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Time-frequency analysis of movement-related spectral power in EEG during repetitive movements: a comparison of methods

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Abstract

During dynamic voluntary movements, power in the α - and β -bands resulting from synchronized neuronal activity is modulated in a manner that is time-locked to movement onset. These signals can be readily recorded from the scalp surface using electroencephalography. Abnormalities in the magnitude and timing of these oscillations are present in a wide variety of movement disorders including Parkinson's disease and dystonia. Most studies have examined movement-related oscillations in the context of single discrete movements, yet marked impairments are often seen during the performance of repetitive movements. For this reason, there is considerable need for analysis methods that can resolve the modulation of these oscillations in both the frequency and time domains. Presently, there is little consensus on which is the most appropriate method for this purpose. In this paper, a comparison of commonly used time-frequency methods is presented for the analysis of movement-related power in the α - and β -bands during repetitive movements. The same principles hold, however, for any form of repetitive or rhythmic input-output processes in the brain. In particular, methods based on band-pass filtering, the short-time Fourier transform (STFT), continuous wavelet transform and reduced interference distributions are discussed. The relative merits and limitations in terms of spectral or temporal resolution of each method are shown with the use of simulated and experimental data. It is shown that the STFT provides the best compromise between spectral and temporal resolution and thus is the most appropriate approach for the analysis and interpretation of repetitive movement-related oscillations in health and disease. (250 words)

Keywords

Event-related desynchronization; Repetitive movements; EEG; Time-frequency analysis; Wavelets; Short-time Fourier Transform; Reduced Interference Distributions

Introduction

The temporal accuracy, cost and relative ease of use of electroencephalography (EEG) has made this particular brain imaging modality the method of choice for many studying motor control and its disorders. The popularity and utility of EEG for understanding normal and disordered movement control has been further enhanced by the fact that time-varying changes in certain spectral bands reflect changes in movement-related oscillatory activity which can be

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used to infer cortical or subcortical activity. For this reason, a large amount of research effort has focused on developing algorithms capable of quantifying these changes and presenting them in a meaningful way.

Oscillatory cortical activity in the upper α -band or μ rhythm (10–12 Hz) and β -band (14–30 Hz), resulting from synchronous current changes in local populations of neurons, is present at rest or during isometric contractions and is considered to reflect steady-state sensorimotor processing or stasis (Baker et al., 1999; MacKay, 2005; Murthy and Fetz, 1992). During dynamic voluntary movements, however, the power within these frequency bands is modulated in a manner that is time-locked to movement onset (Pfurtscheller and da Silva, 1999; Toma et al., 2002). A decrease in movement-related power relative to a resting or baseline state, referred to as event-related de-synchronisation (ERD), occurs before movement onset and is considered to reflect a loss of synchrony in the underlying neuronal population. Conversely, increases in power corresponding to the re-establishment of synchrony post-movement are referred to as event-related synchronisation (ERS) (Pfurtscheller and da Silva, 1999). Although the function of these oscillations is unclear, the balance of evidence suggests that synchronized oscillations of local populations of neurons both within and across specific frequencies acts to bind input-output processing across local and distributed neural networks (Gray et al., 1989). Moreover, it has been shown that abnormalities in the magnitude and timing of oscillations in the α - and β -bands are a hallmark of a variety of neurological disorders, such as Parkinson's disease (Brown and Marsden, 1999) and primary dystonia (Toro et al., 2000). Most studies have examined movement-related oscillations in the context of single discrete movements, yet marked impairments are often seen during the performance of repetitive motor behaviours such as finger tapping and locomotion. It has recently shown that repetitive finger movements are severely impaired in people with Parkinson's disease, but this impairment is movement frequency dependent (Stegemöller et al., 2009). In light of evidence that pharmacological and surgical treatments of movement disorders can have a differential effect on movement-related oscillations in the α - and β -bands (Brown and Marsden, 1999), there is considerable need for analysis methods that can resolve the movement-related oscillations simultaneously in both the frequency and time domains during repetitive motor behaviours.

Since the spectral contents are time varying, EEG collected during dynamic movements are by definition non-stationary. In such circumstances traditional Fourier analysis is not helpful as no indication is given of when frequency changes occurred. Therefore, to determine the time course of ERD/ERS, time-frequency methods capable of estimating time-varying or localised spectra with sufficient temporal and spectral resolution are required. Due to the uncertainty principle (Rioul and Vetterli, 1991), however, it is not possible to have good temporal and spectral resolution simultaneously, as described below. Generally, when performing time-frequency analysis a compromise is made between temporal and spectral resolution and this depends on what information the analyst seeks from the data. In the case of repetitive movements, for example, it could be argued that temporal resolution is more important, since it is knowledge of when changes in α - or β -band power occur that is of interest, rather than at which particular frequencies within those bands the changes occurred. In other experiments different criteria may apply and the relative importance of temporal and spectral resolution may be reversed.

