 mm, and the elements were biased to be larger farther from the transducer's geometric shadow in the lateral-elevation plane.

There was a total of 283,122 nodes and 265,000 elements in the mesh. The spatial extent of the model allowed displacement and stress fields to decay to negligible levels ($<0.1\%$ of their maximum values in the model) before reaching the mesh boundaries for the duration of the simulation. This ensured that waves generated during impulsive tissue excitation were not reflected back into the region of interest. The finer mesh density within the ROE allowed accurate simulation of the spatial distribution and the subsequent tissue response to acoustic radiation force excitation, and allowed for more accurate simulation of ARFI imaging using multiple lateral interrogations. Larger element sizes away from the transducer's geometric shadow allowed for accurate modeling of a tissue continuum, including its inertial contributions while reducing the model's computational overhead.

The model used elevational plane-symmetry. The bottom surface of the model, opposing the transducer, was fully constrained; the top (transducer) surface of the model was constrained to motion in the lateral-elevation plane. All other faces in the model were allowed to have full degrees of freedom. Tissue and spherical inclusions were modeled with varying Young's moduli, a Poisson's ratio of 0.499 that approaches incompressibility conditions, and an attenuation of 0.7 dB/cm/MHz to match the calibrated phantoms (Section III-C,2). A discussion

of why this Poisson's ratio was chosen is provided later in this paper. Eight-noded brick elements were chosen over four-noded tetrahedral elements, at the expense of more computational overhead, to reduce artifacts associated with element locking when modeling nearly incompressible materials.

2. Intensity Measurement—When possible (e.g., for the tracking beams), intensity measurements were made in accordance with FDA guidelines [23]. However, exact quantification of the pushing beam intensity fields was not possible due to saturation effects in water at these relatively high frequencies (6–7 MHz) and transmit pressures [24]. A comparison method was implemented, using linearly characterized, low-amplitude pulses quantified with a calibrated hydrophone. ARFI imaging was performed in a compliant phantom that allowed for a displacement of $0.43\text{ }\mu\text{m}$ when using the low-voltage intensity configuration. When the transmit voltage was maximized and ARFI images were acquired in a typical, high-intensity mode, displacements of $148\text{ }\mu\text{m}$ were generated in the same compliant phantom. Assuming a linear relationship between intensity and phantom displacement, the ratio of the displacements reflected the ratio of the intensities. These empirically determined, linearly extrapolated, small-signal derated fields have been shown to overestimate the true focal intensity fields [24]. For more discussion about the measurement of these intensity fields, along with the simulation of the three-dimensional distribution of the acoustic energy used in ARFI excitations, the reader is referred to Palmeri and Nightingale [25].

3. Simulating Intensity Fields—FIELD II¹ [26], a linear acoustic field simulation software program, simulated the acoustic intensity fields associated with the high-intensity pulses used for ARFI imaging (Siemens VF10-5 linear transducer: one row of acoustic elements, excitation center frequency of 6.67 MHz, aperture width dependent on the number of active elements; Siemens Medical Solutions USA, Inc., Ultrasound Division, Issaquah, WA). The transducer was simulated as F/3.8 in elevation and F/1.3 laterally, with a variable lateral focal depth and a fixed elevation focal depth near 20 mm, corresponding to the experimental ARFI imaging setup used in all models, unless otherwise specified. This same method has been successfully implemented to model the heating associated with ARFI imaging and other acoustic radiation force-based imaging modalities [25].

Three-dimensional intensity fields were computed, normalized, and thresholded at 1% of the maximum, computed intensity to reduce computational overhead of the model. These normalized intensities were scaled to a peak in situ, time-average intensity value of 1000 W/cm^2 , when 69 elements were fired in an F/1.3 focal configuration for $45\text{ }\mu\text{s}$ (300 cycles at 6.67 MHz) at a focal depth of 20 mm. This in situ intensity is an empirically determined value during ARFI imaging using a linear extrapolation of small-signal, derated fields, as outlined in Section III-A,2 [25], [27]. The corresponding radiation body force values, as determined by (27), then were converted to nodal point loads by concentrating the body force contributions over an element volume. For locations within $\pm 10\%$ of the focal depth, point loads were directed purely in the axial direction. For shallower locations closer to the transducer, point loads were directed toward the focal point; and for deeper locations, point loads were directed away from the focal point, consistent with the wave-propagation Poynting vector.

B. FEM Implementation and Postprocessing

The balance of linear momentum (1) was solved numerically with the commercially available FEM package, LS-DYNA3D (Livermore Software Technology Corporation, Livermore, CA), using an explicit, time-domain, integration method. Single-point quadrature was performed with hourglassing control to avoid element locking and to reduce numerical artifacts. For more

¹<http://www.es.oersted.dtu.dk/staff/jaj/field/>

detail regarding the specifics of FEM quadrature rules to solve the equations of motion and hourglass numerical damping methods, the reader is referred to Hughes [22]. Postprocessing of dynamic displacement and stress fields was performed using LS-PREPOST (Livermore Software Technology Corporation, Livermore, CA) and custom-written MATLAB (The Mathworks Inc., Natick, MA) code.

