Published in final edited form as:

Int J Imaging Syst Technol. 2009 September 1; 19(3): 271–276. doi:10.1002/ima.20200.

Attenuation map estimation with SPECT emission data only

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Abstract

Quantitative SPECT can be well achieved if photon attenuation is taken into account in the reconstruction process. Using a transmission scan is a common approach. A dramatic simplification could be made if the attenuation map could be obtained from the emission data. In this paper, we propose a new method to estimate the attenuation map using the data consistency conditions of the attenuated Radon transform. It is based on deriving boundaries of the constant regions of the true attenuation map using an iterative algorithm. This new method is tested by Monte Carlo simulations with the attenuation and scattering effects.

1. Introduction

Single photon emission computed tomography (SPECT) is a widely-used diagnostic imaging technique in which tomography is generated from the emitted gamma photons of a radionuclide distribution. Quantitative SPECT can be well achieved if photon attenuation is taken into account in the reconstruction process. The goal for attenuation correction is to find the attenuation coefficient distribution or attenuation map. Nowadays there are two general methods for obtaining the attenuation map. The first method is to measure and reconstruct the attenuation coefficient map using a transmission scan. The state-of-the-art technology is the use of a combined SPECT/CT scanner (Hasegawa et al., 1993). However, it considerably complicates the scanner design, the data acquisition and processing protocols. Furthermore, the cost of the combined scanner is high and not affordable by all, especially small hospitals. The other method is to estimate the attenuation map from the emission data only. A dramatic simplification could be made if this were possible. An early attempt was to solve the nonlinear equation relating the attenuation map and activity distribution by using an iterative technique. This approach was first proposed by Censor et al. in the late 1970's and has been explored by many researchers (Censor et al., 1979; Nuyts et al., 1999; Ramlaoui et al., 2000; Gourion et al., 2002). A later and more popular approach was to estimate the attenuation map based on the data consistency conditions (DCCs). DCCs are a set of equations relating the Radon transform of the attenuation map to the SPECT emission data. By solving these equations, one can estimate the attenuation map which is most consistent with the emission data. Solving these equations is equivalent to the minimization of an objective function, which is defined later in the next section. However, as the objective function has too many local minima, it is difficult to obtain the attenuation map from it directly. Many researchers have explored the possibilities of using DCCs to estimate the attenuation map (Natterer, 1993; Moore et al., 1997; Welch et al., 1997; Laurette et al., 1999; Mennessier et al., 1999; Panin et al., 1999; Bronnikov, 1995, 2000; Kudo et al., 2000). Natterer (Natterer, 1986) first proposed a method assuming that the true attenuation distribution is approximately an affine distortion of a known prototype attenuation distribution. By doing that, he decreased the number of variables of the attenuation map and therefore narrowed the search space of the DCCs. Welch et al. did similar work and assumed a uniform elliptical attenuator. Moore et al. proposed a

polynomial with a small number of variables to express the attenuation map. Kudo et al. assumed that the topology (*i.e.* the number of regions and the connectivity among regions) of the attenuation map was known. Panin et al. proposed obtaining the attenuation map based on a “knowledge set” and the unknowns were the coefficients of the principle components in a singular value decomposition. All these research efforts use some prior information of the attenuation map and narrow the search space of the DCCs. Further research to produce more accurate methods with less prior information is required.

In this paper, a new approach for obtaining the non-uniform attenuation map from the emission data is proposed. A two-step iterative algorithm is first used to obtain raw reconstructions of the activity image and the attenuation map. There exist crosstalk effects between these two images, from which the boundaries of the constant regions of the attenuation map is then derived and the constant regions are segmented. Next, the attenuation coefficient inside each segmented region is estimated using the DCCs and the default attenuation coefficients of the lung, bone and soft tissue. Finally, a three-variable algorithm is used to fine tune the attenuation coefficient values without altering the boundaries. Our method is shown to work effectively for a non-uniform attenuation map using the Monte Carlo simulated data. Other degradation effects such as Compton scatter are also evaluated.