A question that naturally arises at this point is which method is best at extracting these spectral changes, and, furthermore, are there circumstances that might favour other methods. The aim of this paper is to characterize the relative merits and limitations of a number of time-frequency representations (TFR) for the analysis of ERD/ERS with respect to repetitive movements. Four methods are considered: band-pass filtering, short-time Fourier transform, continuous wavelet transform, and a reduced interference distribution from Cohen's class of quadratic TFR. The

temporal and spectral resolution of each method are assessed with simulated data and EEG collected during a repetitive movement experiment.

Other methods of EEG time-frequency analysis have been reported, most notably non-adaptive and adaptive autoregressive methods (Arnold et al., 1998; Bohlin, 1977; Florian and Pfurtscheller, 1995; Tarvainen et al., 2004), and the matching pursuit algorithm (Durka, 2006; Ginter et al., 2001; Mallat and Zhang, 1993). However, with the exception of the reduced interference distribution, the present authors have concentrated on methods that are found most frequently in the literature. Although the focus of this paper is on movement-related oscillations, the same principles hold for any form of repetitive or rhythmic input-output processes in the brain, such as, a series of somatosensory stimuli.

Methods

Review of time-frequency methods for ERD/ERS

TFR from band-pass filtering—One of the earliest methods for observing ERD/ERS in the time-frequency plane was reported by Pfurtscheller, and involves passing the EEG signal through a parallel bank of narrow band-pass filters centred on a particular frequency (Pfurtscheller and da Silva, 1999). The output of each filter is squared, averaged across trials and smoothed with a moving average (MA) filter to obtain an estimate of the instantaneous signal power at the filter's centre frequency. The band power estimates for all frequencies are combined to form a time-frequency map.

While this method is relatively simple to implement and interpret, it is generally not possible to obtain good temporal and spectral resolution simultaneously. At first glance it would seem that spectral resolution is dependent on the number of filters used, while the temporal resolution is dependent on the amount of smoothing from the MA filter whose window length is set typically to half a cycle of the centre frequency. In reality, however, this is an oversimplification and resolution in both domains is influenced by the type and order of the band-pass filters used. To obtain good spectral resolution, a narrow band-pass filter is required that ideally has the same bandwidth as the increment between filter centre frequencies. Due to the duality of the Fourier transform, however, the narrower a filter's bandwidth, the longer its impulse response is, and consequently the greater the temporal smearing observed at points of discontinuity. Conversely, shortening the impulse response reduces the temporal smearing, but results in a larger bandwidth, and hence a reduction in spectral resolution.

For a given filter type, the length of the impulse response and the slope of the pass-band transition, i.e., its ability to reject frequencies outside of the required pass band, are dependent on the order of the filter. In general, the higher a filter's order, the longer is its impulse response and the steeper its pass-band transition. In the case of ERD/ERS, the type and order of filter selected should reflect the relative importance of temporal or spectral resolution for the application in hand. It can be shown, for example, that the Butterworth filter has a relatively long impulse response and narrow bandwidth, and is therefore a good candidate for high spectral resolution studies. Conversely, the elliptic or Cauer filter has a relatively short impulse response and wide bandwidth, so is more appropriate for high temporal resolution.

A possible drawback of this method is that it is not possible to obtain estimates of event-related coherence, since phase information is lost, i.e., the time-frequency spectrum is real valued.

Short-time Fourier transform—The short-time Fourier transform (STFT) is an extension of conventional Fourier analysis for non-stationary data based on the earlier work of Gabor (Gabor, 1946) that performs a fast Fourier transform (FFT) on consecutive segments or blocks

of data that are assumed stationary, and is equivalent to a sliding window that analyses the local frequency content of the signal (Hlawatsch and Boudreaux-Bartels, 1992).

The STFT for signal $x(\tau)$ windowed by a fixed-length function $w(t - \tau)$ is defined by (1), which is complex-valued, i.e., contains amplitude and phase information. STFT power, or to be strictly correct, energy, $P_x(t, f)$, defined in (2) is the squared modulus of the STFT. A time-frequency plot of STFT energy distribution described by (2) is commonly referred to as a spectrogram.

$$STFT_x(t, f) = \int_{-\infty}^{\infty} x(\tau)w(t - \tau)e^{-j2\pi f\tau} d\tau \quad (1)$$

$$P_x(t, f)_{STFT} = |STFT_x(t, f)|^2 \quad (2)$$

In general, shorter length windows result in a greater ability to localise rapidly changing events in the time domain. However, a well-documented failing of the STFT is that it is not possible to resolve temporal and spectral events simultaneously with arbitrarily high accuracy (Hlawatsch and Boudreaux-Bartels, 1992; Rioul and Vetterli, 1991). Due to the relationship between FFT spectral resolution and window length, high spectral resolution can only be achieved with relatively long windows, but this inevitably results in a loss of temporal resolution. The trade-off between temporal and spectral resolution is a consequence of the uncertainty principle, which states that the time-bandwidth product of the window has a lower bound of $(4\pi)^{-1}$ (Rioul and Vetterli, 1991). It can be shown that the Gaussian window has a time-bandwidth product equal to this lower bound, and is therefore the optimum window function for the STFT. Consequently, the use of any other type of window will result in lower joint time-frequency resolution.