C. Experimental Validation

1. Ultrasound Scanner Configuration—Siemens SONOLINE Antares™ and Siemens SONOLINE Elegra™ ultrasound scanners (Siemens Medical Solutions USA, Inc., Ultrasound Division, Issaquah, WA), using VF10-5 (center frequency of 6.67 MHz) and 75L40 (center frequency of 7.2 MHz) linear arrays were used for all experimental ARFI imaging presented in this paper. Unless otherwise specified, the transducers were laterally focused at 15 mm with an F/1.3 focal configuration, and focused near 20 mm in elevation with an F/3.8 focal configuration.

ARFI imaging uses two types of beams to generate and track displacement in tissue: high-intensity pushing beams and conventional B-mode tracking beams. A reference tracking beam is fired, followed by a pushing beam. The pushing beams were typically generated using the maximum system transmit voltage, with pulses similar to those used for color Doppler; however, apertures were unapodized with longer pulse lengths (e.g., 200 cycles at 6.67 MHz). As mentioned in Section III-A,2, in situ measurements of I_{SPTA} values were made using small-signal, linear-extrapolation techniques; the spatial distribution of these intensity fields has been explored in more detail in the thermal analysis of ARFI excitation by Palmeri and Nightingale [25]. The pushing beam was followed by a series of tracking beams fired at a pulse repetition frequency (PRF) up to 12.5 kHz for up to 5 ms after the ARFI excitation. The pushing and tracking beams were co-located, and adjacent interrogations laterally were separated by 0.2 mm. Summed radio frequency (RF) echo data were stored for off-line processing, for which one-dimensional normalized cross correlation was used between reference and tracking lines with a 0.3 mm kernel and 99% overlap [27], [28].

2. Tissue-Mimicking Phantoms—A computerized imaging reference systems (CIRS) tissue-mimicking phantom (CIRS, Norfolk, VA) and gelatin-based tissue-mimicking phantoms [29] were used for experimental validation of the models. The CIRS phantom was designed to have an ultrasonic attenuation of 0.7 dB/cm/MHz and a sound speed of 1540 m/s. Additionally, gelatin-based, tissue-mimicking phantoms were fabricated using a combination of 1-Propanol and water to adjust sound speed to near 1540 m/s (the sound speed of biological tissues [30]), and 4239 vein graphite powder (<20 μ m particle diameter, Superior Graphite Co., Chicago, IL) to adjust the attenuation of the phantoms to be approximately 0.7 dB/cm/MHz [29]. Varying gelatin bloom number² (Vyse Gelatin Co., Schiller Park, IL) was used to control phantom stiffness, with higher bloom numbers yielding stiffer phantoms [29].

An MTS hydraulic actuator (MTS, Minneapolis, MN) was used to quantify Young's modulus of phantom materials. One circular aluminum plate was attached to a hydraulic actuator, and another plate was attached to a 1 lb Sensotec load cell (Honeywell Sensotec, Columbus, OH). Cylindrical phantom samples, 2.5 cm in height and 5.2 cm in diameter, were placed between the two plates coated with glycerol, with assumed frictionless boundary conditions between the sample and plates. A 5% precompression strain was applied to the samples to ensure uniform contact between plates and sample. Strains up to 15% were applied to samples in six independent ramp functions, ranging from 0.1 to 8 seconds. Linear regressions were performed on the stress-strain data during the ramp, and Young's moduli were determined by the slope

²Bloom is the weight (g) required to push a piston of standard shape 4 mm into a gelatin gel that has set for 18 hours at 10°C.

of these regressed lines. All phantom samples were measured in a single session to reduce variations in temperature between samples. Table II shows the measured Young's moduli of the gelatin-based phantoms and homogeneous CIRS phantom samples.

Shear wave speeds can be reconstructed to measure the Young's modulus of the phantoms (15), and the accuracy of that reconstruction can be validated using the FEM models. To reconstruct the shear wave speeds (c_T) from the experimental and ARFI data, axial displacement data is substituted into (11). This method has been demonstrated by Bercoff *et al.* [6] and Nightingale *et al.* [14].

IV. RESULTS

A. Effects of Poisson's Ratio

The effects of varying Poisson's ratio (ν) for the model were evaluated, as illustrated in Fig. 1. As ν increases, the peak displacement decreases. At ultrasonic excitation frequencies (>1 MHz), the Poisson's ratio must be very close to 0.5 to support a sound speed of 1540 m/s. However, at lower excitation frequencies (<100 kHz), as are used with ARFI imaging, the Poisson's ratio has been measured in muscle and prostate to range from 0.490 to 0.499 [31].

B. ARFI Imaging Displacement Fields in Homogeneous Media

Fig. 2(a) shows the simulated distribution of acoustic radiation force in the axial-lateral plane, centered in elevation, for $\alpha = 0.7$ dB/cm/MHz and an F/1.3 focal configuration focused at 20 mm, at which the transducer would be centered on the top of the image, and brighter pixels correspond to greater radiation force values. Figs. 2(b)–(d) show normalized displacement profiles in this same plane for an 8.5 kPa material 0.6, 1.0, and 2.2 ms after excitation, with brighter pixels indicating greater displacement away from the transducer. Figs. 2(e)–(g) show the corresponding normalized experimental displacement fields in the 100 bloom phantom for the same time steps. Throughout the rest of this paper, displacement profiles will be shown as a function of axial position, centered laterally, and as a function of lateral position at the focal depth.

