Boundary-enhanced region-of-interest image reconstruction in propagation-based x-ray phase-contrast tomography

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Propagation-based phase-contrast tomography is an imaging method that can reconstruct the distribution of the three-dimensional complex-valued refractive index distribution of an object. In many applications, a boundary-enhanced image is sought that reveals the locations of boundaries in the refractive index distribution. In this letter, we demonstrate that within certain regions-of-interest, the three-dimensional Laplacian of the real-valued component of the refractive index distribution can be reconstructed from knowledge of truncated phase-contrast projection data. The proposed reconstruction algorithm is demonstrated by use of experimental measurement data. © 2009 American Institute of Physics. [doi:10.1063/1.3254235]

Propagation-based, or in-line, x-ray phase-contrast imaging methods^{1–4} exploit a contrast mechanism based on differences in an object's x-ray refractive index distribution, and therefore permits visualization of object features that possess very similar x-ray absorption properties. Phase-contrast imaging methods can also operate effectively at relatively high x-ray energies, thereby facilitating low-dose imaging.⁵ Moreover, phase-contrast tomography (PCT) methods^{6–11} are being actively developed for reconstructing information regarding the three-dimensional (3D) complex-valued refractive index distribution of an object, which can facilitate a wide range of imaging applications.

In certain applications of PCT, only information regarding the locations of interfaces in the refractive index distribution, rather than an accurate estimate of the refractive index distribution itself, is sought after and sufficient to accomplish the diagnostic task at hand. We refer to this variant of PCT as *boundary-enhanced PCT*,^{7,13–15} which is conceptually similar to Lambda tomography methods that have been developed for conventional CT.¹⁶ Because the need for phase-retrieval is avoided, image reconstruction algorithms for boundary-enhanced PCT^{12,17} do not possess Fourier space singularities and can be relatively robust to experimental errors and even the effects of wavefield polychromaticity.¹⁵ Data truncation is another important experimental factor that is not accommodated by most conventional tomographic image reconstruction algorithms. In applications of boundary-enhanced PCT, the object will often not fit entirely in the field-of-view of the imaging system and/or the probing x-ray beam is intentionally collimated to minimize radiation exposure to the object, resulting in a set of truncated projection measurements. It has been demonstrated¹² that blurred estimates of 3D Laplacian of the refractive index distribution within a region of interest (ROI) can be reconstructed from truncated projection data. However, the image blurring is undesirable in applications where high spatial resolution is required.

There remains an important need, which we address in this letter, to develop accurate ROI image reconstruction algorithms for boundary-enhanced PCT involving truncated measurement data. In recent years, image reconstruction formulas have been developed for conventional CT that permit the exact reconstruction of an image within an ROI from truncated absorption-contrast projection data.^{18–20} In this work, we adapt and apply one of these algorithms to permit ROI image reconstruction in boundary-enhanced PCT. We demonstrate that, for a weakly absorbing object, the 3D Laplacian of the refractive index distribution can be reconstructed (in principle, exactly) within certain ROIs from knowledge of truncated PCT projection data.

The tomographic scanning geometry of PCT is displayed in Fig. 1, where the z -axis of the fixed coordinate system (x, y, z) defines the axis of rotation of the tomographic scanning. The rotated coordinate system (x_r, y_r, z) is related to the fixed system by $x_r = x \cos \phi + y \sin \phi$, $y_r = y \cos \phi - x \sin \phi$, where the tomographic view angle ϕ is measured from the positive x -axis and y_r denotes the optical axis. The unit vectors along the x and x_r axes will be noted by $\mathbf{s}_0$ and $\mathbf{s}_r(\phi) = (\cos \phi, \sin \phi)$, respectively.

For simplicity, we assume a plane-wave $U_i(x_r, z, \phi)$ with wavelength λ , or equivalently wavenumber $k = 2\pi/\lambda$, propagates in the direction of the positive y_r -axis and irradiates a weakly absorbing object. The results that follow can readily be generalized to the case of an incident spherical wavefield in the paraxial limit and a weakly absorbing object. The ob-

FIG. 1. The tomographic scanning geometry of PCT.

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FIG. 2. (Color online) MIP images of the 3D cylindrical ROI volume corresponding to different viewing angles. The 3D image data were reconstructed from the (a) and (c) truncated data functions and (b) and (d) untruncated data functions. In (b) and (d), the MIP images were produced by extracting the ROI volume from the complete reconstructed image.

ject is characterized by the object function $a(x, y, z) = n(x, y, z) - 1$, where $n(x, y, z)$ is the refractive index distribution. These functions are assumed to be twice differentiable. The intensity of the transmitted wavefield is measured in the (x_r, z) plane located at $y_r = d$ and will be denoted by $I(x_r, z, \phi)$. A tomographic data set is obtained by simultaneously rotating the incident wavefield direction and detector plane (or equivalently, the object) about the z -axis and measuring a collection of such intensity measurements for $\phi \in [0, \pi)$.