Continuous wavelet transform—The continuous wavelet transform (CWT) is a TFR that potentially offers improved temporal and spectral resolution over the STFT by expanding the signal of interest onto a set of variable-length basis functions or wavelets that are scaled and translated versions of a single function, the so-called mother wavelet. By allowing the length of the wavelet to vary, the temporal and spectral resolution of the CWT is no longer fixed, as was the case with the STFT, but is matched to the spectral components of the signal. In general, lower frequencies are better suited to analysis with longer wavelets, whereas high frequencies require shorter wavelets that are better able to capture rapidly changing fluctuations.

The CWT of a signal $x(\tau)$ is defined in (3) where $\psi^*(\cdot)$ is the complex conjugate of the mother wavelet function and b and a are the translation and scale parameters, respectively. It can be seen that, at each time instance τ , the CWT calculates the correlation between $x(\tau)$ and the scaled and shifted wavelet $\psi^*(\cdot)$. The energy density of the CWT or scalogram, $P_x(a, b)$, is given by (4).

$$CWT_x(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau)\psi^*\left(\frac{\tau - b}{a}\right) d\tau \quad (3)$$

$$P_x(a, b)_{CWT} = |CWT_x(a, b)|^2 \quad (4)$$

In practice, selecting the appropriate wavelet type for a given application involves matching the shape of the mother wavelet with the corresponding features of interest in the signal. For example, to detect the oscillatory activity observed in EEG during ERD/ERS, a mother wavelet that has oscillatory features would be appropriate. If coherence is to be determined, then a further requirement is that the wavelet is complex valued. The complex Morlet wavelet, defined in (6), which is equivalent to a complex sinusoid with Gaussian envelope (Zhan et al., 2006), has both these attributes.

$$\psi(t) = \frac{1}{\pi^{1/4}} \exp(j2\pi f_0 t) \exp(-t^2/2) \quad (6)$$

$$\widehat{\Psi}(f) = \pi^{1/4} \sqrt{2 \exp(-1/2(2\pi f - 2\pi f_0)^2)} \quad (7)$$

In wavelet analysis there is no physical relationship between wavelet scale and Fourier frequency. In the case of Morlet or other oscillatory wavelets, however, the central frequency f_0 at each scale can be loosely interpreted as a localised Fourier frequency (Priestley, 1996), i.e., $f = f_0/a$. Thus, a conversion from the time-scale axes of the scalogram to a time-frequency representation is possible. In such cases, the complex-valued CWT of a signal $x(t)$ using wavelet function $\psi(\tau, f_0)$ is defined by (8) (Zhan et al., 2006). Now, the squared modulus of the complex-valued CWT is transformed from a time-scale representation to a time-frequency energy distribution of the signal, as given by (9).

$$CWT_x(t, f) = \sqrt{\frac{f}{f_0}} \int_{-\infty}^{\infty} x(\tau) \psi^* \left(\frac{f(t - \tau)}{f_0} \right) d\tau \quad (8)$$

$$P_x(t, f)_{CWT} = |CWT_x(t, f)|^2 \quad (9)$$

Since the complex Morlet wavelet is characterised by a Gaussian envelope, its Fourier transform is also Gaussian with a standard deviation inversely proportional to wavelet scale, as given by (7). This property of the Morlet wavelet has important implications for spectral resolution, in that the short-scale wavelets required for higher frequency analysis have wider bandwidth, and hence lower spectral resolution, than the longer-scale wavelets for low frequencies. The implication of this is that short-scale wavelets irrespective of their type lead to good temporal resolution but poor spectral resolution at high frequencies, whereas longer scales give good spectral resolution but poor temporal resolution at low frequencies.

It can be seen that the properties of the Morlet wavelet in terms of its resolvability are determined by the width of the Gaussian envelope and the central frequency f_0 . In the original definition of the Morlet wavelet in (6), the width of the Gaussian envelope is fixed. However, this can be made variable by the inclusion of a parameter σ^2 in the denominator of the envelope function, i.e., $(-t^2/2\sigma^2)$.

The value of f_0 determines the number of oscillations or cycles of the Morlet wavelet. In general, higher values of f_0 result in better spectral resolution but lower temporal resolution (De Moortel et al., 2004). In fact, for the Morlet to be admissible as a wavelet, a minimum value of $2\pi f_0 = 6$ is required, i.e., $f_0 \sim 1$ Hz (Farge, 1992). For a given value of f_0 , increasing the variance of the Morlet wavelet leads to increased frequency resolution and decreased temporal resolution, and vice versa if the variance is decreased. Adjusting the parameters of the Morlet wavelet, therefore, leads to time-frequency maps with different properties, and the values chosen should reflect the requirements of the application.