The ARFI imaging displacement fields were compared between FEM models and experimental data in four different homogeneous media: CIRS, 100 bloom, 150 bloom, and 200 bloom phantoms. The MTS-measured Young's moduli of these phantoms are summarized in Table II. Fig. 3, row A shows data from the CIRS phantom (focal depth of 15 mm, F/1.3 focal configuration). Rows B–D show data from the 100 bloom, 150 bloom, and 200 bloom gelatin phantoms, respectively (focal depth of 20 mm, F/1.3 focal configuration). Subplots i–iii show normalized axial displacement profiles, laterally centered in the ROE as a function of depth away from the transducer. Subplots iv–vi show normalized axial displacements at the focal depth of 15 mm, spanning ± 6 mm laterally from the center of the ROE for different specified times after cessation of a 45 μ s ARFI excitation pulse. The solid lines represent the FEM data, and the dashed lines represent experimental data (mean of six independent trials) acquired with the VF10-5 transducer. The shear wave speeds measured from the FEM and experimental ARFI data [14] are summarized in Table III. Table IV shows a comparison of the maximum displacements for the FEM and experimental data for each of the phantoms, along with the normalization factors that were used to generate the plots in Fig. 3. Table V shows the reconstructed Young's moduli using the shear wave speeds in Table III.

C. Impact of Poisson's Ratio, Density, and Attenuation

In addition to elastic modulus, there are several other parameters in this model that could affect the dynamic behavior of tissue in response to an ARFI excitation, including Poisson's ratio, density, and ultrasonic attenuation. Fig. 1 shows the effects of Poisson's ratios on displacement

behavior at the focal point. Fig. 4 shows Poisson's ratios ranging from 0.48–0.4999 affect the displacement profiles in the axial and lateral dimensions, as were shown in Fig. 3. An F/1.3 focal configuration, focused at 20 mm, was simulated, with $E = 8.5$ kPa, $\rho = 1.0$ g/cm³, and $\alpha = 0.7$ dB/cm/MHz.

Fig. 5 shows the impact of small variations in density from 0.9–1.1 g/cm³ on the dynamic behavior of tissue in response to an ARFI excitation, in the axial and lateral dimensions. An F/1.3 focal configuration, focused at 20 mm, was simulated, with $E = 8.5$ kPa, $\nu = 0.499$, and $\alpha = 0.7$ dB/cm/MHz.

Fig. 6 shows the impact of ultrasonic attenuations ranging from 0.5–1.0 dB/cm/MHz on the dynamic behavior of tissue in response to an ARFI excitation in the axial and lateral dimensions. An F/1.3 focal configuration, focused at 20 mm, was simulated, with $E = 8.5$ kPa, $\rho = 1.0$ g/cm³, and $\nu = 0.499$.

D. ARFI Imaging Displacement Fields with a Stiff Spherical Inclusion

The dynamic displacement fields generated in response to ARFI excitation in a medium with a stiff, 3 mm diameter, spherical inclusion were both modeled and measured experimentally in gelatin phantom A. Row A of Fig. 7 shows the B-mode image, the experimental ARFI image, and the FEM axial displacement image (from left to right) 0.4 ms after the cessation of the ARFI excitation with the 75L40 transducer. The displacement images are on the axial-lateral plane, centered in elevation, with the transducer located at the top of the images. Rows B and C show the corresponding images for 0.9 ms and 1.8 ms after cessation of the ARFI excitation. Figs. 8(a)–(c) show corresponding normalized axial displacement profiles laterally centered in the ROE as a function of axial depth away from the transducer. Figs. 8(d)–(e) show normalized axial displacements laterally at a depth of 15.5 mm (center of the spherical inclusion). The FEM data is represented by solid lines, and experimental data is represented by dashed lines. The dotted lines represent the boundaries of the spherical inclusion.

V. DISCUSSION

A. Choice of Modeled Poisson's Ratio & Young's Modulus

Fig. 1 shows that there is a 25% increase in the magnitude of the peak displacement at the focus for a 0.0009 increase in Poisson's ratio, but there are no significant differences in the loading and relaxation times of the tissue. The greater Poisson's ratio, though, demands a much smaller, explicit, time step, as determined by the increase in longitudinal wave velocity (14). This results in run times that are approximately 3.3 times longer for the $\nu = 0.4999$ model than for $\nu = 0.499$. Therefore, a $\nu = 0.499$ was chosen for all of these models, with the knowledge that there may be some discrepancy in the absolute displacement values due to this choice of Poisson's ratio. Additionally, the relaxation of tissue after ARFI excitation is closely related to shear wave speed. As (15) shows, varying ν from 0.490 to 0.499 only varies the shear wave speed by approximately 0.003 m/s for materials with Young's moduli ranging from 1 to 100 kPa.

There were several variables present in making the MTS-measurements of the Young's moduli of the phantom samples shown in Table II. Because all measurements were made in the same session, variations in temperature between samples were minimized. Variations in sample geometry were minimized by using the same phantom molds for all samples. The errors introduced by assumptions in slip boundary conditions at the plates and incompressibility of the phantom materials were constant between all samples; therefore, these assumptions would not have an impact on the relative stiffnesses measured by the MTS system. The ARFI imaging measurements of shear wave speeds also were made within 24 hours of the MTS measurements to reduce the potential for changes in the phantom stiffness that can occur with time.