Because the 3D reconstruction problem can be solved on a 2D slice-by-slice basis, it will be convenient to employ the notation $a(r_2; z) \equiv a(x, y, z)$, where $r_2 = (x, y)$. Note that for a fixed value of z , $a(r_2; z)$, describes a 2D transverse slice through the 3D object function.

For sufficiently small object-to-detector distances d and weakly absorbing samples, $I(x_r, z, \phi)$ can be expressed as^{1,9}

$$I(x_r, z, \phi) = I_0(x_r, z, \phi) \left[1 - \frac{\lambda d}{2\pi} \nabla_{x_r, z}^2 \Phi(x_r, z, \phi) \right], \quad (1)$$

where $I_0(x_r, z, \phi)$ is the intensity of the transmitted beam immediately behind the object, $\nabla_{x_r, z}^2 \equiv \partial^2 / \partial x_r^2 + \partial^2 / \partial z^2$, and

$$\begin{aligned} \Phi(x_r, z, \phi) &= k \int \int_{\mathbb{R}^2} dr_2 a(r_2; z) \delta[x_r - r_2 \cdot s_r(\phi)] \\ &\equiv k R_z a(r_2; z). \end{aligned} \quad (2)$$

In Eq. (2), $\delta(\cdot)$ is the one-dimensional Dirac delta function and the integral corresponds to a 2D Radon transform. We will let R_z denote the 2D radon transform operator that acts on a slice of constant z of $a(r_2; z)$.

In boundary-enhanced PCT, we seek to determine an accurate estimate of $\nabla^2 a(r_2; z)$ within certain ROIs from knowledge of truncated measurements $I(x_r, z, \phi)$, where ∇^2 is the 3D Laplacian operator. The image $\nabla^2 a(r_2; z)$ will contain detailed information regarding the locations of boundaries in the 3D object function. This image can also yield valuable information regarding the magnitude of jump discontinuities in $a(r_2; z)$.¹⁷ Consider the data function

$$g(x_r, z, \phi) \equiv \frac{1}{d} \left[\frac{I(x_r, z, \phi)}{I_0(x_r, z, \phi)} - 1 \right], \quad (3)$$

which is determined from the measured data. It can be verified¹³ that the data function satisfies

$$g(x_r, z, \phi) = R_z \nabla^2 a(r_2; z), \quad (4)$$

where ∇^2 is the 3D Laplacian operator and the right-hand side of this equation corresponds to a stack of 2D Radon transforms of $\nabla^2 a(r_2; z)$ along the z -axis. Below we describe how Eq. (4) can be inverted to determine $\nabla^2 a(r_2; z)$ within certain ROIs from knowledge of truncated measurement data.

In recent years, a class of image reconstruction algorithms^{18,19} for conventional CT have been developed that determine the image values on a collection of chords. A chord is a line that connects any two points in image space. Below we demonstrate that the backprojection filtration (BPF) algorithm¹⁹ can be employed to reconstruct a boundary-enhanced image on a collection of chords that fill an ROI, from knowledge of truncated measurement data.

Consider the backprojection of the differentiated data function

$$b_\theta(r_2; z) = \int_{\theta}^{\theta+\pi} d\phi \frac{\partial}{\partial x_r} g(r_2 \cdot s_r(\theta), z, \phi). \quad (5)$$

The subscript θ on the image $b_\theta(r)$ indicates that the backprojection employs differential projection data corresponding to the angular range $\phi \in [\theta, \theta + \pi)$. From knowledge of $b_\theta(r_2; z)$, we seek to determine the boundary-enhanced image on a collection of lines of constant x_r that we refer to as a “chords.” Note that one can select chords with different orientation θ . For example, within a plane of constant z , the chords with $\theta = 0$ are parallel to the y -axis.

The 3D Laplacian of the object function over the interval $[L_{x_r}, U_{x_r}]$ of a chord can be determined as^{19,21}

$$\begin{aligned} \nabla^2 a(x_r s_r(\theta) + y_r s_r^\perp(\theta); z) \\ = -\frac{1}{4\pi^2} \sqrt{\frac{y_r - U_{x_r}}{y_r - L_{x_r}}} \int_{L_{x_r}}^{U_{x_r}} \frac{dy'_r}{y'_r - y_r} \sqrt{\frac{y'_r - L_{x_r}}{y'_r - U_{x_r}}} \\ \times b_\theta(x_r s_r(\theta) + y'_r s_r^\perp(\theta); z). \end{aligned} \quad (6)$$