It should be noted that wavelet functions are subject to the same uncertainty principle applicable to STFT windows, i.e., a lower bound of time-bandwidth product equal to $(4\pi)^{-1}$. Therefore, it follows that the Morlet wavelet with its Gaussian envelope is optimal in this respect and that no other wavelet offers better joint time-frequency resolution.

Cohen's class of quadratic TFR—The complex-valued STFT and CWT are examples of linear TFR, since both satisfy the principle of superposition, the cornerstone of linear systems theory. The corresponding spectrogram and scalogram are not classed as linear, however, since both contain squared terms, such that adding the spectrograms of two signals $x(t)$ and $y(t)$, for example, is not the same as the spectrogram obtained from $x(t) + y(t)$ (Hlawatsch and Boudreaux-Bartels, 1992). Because of the squared terms both methods are referred to as quadratic TFRs.

The spectrogram and scalogram belong to Cohen's class of quadratic TFR (Cohen, 1989), which have the property of being time-frequency shift invariant, which means that if a signal $x(t)$ is delayed in time or shifted in frequency, then the corresponding TFR is invariant under these conditions and is delayed or shifted by an equivalent amount (Hlawatsch and Boudreaux-Bartels, 1992).

In general, a quadratic TFR of a multiple frequency signal consists of a component for each frequency component of the signal, referred to as an auto-component, and cross-component or interference terms for all pairs of frequency components. In the case of the spectrogram or scalogram, interference terms occur where the auto-components overlap in frequency, and therefore do not normally cause a problem if the component frequencies are spaced sufficiently far apart. The spectrogram and scalogram are thus described as having good interference rejection properties, but neither has particularly good time-frequency resolution due to the uncertainty principle, as stated above.

Other members of Cohen's class of quadratic TFR have improved time-frequency resolution compared to that of the spectrogram or scalogram. The cost of improved resolution, however, is an increase in the number and magnitude of interference terms. For example, the Wigner distribution $W_x(t, f)$, given by (10), exhibits ideal time-frequency resolution for single linear chirp-like signals, but suffers poorer interference rejection than other members of Cohen's class when multiple signal components are present (Hammond and White, 1996). The resolution of other quadratic TFRs falls between the two extremes of the spectrogram and Wigner distribution.

$$W_x(t, f) = \int_{-\infty}^{\infty} x\left(u + \frac{\tau}{2}\right) x^*\left(u - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau \quad (10)$$

In (10), $x(\cdot)$ is the analytic form of a real-valued signal obtained via the Hilbert transform and $x^*(\cdot)$ its complex conjugate from which the instantaneous magnitude or power and

instantaneous phase can be determined (Bendat and Piersol, 2000). The Wigner distribution described in (10) can be visualised as a folding of $x(t)$ about a point in time u , cross-multiplying the folded parts and Fourier transforming the result. If the folded signals contain overlapping non-zero segments, then the function at time u is non-zero. Otherwise the function will be equal to 0. The interference terms associated with the Wigner distribution occur, for example, when the value of $x(t)$ at time u is zero, i.e., during quiet periods of the signal, but overlapping non-zero activity occurs in the signal before and after point u , which cause the cross-product to be non-zero. Similarly, the presence of noise in one or both of the folded signals may result in artefacts at u even though noise may not be present in the signal at that time (Cohen, 1989). In such cases, it is also possible for the interference term to be of longer duration than the actual signal noise.

In the case of single-component signals, the Wigner time-frequency distribution does not contain interference terms and the TFR has optimum resolution. If a signal consists of two sinusoids at frequencies f_1 and f_2 , however, an interference term occurs in the Wigner time-frequency distribution at a frequency equal to $(f_1 + f_2)/2$. In general, for n sinusoids there exists $n(n - 1)/2$ such interference terms. Furthermore, the amplitude of the interference terms can be up to twice that of genuine frequency components. Consequently, the Wigner distribution is difficult to interpret and can be misleading when two or more movement-related oscillation frequencies are present, as is the case for most movement behaviours. In fact it has been suggested that the Wigner distribution is not well suited to time-frequency analysis of EEG or other physiological signals (van Hoey et al., 1997). Another problem with this particular distribution is the fact that energy estimates may take negative values, which have no physical interpretation.