The stress-strain relationships determined by the MTS system were made under strain rates of 0.2 to 1.5 Hz, with standard deviations of <0.1 kPa for all phantom samples, indicating little frequency-dependence in the Young's modulus at these strain rates. Although ARFI excitations occur in less than 50 μ s, the dynamic behavior of the phantoms after the ARFI excitation appears to be dictated by the shear wave speed of the tissue. The absence of viscous material properties in the model may cause overestimation of the maximum displacement achieved after ARFI excitation in tissue. The presence of viscosity in tissue also may lead to phase dispersion in the propagating shear wave that the models also would not predict.

B. Comparison of Dynamic Behavior in FEM Model and Experimental Studies

Fig. 3 demonstrates good temporal agreement between FEM models and experimental ARFI data in homogeneous phantoms of varying stiffness. Notice that, immediately after ARFI excitation, displacement is localized around the focal depth, and it is an accurate representation of the radiation force field, as demonstrated in Fig. 2. Later in time [Fig. 3(ii)–(iii)] greater relative displacements are observed in the near field as compared with the focal point, again with good agreement between models and experiments. This shift of displacement into the near field is dictated by shear wave propagation away from the ROE. As shown in Fig. 2(b) and (e), the lateral extent of the displacement field is larger in the near field than at the focus. As shear waves propagate away from the ROE, the tissue recovers quicker at the focus than it does in the near field due to these differences in lateral extent of the initial displacement field.

Within 1.5 ms after ARFI excitation, the axial position of the peak displacement in the FEM models and the experimental data agree to within 1 mm. For the later time steps portrayed in Fig. 3(A)–(D,iii), the locations of the peak displacements differ by up to 3 mm, but the overall shape of the displacement curves still agree. Absolute magnitudes of axial displacement in the FEM data (Table IV) deviated by a mean of 32.0% from the experimental ARFI data for the four phantoms, with the greatest error being 1.9 μ m in the 200 bloom gelatin phantom and the smallest error being 0.4 μ m in the CIRS phantom.

Fig. 3(iv)–(vi) shows that, as tissue relaxes at the focal point, shear waves propagate away from the ROE. The Young's moduli reconstructed from these shear wave speeds using (15) are summarized in Table V. Notice that the relative decay in shear wave magnitude as they propagate away from the ROE in the FEM and experimental data sets are similar, suggesting that this decay is due to geometric spreading of the waves, versus viscous effects that are not taken into account in the FEM models. The FEM data sets agree within a mean of -0.8 kPa of the specified Young's moduli, with the greatest deviation being 2.0 kPa in the 150 bloom gelatin phantom. The experimentally reconstructed Young's moduli from the shear wave speeds deviated by a mean value of 1.7 kPa from the MTS measurements, with the greatest deviation being 3.9 kPa in the 150 bloom gelatin phantom.

Fig. 4 reinforces the trend shown in Fig. 1 that Poisson's ratio can have an impact on the peak magnitude of displacement, but overall the dynamics of the displacement field in the axial dimension are not changed [Fig. 4(a)–(c)]. In the lateral dimension [Fig. 4(d)–(f)], again, the displacement magnitudes are impacted by the Poisson's ratio, with less compressible materials displacing less; however, as expected, the speed at which the shear waves propagate away from the ROE is not affected.

Fig. 5 shows that small variations in density do not greatly impact the displacement profiles. As expected, the less dense material is able to displace slightly more than the more dense materials in response to the same excitation, but the overall dynamic response of the material in the axial dimension Figs. 5(a)–(c) remains the same. As expected by (13), shear wave speed is impacted by density; when compared with the shear wave speed for $\rho = 1.0$ g/cm³, there is an expected decrease in c_T of 4.66% for $\rho = 1.1$ g/cm³, and an increase of 5.4% for $\rho = 0.9$ g/cm³.

cm^3 . The computed shear wave speeds changes from Figs. 5(d)–(f) are $\pm 7.7\%$ for $\rho = 0.9 \text{ g/cm}^3$ and 1.1 g/cm^3 , respectively. Upon further inspection, changes of 7.7% in the shear wave speed in the FEM models correspond to displacement differences in the lateral dimension of 0.2 mm, which is the node spacing. This represents a limitation in the resolution of tracking the shear wave speeds from the FEM data because the peaks always will occur at nodal locations. Improved accuracy, therefore, could be achieved by refining the mesh to have smaller node spacing.

Ultrasonic attenuation of the material did not have an impact on the shear wave speed, as shown in Figs. 6(d)–(f), but does show differences in the displacement profiles in the axial dimension Figs. 6(a)–(c). As Figs. 6(a)–(c) show, higher attenuations lead to greater relative displacements in the near field compared to the displacement at the focal depth, although the absolute magnitude of the displacement field is less. As demonstrated in Fig. 2, the near field displacement field has a greater lateral extent than at the focal depth, which causes a greater relative displacement in the near field at later times for higher attenuations, as demonstrated by Fig. 6(b) and (c). Regardless of the relative displacement profiles in the axial dimension, the relaxation rate is determined by the shear wave speed and is independent of ultrasonic attenuation.

The temporal behavior of gelatin phantom A with stiff spherical inclusion A is complicated (Fig. 7 and Fig. 8). Initially, the displacements are inversely related to the material stiffness (i.e., the stiffer lesion does not move as far as the softer background) and the lesion appears to be the actual diameter (3 mm), as shown in Fig. 8(a) and Fig. 7(A). As waves propagate away from the ROE, reflections and interference within the lesion and at its boundaries make the lesion boundaries appear to spread, as shown in Fig. 7(B) and (C) and Fig. 8(b) and (c) [14]. The plots of axial displacement laterally through the center of the inclusion [Fig. 8(d)–(f)] also qualitatively match the experimental data, including the inversion of the location of the peak displacement from outside of the lesion to within the lesion by 1.8 ms after ARFI excitation. Qualitatively, the models match the shapes of the experimental displacement fields through time in Fig. 7.