2. Method

2.1. Data Consistency Conditions

The two dimensional attenuated Radon transform is defined as

$$g(\theta, s) = (R_\mu f)(\theta, s) = \int_{x \cdot \theta = s} e^{-(D\mu)(x, \theta^\perp)} f(x) dx \quad (1)$$

where $f(x)$ is the activity distribution of a radiopharmaceutical inside the body, $D\mu$ is the divergent beam transform of the attenuation map $\mu(x)$, defined as $(D\mu)(x, \theta) = \int_0^\infty \mu(x+t\theta) dt$. The noise-free measured emission data are $g(\theta, s)$ which is in the range of R . Therefore, $g(\theta, s)$ must satisfy the range conditions of this attenuated Radon transform, known as the data consistency conditions (DCCs) (Natterer, 1986). They can be written as a set of equations

$$\int_0^{2\pi} \int_{-\infty}^{\infty} s^m e^{ik\phi} e^{\frac{1}{2}(I+iH)R\mu(\theta, s)} g(\theta, s) ds d\theta = 0 \quad (2)$$

for integers k and m such that $0 \leq m < k$. In (2) $R\mu(\theta, s)$ represents the transmission sinogram of the attenuation map, H represents the Hilbert transform with respect to s , and I is the identity transform. With given emission data $g(\theta, s)$, which are obtained by a SPECT system, the previously suggested approach (Natterer, 1986) was to estimate the attenuation map by solving the above DCC equations. Solving these equations is equivalent to the minimization problem with the following objective function:

$$F(\mu) = \sum_{0 \leq m < k} \left\| \int_0^{2\pi} \int_{-\infty}^{\infty} s^m e^{ik\phi} e^{\frac{1}{2}(I+iH)R\mu(\theta, s)} g(\theta, s) ds d\theta \right\|^2 \quad (3)$$

for integers k and m such that $0 \leq m < k$. However, this objective function has too many local minima and is sensitive to noise. Therefore, more information of the attenuation map is necessary to reduce the search space. In our method, we take advantage of the crosstalk

effects to obtain the boundaries of the constant regions of the attenuation map and therefore avoid the local minima.

2.2. Crosstalk Effects

It has been shown (Krol et al., 2001) that by using an iterative algorithm to estimate the activity image and attenuation map simultaneously, there exist crosstalk effects between these two images. The crosstalk effects are as follows: The negative attenuation map is superimposed on the activity reconstruction, and the negative activity distribution is superimposed on the attenuation map reconstruction. We show a simple illustration of the crosstalk effects in Figure 1.

To get the crosstalk effects, we propose a two-step iterative reconstruction algorithm in which the activity image x and attenuation map μ are updated alternately. Holding μ at its current value, the activity image can be reconstructed using the first step: the well-known ML-EM algorithm (Shepp and Vardi, 1982), which can be expressed in the following

$$x_i^{new} = \frac{x_i^{old}}{\sum_j a_{ij}} \sum_j a_{ij} \frac{p_j}{\sum_k a_{kj} x_k^{old}} \quad (4)$$

where p_j is the measured emission data, x_i is the estimated activity in image pixel i , and a_{ij} is the known coefficient that represents the contribution of image pixel i to projection bin j with the current attenuation map μ . The summation over k is the projector and the summation over j is the backprojector.

Holding x at its current value, the attenuation map μ is then updated with an ML-EM-like algorithm (Hwang and Zeng, 2005):

$$\mu_i^{new} = \frac{\mu_i^{old}}{\sum_j a_{ij}} \sum_j a_{ij} \frac{\sum_k a_{kj} x_k^{new}}{p_j} \quad (5)$$

where μ_i is the estimated attenuation map pixel i , and the x^{new} is the estimated image in the same iteration from equation (4). a_{ij} is the known coefficient that represents the contribution of image pixel i to projection bin j with the old attenuation map μ .

Our method iterates between updates of the activity image and the attenuation map using (4) and (5), respectively. With this two-step algorithm, both the activity image and the attenuation map can be roughly reconstructed with clear crosstalk effects. Because of the crosstalk effects, none of the reconstructed images is accurate. However, the boundaries of the constant regions of the attenuation map can be derived from the crosstalk effects. With this boundary information, we can reduce the search space of the DCCs.

Alternatively, we can reconstruct the activity image using the general ML-EM algorithm alone from (4), under the assumption of zero attenuation. The reconstructed image also shows crosstalk effects. However, because there is no attenuation compensation, the intensity of the reconstructed image decreases towards the center (See Simulation results Section 3.1). This will introduce difficulties in region segmentation, therefore the two-step algorithm is a better choice.

2.3. Proposed Method

Our proposed method consists of the following three steps.

Step 1.

Use the two-step iterative reconstruction algorithm to perform a raw estimation of the activity image and the attenuation map simultaneously from the emission data. The reconstruction alternately updates the activity image and the attenuation map.

Step 2.

Segment boundaries from either the activity image or the attenuation map, and form constant regions in the attenuation map.

Step 3.