Since interference terms are oscillatory, however, it is possible to remove, or at least reduce their amplitude, with the use of a suitable smoothing function or kernel $\phi(t, f)$ applied to the Wigner distribution via the double integration given by (11). It can be shown that any TFR that is a member of Cohen's class can be derived from the Wigner distribution by applying the appropriate kernel in this manner (Cohen, 1989). For example, the spectrogram can be obtained by substituting the Wigner distribution of the window function $w(t - \tau)$ of (1) into (11).

$$W_x^s(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(t - t', f - f') W_x(t', f') dt' df' \quad (11)$$

The cost of removing interference terms by the addition of a kernel function, however, is a smoothing of the auto-component terms, which leads to a reduction in time frequency resolution. This accounts for the inferior time-frequency resolution of the spectrogram and other members of Cohen's class compared to the Wigner distribution. The choice of kernel used, therefore, is a compromise between the desirable properties of interference rejection and the detrimental effects of reduced resolution from smoothing. The exponential kernel used by the Choi-Williams distribution (Choi and Williams, 1989), for example, is one such function and belongs to the class of so-called Reduced Interference Distributions (RID), which has been applied to event-related potentials (Williams, 1996).

RID generally, though not necessarily, apply equal smoothing to the time and frequency domains, and have been shown to be the best choice when analysing multicomponent signals that have constant frequencies. Like the Wigner distribution, however, RID can result in negative energy values (Jones and Parks, 1992). Their application to broadband, frequency-varying signals such as EEG, therefore, would appear to be limited.

Experimental Procedure

Spectral resolution test data—In order to test the spectral resolution of each TFR in the α - and β -bands, two sets of simulated data were generated each consisting of 10 signals with two frequency components. Signals in the first set, intended for α -band testing, consisted of a sinusoid with frequency $f_1 = 10$ Hz to which another sinusoid $f_2 \in \{11, 12, \dots, 20\}$ Hz was added. Similarly, for β -band testing, signals in the second set consisted of a sinusoid $f_1 = 20$ Hz with an additional sinusoid $f_2 \in \{21, 22, \dots, 30\}$ Hz. The sampling rate for all signals was 256 Hz, epoch length was 4 s (from 2 s before to 2 s after movement onset), and the amplitude of all frequency components was 4.0 μ V. Both data sets were presented to the time-frequency methods under consideration, and the spectral resolution of each signal determined according to the criteria described below. Gaussian noise with zero mean and unit variance was then added to both data sets and the spectral resolution determined once more. The signal-to-noise ratio was ~ 4.5 dB, which is consistent with a noise variance 3.5 times that of the signal for a single epoch averaged over 100 epochs. An example of a noise-free signal is shown in Fig. 1a.

Temporal resolution test data—To test temporal resolution in the α - and β -bands, two sets of data were generated each containing 10 signals consisting of short bursts of a sinusoidal signal interspersed with periods of inactivity. An example of such a signal is shown Fig. 1b. The frequencies of the signal bursts were 10 and 20 Hz for α - and β -band data sets, respectively. The sampling rate, epoch duration and signal amplitudes were the same as those for spectral resolution test data. The length of the bursts was equal to 0.5 s, and the interval varied from 1 to 10 samples, i.e., 3.9 to 39.1 ms. Temporal resolution was determined by the method described below. In addition to noise-free data, Gaussian noise ($\mu = 0$, $\sigma = 1.0$) was added to both sets of data, signal-to-noise ratio ~ 4.5 dB, and temporal resolution recalculated.

Combined temporal and spectral resolution data—A third data set was generated to enable a visual assessment of each time-frequency method. The data consisted of a single signal with 0.25 s bursts of summed 10 and 20 Hz sinusoids followed by intervals of 0.25 s, as shown in Fig. 1c. The signal was intended to simulate α - and β -band activity during repetitive movements at 2 Hz. The sampling rate and duration for this signal were the same as for the data described above.

Experimental data—In addition to tests of resolution on simulated data, each TFR was applied to experimental data to contrast the effects of these techniques on a physiological data set. EEG were obtained from a series of finger tapping experiments using an acoustic pacing tone at a rate of 2 Hz from a non-impaired adult as part of a study examining the effects of movement frequency on movement performance and movement-related cortical oscillations in people with Parkinson's disease (Stegemöller et al., 2009). Data were acquired with a Neuroscan SynAmps system, band-pass filtered (DC to 250 Hz), sampling rate 1000 Hz (subsequently downsampled to 250 Hz), and digitised with a 16-bit analogue-to-digital converter. Movement onsets were determined from kinematic data and used to epoch the EEG into segments aligned at the center to movement onset. The data consisted of 197 artifact-free epochs from a single surface Laplacian-derived channel covering the contralateral sensorimotor hand region (C3 on the International 10–20 System). All epochs were processed by each of the four TFR methods and an across-trial average for each method calculated.

TFR experimental parameters—Band-pass filtered ERD/ERS of the experimental and simulated data was performed both with Butterworth and elliptic filters. The range of centre frequencies for either type of filter was 6 to 40 Hz in 1 Hz increments, with bandwidths of 1 Hz and 0.5 Hz for the Butterworth and elliptic filters, respectively. First-order elliptic filters were implemented with 0.1 dB ripple and 40 dB attenuation in the stop band, whereas the Butterworth filters were second order.