C. Limitations of the FEM Model

The use of single point quadrature and the addition of hourglassing control, instead of using full eight-point quadrature rules, reduces computation overhead considerably; but it introduces some degree of numerical damping into the model. This numerical damping, though, is believed to have minimal impact on the dynamic response of the model, as suggested by extrapolations performed to study the impact of varying degrees of numerical damping in the model.

The magnitudes of peak displacement in these models, in general, overestimated the experimentally measured displacements. This could be related to the choice of Poisson's ratio or the peak in situ intensity of 1000 W/cm^2 chosen to scale the intensity fields. Therefore, although the relative trends presented in this paper are not affected by inaccurate choice of these values for the models, the absolute values of displacement may be impacted and could be corrected in a future version of the model if more accurate parameters become available. Additionally, the displacements measured by ultrasonic, correlation-based, tracking methods have a negative bias in the presence of displacement gradients occurring within the imaging point spread function (PSF) [32]. Thus, the experimental data may underestimate the actual tissue displacement by up to 33% [32].

The limited spatial extent of the mesh restricts the maximum time that can be modeled. Once displacement waves reach the boundary closest to the ROE, they are reflected back into the ROI and corrupt the displacement fields. Therefore, stiffer materials and larger ROEs will

suffer from reduced times over which they can be modeled. This can be overcome by increasing the size of the mesh, but with a computational penalty.

D. Future Applications of the FEM Model

Ultrasonic tracking of the FEM displacement fields will be added to future versions of this model to evaluate the degree of displacement underestimation (bias) that occurs due to lateral speckle shearing under the track (PSF), and to evaluate jitter magnitudes. These models assumed that spherical inclusions, in an otherwise homogeneous background medium, share the same attenuation (α) and sound speed (c) as the background medium. This assumption cannot be made for accurate modeling of clinical lesions (e.g., tumors and ablated tissue). Instead, simulation of intensity fields and force distributions using Field II must be modified to accommodate heterogeneous distributions of ultrasonic attenuation and sound speed in the ROE. Viscoelastic material properties consistent with soft tissue also need to be incorporated into the model to evaluate viscosity's impact on tissue dynamics associated with ARFI imaging. These models also will be extended to incorporate biological structures other than spherical inclusions, such as layered organs and blood vessels with fluid flow. The impact of boundary conditions between inclusions and layers also will be studied in future models.

VI. CONCLUSIONS

The FEM model presented in this paper allows for the simulation of the dynamic response of tissue to an impulsive excitation with acoustic radiation force, as is done in ARFI imaging. The model is able to take into account acoustic attenuation and tissue material properties, in addition to the transducer parameters, for generating the acoustic radiation force. The dynamic response of this model has been validated using four calibrated, tissue-mimicking phantoms, with and without spherical inclusions, providing good agreement in reconstructed shear wave speeds. Shear wave speed and, therefore, elasticity appear to be the dominant factor affecting tissue relaxation following ARFI excitation within the ROE. But Poisson's ratio and density do not significantly alter relaxation rates in tissue. Increased acoustic attenuation in tissue increases the relative amount of tissue displacement in the near field as compared with the focal depth, but relaxation rates are not altered. Applications of these validated ARFI imaging models include improving image quality and distilling material and structural information from tissue's dynamic response to ARFI excitation. These models will be extended to include viscous material properties, variable attenuation and sound speed in inclusions, and ultrasonic displacement tracking.

Acknowledgments

The authors would like to thank Dr. Gregg Trahey for his valuable insight, Siemens Medical Solutions USA, Inc. Ultrasound Division for their technical assistance, Altair Computing, Inc., for providing FEM preprocessing software, Paul Toomey for writing postprocessing code, and Joshua Baker-LePain for his technical assistance.

This work was supported by NIH grant 8 R01 EB002132, the Whitaker Foundation, and the Medical Scientist Training Program grant T 32 GM-07171.

REFERENCES

1. Alizad A, Fatemi M, Nishimura RA, Kinnick RR, Rambod E, Greenleaf JF. Detection of calcium deposits on heart valve leaflets by vibro-acoustography: An in vitro study. *J. Amer. Soc. Echocardiogr* 2002 Nov.;vol. 15:1391–1395. [PubMed: 12415234]
2. Calle S, Remenieras J, Matar O, Defontaine M, Patal F. Application of nonlinear phenomena induced by focused ultrasound to none imaging. *Ultrasound Med. Biol* 2003;vol. 29(no 3):465–472. [PubMed: 12706198]