The limits of integration L_{x_r} and U_{x_r} depend implicitly on the value of x_r (i.e., the particular chord) and satisfy $L_{x_r} \in (-\infty, L_{x_r}^s]$ and $U_{x_r} \in [U_{x_r}^s, \infty)$, where $L_{x_r}^s$ and $U_{x_r}^s$ ($L_{x_r}^s \leq U_{x_r}^s$) denote the end points of the intersection of the chord with the support of the object. This is referred to as the support segment of the chord. Equation (6) represents a BPF reconstruction formula for determination of the image $\nabla^2 a(r_2; z)$ on a chord. This formula requires knowledge of the backprojection image over the interval $y_r \in [L_{x_r}, U_{x_r}]$ on the chord. Consequently, the data function $g(x_r, z, \phi)$ must be known at detector locations corresponding to the projections of the support segment onto the detector line at all view angles over a π -angular interval, but this does not necessarily require the data function to be untruncated. By use of this reconstruction formula, an ROI that is covered by a collection of chords can be determined from knowledge of $g(x_r, z, \phi)$ containing certain types of truncation.

A data-sufficiency condition for exact determination of $\nabla^2 a(r_2; z)$ within an ROI can be readily identified. A specified ROI can always be decomposed into a set of parallel line segments, each of which can be interpreted as an interval on a chord. We assume that these intervals on the chords contain the support segments on the corresponding chords. The data-sufficiency condition for exact ROI image reconstruction is that the data function $g(x_r, z, \phi)$ is known at detector locations corresponding to the projections of the ROI onto the detector plane at all views over a π -angular interval. Because the ROI is assumed to contain all of the support segments, the data-sufficiency condition for reconstruction on each chord is automatically satisfied. This ROI data-sufficiency condition can be satisfied as long as the ROI is illuminated over a π -angular interval despite the fact that the entire object may not be illuminated at each view, resulting in truncated data. This feature of the BPF algorithm may greatly benefit many applications of boundary-enhanced PCT by reducing the data-acquisition time and minimizing the exposure of the sample to damaging radiation. Note that the data-sufficiency condition for exact ROI image reconstruction in boundary-enhanced PCT is identical in form to that identified for ROI image reconstruction in parallel-beam absorption-based CT.^{18,19} This is because, although the data and object functions differ, the underlying imaging model in both cases is a 2D Radon transform.

We numerically implemented Eq. (6) and employed it to reconstruct ROI images from experimental data. When computing the differentiated data function in Eq. (5), a simple two-point approximation to the derivative $\partial/\partial x_r$ was employed. In future studies, alternative implementations of the discrete derivative operation and their effects on the noise properties of the reconstructed images can be investigated. Experiments were conducted at the 2-BM microtomography beamline at the Advanced Photon Source (Argonne National Laboratory). A detailed description of this imaging system has been published elsewhere.²² A piece of foam was imaged with $\lambda = 1.0 \times 10^{-10}$ m and untruncated PCT projection data were acquired at 1440 view angles. The detector contained 2048×2048 pixels with an effective pixel dimension of $6.5 \mu\text{m}$. A truncated data set was created by discarding the values of $g(x_r, z, \phi)$ for $|x_r| > 7540$ microns. Two-dimensional images corresponding to slices of constant z were reconstructed by use of Eq. (6) on a 512×512 grid with pixel dimension of $26 \mu\text{m}$.

Images reconstructed from the untruncated and truncated data were found to be virtually identical within the ROI, which was confirmed by examining the reconstructed image values. Full 3D estimates of $\nabla^2 a(r_2; z)$ were formed by stacking 1500 reconstructed 2D slices along the z -axis, reconstructed from the untruncated and truncated data. Figures 2(a) and 2(c) show maximum intensity projection (MIP) images, corresponding to different viewing angles through the 3D object, for the case of the ROI images reconstructed from the truncated data function. To verify that these images accurately depict the object boundaries within the ROI, the corresponding cylindrical ROI volume was extracted from the 3D image data reconstructed from the untruncated data, and MIP images of the extracted volume were produced. The MIP images of the extracted ROI volume are shown in Figs. 2(b) and 2(d), and appear nearly identical to the corresponding images produced from the truncated data in Figs. 2(a)

and 2(c). Moreover, within the entire cylindrical ROI volume, the image data produced by use of the truncated and untruncated data were compared and found to be virtually identical as expected.

In summary, we have investigated a reconstruction algorithm for 3D ROI image reconstruction in boundary-enhanced PCT. Our algorithm is based on the BPF algorithm that was originally developed for conventional absorption-based CT. The key feature of this reconstruction algorithm is that it permits determination of $\nabla^2 a(r_2; z)$ within certain types of ROIs from knowledge of truncated PCT projection data. The proposed reconstruction algorithm may benefit numerous biomedical imaging and nondestructive testing applications due to its ability to reduce the data-acquisition time and minimize the exposure of the sample to damaging radiation.

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