STFT analysis of simulated and experimental data was performed with two different window lengths, i.e., at two different levels of Fourier resolution. The window lengths were equal to the epoch length divided by 8 or 16, i.e., 128 and 64 samples, which have Fourier transform frequency resolutions of 2 and 4 Hz, respectively. These values suggest the limits of spectral resolution that this method can achieve. Gaussian window functions were used because of their optimum time-bandwidth product. The temporal increment of the window for both resolution levels was 1 sample, i.e., 3.9 ms. Henceforward, processing with the two window lengths are referred to as STFT-128 and STFT-64.

Wavelet-based time-frequency analysis was performed with a complex Morlet wavelet ($f_0 = 1$ Hz, $\sigma^2 = 4$) which has approximately 7 damped oscillations and offers a good compromise between spectral and temporal resolution. A total of 30 scales were used that ranged logarithmically from approximately 4.7 to 50.1. When converted to frequency, this resulted in a range of frequencies from 4.99 to 54.1 Hz. The use of logarithmic scale increments ensures bins that are more uniformly spaced to give better frequency resolution at high frequencies than linear increments.

In the case of RID-based time-frequency analysis, smoothing was achieved with a Hamming kernel that had time and frequency window lengths equal to 64 samples.

Simulation of data and all signal processing were performed with MATLAB® Version 7.5 (The MathWorks Inc. Natick, MA).

Spectral and temporal resolution criteria—A statistical method of comparing the spectral and temporal resolution of time-frequency coherence estimates for the CWT and STFT was presented by Zhan et al. (2006). An alternative measure of resolution is presented here based on the Rayleigh criteria for spectral resolution (Griffiths and de Haset, 2007) that is more suited to comparing changes in power in the simulated signals described above. For spectral resolution tests, frequency components f_1 and f_2 were classed as resolved if power in the band between their spectral peaks was less than half (or -3 dB) that of the lower of the two peaks, as given by (12) in which A , B and C are defined in Fig. 2.

Similarly, for the temporal resolution tests, intervals were deemed to be resolved if the power in the interval between successive bursts fell to half that of the bursts.

$$\text{Resolved}(f_1, f_2) = \begin{cases} \text{True:} & B < \frac{1}{2} \min\{A, C\} \\ \text{False:} & \text{Otherwise} \end{cases} \quad (12)$$

Maximum resolution in the spectral and temporal domains, therefore, was defined as the minimum frequency or interval for which these conditions were true. When testing for resolution only the peaks immediately prior and post movement onset are considered, i.e., those at the centre of the epoch.

Results

Simulated data -Spectral resolution

Table 1 lists the spectral resolution of each time-frequency method when applied to simulated data consisting of two sinusoids at different frequencies. The table shows the minimum difference between two frequency components that can be resolved in the α and β -bands using the method defined by (12) for noise-free and noisy data. It is interesting to note that for all methods, with the exception of the wavelet transform, the resolution does not change across

frequency bands in the noise-free case. Moreover, the frequency resolution also does not change by more than 1 Hz for any method when the signal-to-noise ratio is ~ 4.5 dB.

It can be seen that the best spectral resolution is 3 Hz, which was achieved by band-pass filtering with the Butterworth. In the α -band this is matched by the wavelet transform and is only marginally better than the RID. In contrast, the elliptic filter has a resolution of 9 Hz. The resolution of the STFT depends on window length, that is, the longer the window, the greater the resolving power.

Fig. 3 shows examples of spectra formed by cross-sectioning the time-frequency plane generated by each method at $t = 0$ s for a signal consisting of two frequencies, $f_1 = 20$ Hz and $f_2 = 26$ Hz. The plots show that the Butterworth filter, STFT-128, CWT and RID all resolved the two frequencies, whereas the elliptic filter and the STFT-64 did not. The relative widths and amplitudes of the two peaks in Fig. 3e reflect the reduced spectral resolution with increasing frequency that can occur with the wavelet transform. This is in contrast to the other methods whose peaks are more or less the same amplitude and width. Fig. 3f also reveals the negative power that can occur with RID.

Simulated data -Temporal resolution

Maximum temporal resolution was defined as the minimum resolved interval (in samples) between adjacent bursts of a sinusoidal signal. Table 2 lists the temporal resolution of each time-frequency method for signals having frequencies in the α - and β -bands with and without added noise. It can be seen that, for all methods, the temporal resolution did not decrease by more than 1 sample when noise was added. However, improved resolution was observed for all methods in the β -band compared to the α -band. In the α -band, the Butterworth filter method and the CWT performed slightly worse than the other methods, whereas in the β -band the CWT and the elliptic filter were the best performing methods.

Fig. 4 shows examples of time-varying amplitudes formed by cross-sectioning the time-frequency plane of each method at a frequency of 10 Hz for an interval length of 8 samples (31.3 ms). The plots show that only the Butterworth filter method and CWT are not resolved according to the above criteria. It can also be seen that STFT-64 exhibits the least rounded peaks. The plots in Fig. 4 also show the extent of edge effects of each TFR.