3. Fatemi M, Greenleaf J. Vibro-acoustography: An imaging modality based on ultrasound-stimulated acoustic emission. *Proc. Natl. Acad. Sci* 1999;vol. 96:6603–6608. [PubMed: 10359758]
4. Konofagou E, Thierman J, Hynynen K. A focused ultrasound method for simultaneous diagnostic and therapeutic applications—A simulation study. *Phys. Med. Biol* 2001 Nov.;vol. 46(no 11):2967–2984. [PubMed: 11720358]
5. Viola F, Walker WF. Radiation force imaging of viscoelastic properties with reduced artifacts. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 2003 June;vol. 50(no 6):736–742.
6. Bercoff J, Tanter M, Fink M. Supersonic shear imaging: A new technique for soft tissue elasticity mapping. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 2004;vol. 51(no 4):396–409.
7. Barannik EA, Girnyk A, Tovstiyak V, Marusenko AI, Emelianov SY, Sarvazyan AP. Doppler ultrasound detection of shear waves remotely induced in tissue phantoms and tissue in vitro. *Ultrasonics* 2002 May;vol. 40:849–852. [PubMed: 12160057]
8. Sarvazyan A, Rudenko O, Swanson S, Fowlkes J, Emelianov S. Shear wave elasticity imaging: A new ultrasonic technology of medical diagnostics. *Ultrasound Med. Biol* 1998;vol. 24(no 9):1419–1435. [PubMed: 10385964]
9. Lizzi F, Muratore R, Deng C, Ketterling J, Alam K, Mikaelian S, Kalisz A. Radiation-force technique to monitor lesions during ultrasonic therapy. *Ultrasound Med. Biol* 2003;vol. 29(no 11):1593–1605. [PubMed: 14654155]
10. Dayton P, Klibanov A, Brandenburger G, Ferrara K. Acoustic radiation force in vivo: a mechanism to assist targeting of microbubbles. *Ultrasound Med. Biol* 1999 Oct.;vol. 25:1195–1201. [PubMed: 10576262]
11. Hancock A, Insana MF, Allen JS. Microparticle column geometry in acoustic stationary fields. *J. Acoust. Soc. Amer* 2003 Jan.;vol. 113(no 1):652–659. [PubMed: 12558301]
12. Shortencarier M, Dayton P, Bloch S, Schumann P, Matsunaga T, Ferrara K. A method for radiation-force localized drug delivery using gas-filled lipospheres. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 2004;vol. 51(no 7):822–831.
13. Nightingale K, Bentley R, Trahey G. Observations of tissue response to acoustic radiation force: Opportunities for imaging. *Ultrason. Imag* 2002;vol. 24:100–108.
14. Nightingale K, McAleavey S, Trahey G. Shear wave generation using acoustic radiation force: In vivo and ex vivo results. *Ultrasound Med. Biol* 2003;vol. 29(no 12):1715–1723. [PubMed: 14698339]
15. Nightingale K, Soo M, Nightingale R, Trahey G. Acoustic radiation force impulse imaging: In vivo demonstration of clinical feasibility. *Ultrasound Med. Biol* 2002;vol. 28(no 2):227–235. [PubMed: 11937286]
16. Lai, W.; Rubin, D.; Kreml, E. *Introduction to Continuum Mechanics*. Woburn, MA: Butterworth-Heinemann; 1999.
17. Achenbach, J. *Wave Propagation in Elastic Solids*. New York, NY: Elsevier Science Publishers; 1999.
18. Nyborg, W. Acoustic streaming, ch. 11. In: Mason, W., editor. *Physical Acoustics*. Vol. vol. IIB. New York: Academic; 1965. p. 265-331.
19. Kino, G. *Acoustic Waves: Devices, Imaging, and Analog Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, Inc.; 1987.
20. Fox, R.; McDonald, A. *Introduction to Fluid Mechanics*. New York: Wiley; 1992.
21. Davis, J. *Introduction to Dynamics of Continuous Media*. New York, NY: Macmillan; 1987.
22. Hughes, T. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Mineola, NY: Dover; 2000.
23. Center for Devices and Radiological Health (CDRH). 510(k) guide for measuring and reporting acoustic output of diagnostic ultrasound medical devices. Rockville, MD: 1994 Dec..
24. Report No. 140, National Council on Radiation Protection and Measurements. Bethesda, MD: NCRP Publications; 2002. *Exposure Criteria for Medical Diagnostic Ultrasound: II. Criteria Based on All Known Mechanisms*. p. 20814
25. Palmeri M, Nightingale K. On the thermal effects associated with radiation force imaging of soft tissue. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 2004;vol. 51(no 5):551–565.

26. Jensen J, Svendsen N. Calculation of pressure fields from arbitrarily shaped, apodized, and excited ultrasound transducers. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 1992;vol. 39(no 2):262–267.
27. Nightingale K, Palmeri M, Nightingale R, Trahey G. On the feasibility of remote palpation using acoustic radiation force. *J. Acoust. Soc. Amer* 2001;vol. 110(no 1):625–634. [PubMed: 11508987]
28. Trahey G, Allison J, VonRamm O. Angle independent ultrasonic detection of blood flow. *IEEE Trans. Biomed. Eng* 1987;vol. BME-34(no 12):965–967. [PubMed: 2961682]
29. Hall TJ, Bilgen M, Insana MF, Krouskop TA. Phantom materials for elastography. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 1997;vol. 44(no 6):1355–1365.
30. Report No 113, National Council on Radiation Protection and Measurements. Bethesda, MD: NCRP Publications; 1992. Exposure Criteria for Medical Diagnostic Ultrasound: I. Criteria Based on Thermal Mechanisms. p. 20814
31. Parker K, Huang S, Musulin R, Lerner R. Tissue response to mechanical vibrations for sonoelasticity imaging. *Ultrasound Med. Biol* 1990;vol. 16(no 241)
32. McAleavey S, Nightingale K, Trahey G. Estimates of echo correlation and measurement bias in acoustic radiation force impulse imaging. *IEEE Trans. Ultrason., Ferroelect., Freq. Contr* 2003;vol. 50(no 6):631–641.
33. Rudenko O, Sarvazyan A, Emelianov S. Acoustic radiation force and streaming induced by focused nonlinear ultrasound in a dissipative medium. *J. Acoust. Soc. Amer* 1996;vol. 99(no 5):2791–2798.
34. Walker W. Internal deformation of a uniform elastic solid by acoustic radiation force. *J. Acoust. Soc. Amer* 1999;vol. 105(no 4):2508–2518. [PubMed: 10212432]
35. Chu BT, Apfel RE. Acoustical radiation pressure produced by a beam of sound. *J. Acoust. Soc. Amer* 1982;vol. 72(no 6):1673–1687.