Assign each segmented region i a constant value μ_i , which can be the linear attenuation coefficient of the bone, soft tissue, or lung. The SPECT DCCs are used to select the optimal constant value μ_i^* for each region i . The optimal $\mu^* = \{\mu_i^*\}$ is determined such that

$$F(\mu^*) = \min_{\{\mu_i\} = \{0.04, 0.15, 0.25\}} F(\mu) \quad (6)$$

Two important assumptions are made in this step: (1) The attenuation distribution is uniform in each segmented region. (2) All the possible attenuation coefficients are known. This is reasonable because we usually know the specific organs involved and the attenuation coefficients for regular organs are available for given photon energies.

With the estimated attenuation map, an accurate activity distribution can be reconstructed using either an analytical formula (Novikov, 2002) or an iterative algorithm.

3. Simulation Results

3.1. Noiseless Simulation

Our method has been tested for Monte Carlo simulated data with the non-uniform attenuation map. The SIMSET software package was used to generate the data (Harrison et al., 1993). A simplified thorax phantom was used for our simulation, as shown in Figure 2 (a) and (b). The $40\text{cm} \times 40\text{cm}$ object region was digitized onto a 256×256 array with pixel size of 0.156 cm. The attenuation coefficients for the attenuation map were $\mu_{lung} = 0.04\text{cm}^{-1}$, $\mu_{softtissue} = 0.15\text{cm}^{-1}$ and $\mu_{bone} = 0.25\text{cm}^{-1}$. The detected photons were binned into a 256×300 projection array, with 0.156-cm bins at each of 300 angles uniformly distributed over 360° . The detection energy window was centered at 140 keV with a width of 10%.

A raw estimate of the activity image and attenuation map was first obtained after performing 15 iterations of the two-step iterative algorithm which alternately updates the activity image and the attenuation map. Figure 3(a) and (b) shows the raw reconstructions of the attenuation map μ and the activity image f , respectively, where crosstalk effects between the activity image and the attenuation map are obvious. That is, the negative attenuation map was superimposed on the raw activity image, and the negative activity image was superimposed on the raw attenuation map. Figure 3(c) shows the raw reconstruction of the activity image using the ML-EM method. The attenuation map in this case was assume to be zero. This raw image f^* did show crosstalk effects, however, the intensity of the image decreased towards the center and it introduced difficulties in region segmentation. Therefore, the two-step algorithm was used in our method. Five constant regions (see Figure

3(d)) were extracted from either the raw activity image or raw attenuation map, using the watershed segmentation method (Vincent et al., 1991).

For each region i ($i = 1, 2, 3, 4, 5$), an attenuation coefficient μ_i was assigned. Let each μ_i take the potential attenuation coefficient values of 0.04cm^{-1} , 0.15cm^{-1} , 0.25cm^{-1} , one at a time and evaluate the objective function (3). The optimal solution was chosen for the smallest objective function value. With the optimal value μ_i^* , we combined five regions to form the new attenuation map, as shown in Figure 3(e). The accuracy of the estimation of attenuation map depended on the accuracy of the boundaries of the segmented regions. Figure 3(f) shows the error between the estimated and the true attenuation map.

In practice, the attenuation coefficients of the lung, tissue and bone are not constant but vary in a small range between different patients and different measurements. Therefore, the different potential attenuation coefficient values around 0.04cm^{-1} , 0.15cm^{-1} , 0.25cm^{-1} were also evaluated in our computer simulation. Table 1 showed that when deviated attenuation coefficient values were used to generate the projection data, a *correct-boundary* attenuation map could still be obtained. By saying the *correct-boundary* attenuation map, we mean the following: if we assume $\mu_{\text{lung}} = 0.04\text{cm}^{-1}$, $\mu_{\text{softtissue}} = 0.15\text{cm}^{-1}$ and $\mu_{\text{bone}} = 0.25\text{cm}^{-1}$, the objective function reaches the minimum when the lung region is assigned with μ_{lung} , the tissue region is assigned with $\mu_{\text{softtissue}}$, and the bone region is assigned with μ_{bone} . It was found that a stable *correct-boundary* attenuation map could be obtained when an approximate 20% deviation of the assumed attenuation coefficients was used in the data generation, that is, $\mu_{\text{lung}} = 0.04 \pm 0.008\text{cm}^{-1}$, $\mu_{\text{softtissue}} = 0.15 \pm 0.030\text{cm}^{-1}$ and $\mu_{\text{bone}} = 0.25 \pm 0.050\text{cm}^{-1}$. To get the actual correct attenuation map (both correct-boundary and correct-value), we proposed a simple approach to fine tune the attenuation coefficient values: (1) With the estimated *correct-boundary* attenuation map, reconstruct the activity image using the analytical inverse formula. (2) With this estimated activity image, reconstruct the attenuation map using the following three-variable iterative algorithm:

$$\mu_n^{\text{new}} = \frac{\mu_n^{\text{old}}}{\sum_{n,j} a_{ij}} \sum_{n,j} a_{ij} \frac{\sum_k a_{kj} x_k^{\text{new}}}{p_j} \quad (7)$$

where $n = 1, 2, 3$, and $\sum_{n,j}$ represents the summation over j for each n . A support of the exact elliptical attenuation distribution was chosen outside of which the attenuation coefficients were set to zero. Compared to (5), we assumed that there were only three variables in the attenuation map instead of taking every pixel i as a variable, that is $\mu_n = \mu_1, \mu_2, \mu_3$ for the constant regions of the bone, lung and soft tissue, respectively. The bone, lung and soft tissue regions were determined in Step 2 of the proposed method, and in (7) the boundaries were fixed. This three-variable reconstruction algorithm was implemented to obtain the attenuation coefficient values of the bone, lung and soft tissue. After 15 iterations of (7), the reconstructed attenuation map was close to the truth.

3.2. Noisy Data Simulation

We modelled noise in the emission data with a Poisson distribution. The first simulation had a total count of 3×10^6 . The attenuation map was successfully estimated by performing our proposed method, as shown in Figure 4(a). We then increased the noise level at which the total count was 3×10^5 . The boundaries in the raw reconstruction of the activity image were not clear enough to be detected, shown in Figure 4(b), leading to inaccurate region segmentation. As the accuracy of the estimation of the attenuation map depends on the

accuracy of the segmented region boundaries, inaccurate region segmentation will cause the estimation to fail.

To test the validity of our method with noisy data, we assumed that the segmentation was perfect (in reality, it is never perfect): all five regions were clearly separated as they were in the noise-free case. The relation between different noise levels and the results of the convergence of the objective function to the correct attenuation map is shown in Table 2. It was found that 1×10^5 or less counts were too noisy for DCCs to find the correct attenuation map.

3.3. Effect of Scatter

One way to allow for the scatter effect on attenuation map estimation is to use a reduced (i.e., a “broad beam”) attenuation coefficient (Harris, 1984). Using Monte Carlo simulation, we added scattered photons to the noise-free primary photons. Attenuation coefficients were decreased by around 15% of the original value to compensate for the effect of scatter in equation (6). The smallest objective function value was obtained at which the solution for the attenuation map was the corresponding scaled-down attenuation map.

3.4. A Failed Case

Suppose we have an activity image which has zero background activity. Then the reconstructed raw attenuation map and raw activity image do not show any crosstalk effects, as shown in Figure 5. Therefore, it is impossible to derive the boundary information from them. However, in practice, the absorption of the radiopharmaceuticals by normal tissues is not zero but a smaller amount compared to the concentrated area. That is, in most practical cases the activity distribution has some background activities. Fortunately, the zero background situation rarely happens.

4. Discussion & Conclusion

In conclusion, an accurate determination of the attenuation map can be achieved with only emission data using our new method. This new method is based on estimating the boundaries of the constant regions of the attenuation map from the raw reconstructions then using the DCCs to select a set of optimal attenuation coefficients in the segmented regions. All the possible attenuation coefficients for the attenuation map are assumed to be known, which significantly reduces the search space. A small deviation of the possible attenuation coefficients is allowed in the estimation. The estimated attenuation coefficients can be fine tuned toward to the truth using our proposed method.

Our method has been shown to work effectively for non-uniform attenuators using Monte Carlo simulated data, including noise and scattering. The critical part of our method is to find the exact boundaries of the constant regions of the attenuation map. The estimated attenuation map is shown to be very accurate if the boundary information can be accurately obtained from the raw reconstructed images. However, when data are too noisy and blurred, the crosstalk and the boundaries are difficult to detect. A more powerful segmentation method may be required. More work needs to be done to get accurate boundary information from the noisy and blurred data before our method can be applied to experimental data.

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Fig 1.

Fig 2.

Fig 3.

Fig 4.

Fig 5.

Attenuation coefficients v.s. Converge results

Table 1

Attenuation Coefficients (cm ⁻¹)	0.04 (lung)	0.05	0.04	0.045	0.05
	0.15 (tissue)	0.15	0.15	0.14	0.16
	0.25 (bone)	0.25	0.27	0.23	0.20
Converge to the correct boundary attenuation map	Yes	Yes	Yes	Yes	No

Table 2

Noise levels v.s. Converge results

Photon Counts	5×10 ⁶	1×10 ⁶	5×10 ⁵	3×10 ⁵	1×10 ⁵
Converge to the correct boundary attenuation map	Yes	Yes	Yes	Yes	No