Simulated and experimental data - Qualitative time-frequency analysis

Fig. 5 shows time-frequency maps generated by each method for a signal with repeated bursts of activity interspersed with quiet periods, in which the bursts consisted of two frequency components with the same amplitude. The plots show the spread of energy density in the time and frequency domains. It can be seen that the elliptic filter and STFT-64, shown in Fig. 5a and 5d, display the best temporal resolution. The RID plot in Fig. 5f also shows good temporal resolution, but interference is clearly visible. The Butterworth filter (Fig. 5b) and the wavelet transform (Fig. 5e) do not provide particularly good temporal resolution, especially at low frequencies in the case of the latter. In the frequency domain, however, the Butterworth filter and STFT-128 perform best, whereas the elliptic filter and wavelet transform perform relatively poorly. The plot in Fig. 5e highlights the other shortcoming of the wavelet transform, that is, the decrease in spectral resolution at higher frequencies caused by the spreading of the distribution.

Fig. 6 shows across-trial averaged time-frequency maps of the experimental data for each method. These plots reemphasize that differences in time-frequency processing methods can have a marked effect on the profiles obtained and are thus subject to differences in the physiological interpretation of the findings. Each method detected power in the α - and low β -

band. It can be seen that the RID, elliptic filter and STFT-64 display clearly separated ERD/ERS activity in the time axis. In terms of spectral resolution, however, the elliptic filter and STFT-64 are not capable of discriminating between the α - and β -bands. Conversely, the RID, CWT, Butterworth filter and STFT-128 all exhibit all distinct α - and β -bands activity. In fact, the Butterworth filter and RID appear to show the presence of 2 frequencies within the β -band that are indistinguishable with the other methods. The CWT and STFT-128 both have generally good joint time-frequency resolution, with the latter being slightly the better of the two.

Discussion

The aim of this study was to compare the temporal and spectral resolution of a number of common time-frequency representations when presented with simulated and experimental EEG representative of repetitive movements. It has been shown both theoretically and empirically that the various methods considered have different properties and that the energy density map obtained depends very much on the method used.

ERD/ERS is commonly quantified either by band-pass filtering as described by Pfurtscheller and da Silva (1999) or with wavelets (Alegre et al., 2002; Graimann and Pfurtscheller, 2006). Both of these methods have proven adequate for slow-paced or discrete movements with several seconds rest between movements (Gerloff et al., 1998; Pfurtscheller and da Silva, 1999). If faster rate repetitive movements are to be analysed in the time-frequency plane, however, the issues of temporal and spectral resolution become more important, since the signal is alternating from one state to another at the same rate as the movement, and uncertainty due to spectral and temporal smearing makes analysis of the transition points more difficult.

It is also important to note that differences in the parameters of a particular method can lead to different energy densities and hence a different interpretation of the results. This can be seen most vividly with the choice of filter for the band-pass filter method. The elliptic filter has been shown to give better temporal resolution than the Butterworth filter when applied to simulated data. This is evidenced by the smearing of energy between successive bursts in Fig. 5b, which is not present to the same extent with the elliptic filter. The Butterworth does have superior spectral resolution to the elliptic filter, though, and does not exhibit any of the spectral smearing of the latter.

Changes in parameters also affect the results of STFT analysis, where it has been shown that the relative temporal and spectral resolution in the time-frequency plane is dependent on window length. As expected, a relatively long window results in higher spectral resolution, whereas the opposite effects are observed with the shorter window.

It is commonly held that the wavelet transform offers superior resolution to the STFT. This is only partly true in the sense that it is possible to get improvements to temporal resolution at high frequencies and spectral resolution at low frequencies in the same time-frequency plane. It has been shown here and elsewhere (Bruns, 2004; Zhan et al., 2006), however, that this is not the case when considering joint time-frequency resolution. The plots of Fig. 5c and 5e would suggest that STFT-128 offers better joint time-frequency resolution than the CWT for this type of data. It can be seen that the STFT does not exhibit as much spectral smearing as the CWT at higher frequencies, nor does it exhibit the same temporal smoothing at low frequencies. It should be noted, however, that in other applications where different criteria apply, these conclusions and recommendations may not necessarily apply.

The results also show that RID has little practical use for analysis of EEG, especially when considering fast repetitive movements. The time-frequency map of the simulated data in Fig. 5f shows a considerable number of high-magnitude interference terms in both domains. Although energy can be seen in the α - and low β -bands for the experimental data in Fig. 6f, the

amount of interference present means that it is more difficult to determine the transitions between ERD and ERS than, for example, the STFT64 or elliptic filter.