Biographies

Mark L. Palmeri (S'99) received his B.S. degree (Biomedical and Electrical Engineering) in 2000 from Duke University. He is currently a student in the Medical Scientist Training Program and a James B. Duke graduate fellow in the Biomedical Engineering Department at Duke University.

Amy C. Sharma received her B.S.E. degree in biomedical and electrical engineering from Duke University, Durham, NC, in 2001. She is currently a graduate student in the Biomedical Engineering Department at Duke University, studying imaging under Dr. Carey Floyd.

Richard R. Bouchard was born in Winnipeg, Manitoba, Canada, in 1981. He received his B.S. degree in biomedical and electrical engineering in 2004 from Duke University, Durham, NC. He is currently a National Science Foundation graduate fellow pursuing the Ph.D. degree in biomedical engineering from Duke University.

Roger W. Nightingale received his B.S. degree in biomedical engineering in 1987 from Duke University, Durham, NC. He received a Ph.D. degree in biomedical engineering from Duke University in 1993 and is currently an associate research professor of biomedical engineering at Duke University.

His research interests are in the biomechanics of trauma, automotive safety, acoustic radiation force-based imaging, and craniofacial morphology.

Kathryn R. Nightingale (M'93) received her B.S. degree in Electrical Engineering in 1989 from Duke University. She served in the United States Air Force as a program engineer from 1989 to 1992. She received her Ph.D. degree in Biomedical Engineering from Duke University in 1997. She is currently an assistant research professor of Biomedical Engineering at Duke University with research interests in radiation force-based imaging, ultrasonic breast imaging, ultrasonic flow detection, and the bioeffects associated with diagnostic ultrasonic imaging.

Fig. 1.

Comparison of the simulated axial displacement at the focal point through time in response to a $45\ \mu\text{s}$ ARFI excitation. Notice that there is a difference in the magnitude of the peak displacement, but there are no significant differences in the loading and relaxation times of the tissue.

Fig. 2.

Subfigure (a) shows the simulated distribution of acoustic radiation force in the axial-lateral plane, centered in elevation, for an F/1.3 focal configuration, focused at 20 mm, with $\alpha = 0.7$ dB/cm/MHz. The transducer would be centered at the top of these images. Brighter pixels indicate greater radiation force magnitude. Subfigures (b)–(d) show the normalized FEM displacement fields resulting from this acoustic radiation force distribution, 0.6, 1.0, and 2.2 ms after the excitation, respectively, in an 8.5 kPa medium. Subfigures (e)–(g) show the normalized experimental ARFI displacement data from the 100 bloom gelatin phantom, again, for the same time steps. Brighter pixels in the displacement images represent greater displacement away from the transducer.

Fig. 3.

Comparison of FEM models and experimental normalized displacement profiles in the CIRS and gelatin phantoms. Row A shows the CIRS phantom, with a focal depth of 15 mm and an F/1.3 focal configuration. Rows B–D show 100, 150, and 200 bloom gelatin phantoms, respectively, with a focal depth of 20 mm with an F/1.3 focal configuration. The solid lines represent FEM data, and the dashed lines represent experimental phantom data (mean of six independent trials). Subplots i–iii represent displacement profiles laterally centered in the ROE as a function of axial depth away from the transducer, for the indicated times after cessation of a $45 \mu\text{s}$ ARFI excitation. Note the different time steps being represented by the subfigures between rows. Subplots iv–vi represent normalized axial displacement profiles at the focal

depth (15 mm for row A, 20 mm for rows B–D), spanning ± 6 mm laterally from the center of the ROE. The shear wave speeds and reconstructed Young's moduli from this data are summarized in Table III and Table V.

Fig. 4.

Comparison of normalized FEM displacement values laterally centered in the ROE as a function of axial depth away from the transducer (a)–(c) and spanning ± 7 mm laterally from the center of the ROE at the focal depth of 20 mm (d)–(f), for varying material Poisson's ratios (solid line = 0.499, dashed line = 0.4999, dotted line = 0.49, dashed-dotted = 0.48). The times shown refer to times after the ARFI excitation. The material was modeled with $E = 8.5$ kPa, $\rho = 1.0$ g/cm³ and $\alpha = 0.7$ dB/cm/MHz. Notice that the displacement profiles for $\nu = 0.49$ and $\nu = 0.48$ are almost identical.

Fig. 5.