The magnitudes for each time-frequency energy density were not normalised, and the values shown are those obtained from the various MATLAB[®] algorithms. It can be seen that none of the magnitudes for the simulated data in Fig. 5 accurately reflect the power of the signal components, which have amplitude of 1.0 μ V. However, it should be noted that ERD/ERS is commonly normalised to a baseline or resting condition recorded prior to movement onset, since it is relative changes in oscillations that are of interest (Pfurtscheller and da Silva, 1999). In such circumstances, the actual magnitudes of baseline and movement conditions returned by a particular TFR are irrelevant as movement-related changes in oscillations are expressed as a ratio of the baseline condition. The simulated and experimental signals in the current study were not normalised in this manner as the intention was to show the effects of each TFR on temporal and spectral resolution, which cannot be demonstrated unambiguously when normalisation is applied.

It should also be noted that the edge effects of the TFRs revealed in Figs. 4, 5 and 6 should not cause a problem if the size of the epoch is made sufficiently large. Since the central region of interest is the portion of data that is time-locked to movement onset (± 1 movement), the epoch length can be set longer to accommodate the edge effects which can then be discarded. This then circumvents the issue of the cone of influence, a problem associated with the CWT, which occurs at low frequencies when relatively large scales extend beyond the limits of the data window (Torrence and Compo, 1998).

In summary, selecting a time-frequency representation for analysis of EEG from fast repetitive movements depends on a number of factors including the spectral and temporal resolution, consistency of energy magnitudes across the spectrum, and whether phase information is required or not, i.e., if the TFR needs to be complex or real valued. The results from simulated and experimental data presented in this current work suggest that the CWT is not superior to the STFT in terms of spectral and temporal resolution. It can also be seen that ERD/ERS from band-pass filtering is inferior to the STFT if one considers time and frequency resolution jointly. These facts, coupled with the added advantage that it is complex valued, would suggest that the Gaussian-window STFT incremented at each point along the time axis is the method of choice for analysing time-varying spectral changes during fast repetitive movements.

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Figure 1.

Examples of simulated signals used to determine temporal and spectral resolution. (a) spectral resolution test signal consisting of two sinusoids at frequencies of 20 and 24 Hz. (b) temporal resolution test signal showing 20 Hz bursts separated by intervals of inactivity of 39 ms duration (i.e., 10 samples). (c) test signal simulating repetitive movement at 2 Hz for qualitative analysis of TFR consisting of 0.25 s bursts of 10 and 20 Hz activity followed by 0.25 s of inactivity.

Figure 2. Plot showing criteria for spectral resolution. f_1 and f_2 are resolved if power between the two frequencies B is less than half that of the lower of A or C .

Figure 3.

Examples of spectra for a pair of frequencies at 20 and 26 Hz at time $T = 0$ s (a) band-pass filter method (elliptic filter); (b) band-pass filter method (Butterworth); (c) STFT-128 (0.5 s window); (d) STFT-64 (0.25 s window); (e) CWT; (f) RID.

Figure 4.

Plots showing time course of a signal with 31.3 ms (8 samples) intervals between activity bursts at $f = 10$ Hz: (a) band-pass filter method (elliptic filter); (b) band-pass filter method (Butterworth); (c) STFT-128 (0.5 s window); (d) STFT-64 (0.25 s window); (e) CWT; (f) RID.

Figure 5.

Plots showing time-frequency energy distribution of each method for signal with frequency components at 10 and 20 Hz comprising 0.25 s repeated bursts of activity followed by a similar period of inactivity. (a) band-pass filter method (elliptic filter); (b) band-pass filter method (Butterworth); (c) STFT-128 (0.5 s window); (d) STFT-64 (0.25 s window); (e) CWT (Solid black line = Cone of Influence); (f) RID.

Figure 6.

Plots showing time-frequency energy distribution of each method for experimental EEG recorded over the primary motor cortex during repetitive movements of the index finger at a frequency of 2 Hz. (a) band-pass filter method (elliptic filter); (b) band-pass filter method (Butterworth); (c) STFT-128 (0.5 s window); (d) STFT-64 (0.25 s window); (e) CWT (Solid black line = Cone of Influence); (f) RID.

Table 1

Frequency resolution (in Hz) in the α -and β -bands of each TFR with signal-to-noise ratio of ∞ dB and 4.5 dB.

	10 Hz		20Hz	
	∞ dB	4.5 dB	∞ dB	4.5 dB
BP Filter - Elliptic	8	8	8	9
BP Filter - Butterworth	3	3	3	3
STFT - 128	5	5	5	5
STFT - 64	10	10	10	10
Morlet Wavelet	3	3	6	6
RID	4	4	4	4

Table 2

Temporal resolution (in samples) in the α - and β -bands of each TFR with and without added noise, i.e., signal-to-noise ratio of 4.5 dB and ∞ dB, respectively. (Sample interval = 3.9 ms).

	10 Hz		20Hz	
	∞ dB	4.5 dB	∞ dB	4.5 dB
BP Filter - Elliptic	6	6	3	4
BP Filter - Butterworth	9	9	5	5
STFT - 128	7	8	4	5
STFT - 64	6	7	4	5
Morlet Wavelet	10	10	3	3
RID	6	6	3	4