Comparison of normalized FEM displacement values laterally centered in the ROE as a function of axial depth away from the transducer (a)–(c) and spanning ± 7 mm laterally from the center of the ROE at the focal depth of 20 mm (d)–(f), for varying material densities (solid line = 1.0 g/cm^3 , dashed line = 0.9 g/cm^3 , dotted line = 1.1 g/cm^3). The times shown refer to times after the ARFI excitation. The material was modeled with $E = 8.5 \text{ kPa}$, $\nu = 0.499$, and $\alpha = 0.7 \text{ dB/cm/MHz}$.

Fig. 6.

Comparison of normalized FEM displacement values laterally centered in the ROE as a function of axial depth away from the transducer (a)–(c) and spanning ± 7 mm laterally from the center of the ROE at the focal depth of 20 mm (d)–(f), for varying material ultrasonic attenuations (solid line = 0.5 dB/cm/MHz, dashed line = 0.7 dB/cm/MHz, dotted line = 1.0 dB/cm/MHz). The times shown refer to times after the ARFI excitation. The material was modeled with $E = 8.5$ kPa, $\nu = 0.499$, and $\rho = 1.0$ g/cm³.

Fig. 7.

Matched simulation and experimental displacement profiles in the axial-lateral plane, centered in elevation, demonstrating similar temporal behavior for a stiff spherical inclusion in an otherwise homogeneous medium. The images at each time step are normalized by the maximum displacement value for that time step. Each row represents a different time step: 0.4, 0.9, and 1.8 ms for the top, middle, and bottom rows, respectively. The Young's moduli for the background and lesion in both simulation and experiment were 4.0 and 15 kPa, respectively, with a lesion diameter of 3 mm. Initially, the displacements are indicative of differences in stiffness; however, later wave propagation effects are portrayed, and a reversal in contrast and apparent spreading of the lesion is observed in the displacement profiles. Note that the

simulation images do not include the effects of speckle tracking the ARFI-induced displacements.

Fig. 8.

Subplots (a)–(c) compare FEM model (solid line) and experimental (dashed line) normalized axial displacement profiles in gelatin phantom A with spherical inclusion A. The displacement profiles are laterally centered in the ROE as a function of axial depth away from the transducer, for 0.4 ms, 0.9 ms, and 1.8 ms after the cessation of a 45 μ s ARFI excitation. The ARFI excitation was focused at 15 mm with an F/1.3 configuration. Subplots (d)–(f) show normalized axial displacement profiles at the focal depth of 15.5 mm, spanning ± 2.5 mm laterally from the center of the ROE. The dotted lines in all subplots represent the boundaries of the spherical inclusion.

TABLE I

Nomenclature.¹

i, j	Spatial tensor subindices
u_i	Particle displacement
v_i	Particle velocity
$\nabla \cdot \equiv u_{i,i}$	Divergence operator
$\nabla \equiv u_{,i}$	Gradient operator
$\nabla^2 \equiv u_{,ii}$	Laplacian operator
$\Delta \equiv \epsilon_{ii}$	Dilatation
$\ddot{u}$	Second time derivative (e.g., acceleration)
δ_{ij}	Kronecker delta
$\langle \rangle$	Time average
α	Attenuation
c	Sound speed
c_L	Longitudinal wave speed
c_T	Transverse wave speed
d	Active aperture width
z	Acoustic focal length
σ_{ij}	Stress tensor
ϵ_{ij}	Strain tensor
b_i	Body force
f_i	Applied load
ρ	Material density
λ	Lame constant
μ	Lame constant (shear modulus)
E	Young's modulus
ν	Poisson's ratio
ψ	Dilatational displacement
$\vec{W}$	Equivoluminal displacement
λ_f	Lame constant (fluids)
μ_f	Viscosity (fluids)
μ'_f	Bulk viscosity (fluids)
κ	Wave number

¹The Einstein convention for tensor subindices will be used throughout this paper (i.e., repeated subindices imply summation over that index).

TABLE II

MTS-Measured Young's Moduli of Gelatin and CIRS Phantom Materials.

Phantom	Young's Modulus (kPa)
Phantom A (gelatin)	4.0 ± 0.1
Inclusion A (gelatin)	15 ± 0.1
100 bloom gelatin	8.5 ± 0.02
150 bloom gelatin	18 ± 0.1
200 bloom gelatin	23 ± 0.04
CIRS phantom	4.0 ± 0.1

TABLE III

Shear Wave Speeds.

Phantom	FEM c_T (m/s)	Experimental c_T (m/s)
CIRS	1.1	1.2 ± 0.13
100 bloom	1.7	1.5 ± 0.20
150 bloom	2.3	2.2 ± 0.20
200 bloom	2.7	2.8 ± 0.30

TABLE IV

Comparison of FEM and Experimental Displacement Magnitudes.

Phantom	FEM max. disp. (μm)	ARFI max. disp. (μm)	Normalization factor
CIRS	5.0	5.4 ± 0.13	1.08
100 bloom	11.3	10.1 ± 0.35	0.894
150 bloom	6.1	4.2 ± 0.59	0.689
200 bloom	4.9	3.0 ± 0.46	0.612

TABLE V

Differences in Shear Wave-Reconstructed Young's Modulus.

Phantom	MTS E (kPa)	FEM E (kPa)	ARFLE (kPa)
CIRS	4.0	3.9	4.3
100 bloom	8.5	8.4	6.6
150 bloom	18.0	16.0	14.1
200 bloom	23	21.9	23.5